Introduction
This activity will introduce basic concepts of problem formulation.

Before you start, complete the form below to assign a role to each member.
If you have 4 people, combine Speaker & Reflector. If you have 3, combine Manager & Technician.

<table>
<thead>
<tr>
<th>Group Number</th>
<th>Date</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Team Roles</th>
<th>Team Member</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recorder: records all answers &amp; questions, and provides copies to team &amp; facilitator.</td>
<td></td>
</tr>
<tr>
<td>Speaker: talks to facilitator and other teams.</td>
<td></td>
</tr>
<tr>
<td>Technician: performs calculations &amp; completes worksheets.</td>
<td></td>
</tr>
<tr>
<td>Facilitator: keeps track of time and makes sure everyone contributes appropriately.</td>
<td></td>
</tr>
<tr>
<td>Reflector: considers how the team could work and learn more effectively.</td>
<td></td>
</tr>
</tbody>
</table>

Reminders:
1. **Note the time whenever your team starts a new section or question.**
2. **Write legibly & neatly so that everyone can read & understand your responses.**

<table>
<thead>
<tr>
<th>Question number</th>
<th>Target completion time</th>
<th>Actual completion time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
I. Usable and Feasible Directions

A critical aspect of optimization is how the design space is searched. The search needs to efficiently improve the design while ensuring that all requirements are met. Optimization searches through the design space, using information derived from the objective function and constraints to select search directions. Recall from calculus that a gradient is found by taking the partial derivative of a function with respect to each of the variables in the function.

Model 1

Answer the following questions using the problem and plot below.

\[ F(\vec{X}) = X_1^2 + X_2^2 \]
\[ g(\vec{X}) = (X_2 - 1)^2 - X_1 + 1 \leq 0 \]
1. (2 min) Does the point (2,1) satisfy the constraint (is it feasible)?

\[ g(X) = (X_2 - 1)^2 - X_1 + 1 \leq 0 \]

\[ g(X) = ((1) - 1)^2 - (2) + 1 = -1 \leq 0 \]

The point is feasible.

2. (2 min) Does the point (1,1) satisfy the constraint (is it feasible)?

\[ g(X) = (X_2 - 1)^2 - X_1 + 1 \leq 0 \]

\[ g(X) = ((1) - 1)^2 - (1) + 1 = 0 \leq 0 \]

The point is feasible.

3. (3 min) What is the gradient of the objective function (F) at (1,1)? Draw the gradient on the plot.

\[ F(X) = X_1^2 + X_2^2 \]

\[ \nabla F(X) = \begin{bmatrix} 2X_1 \\ 2X_2 \end{bmatrix} = \begin{bmatrix} 2(1) \\ 2(1) \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \]

4. (3 min) If this is a minimization problem, does the gradient of the objective function (F) at (1,1) point in the direction of improving designs (a usable direction)?

No.

5. (3 min) What is the gradient of the constraint (g) at (1,1)? Draw the gradient on the plot.

\[ g(X) = (X_2 - 1)^2 - X_1 + 1 \leq 0 \]

\[ \nabla g(X) = \begin{bmatrix} -1 \\ 2(X_2 - 1) \end{bmatrix} = \begin{bmatrix} -1 \\ 2(1-1) \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \]

6. (3 min) Does the gradient of the constraint (g) at (1,1) point in the direction of design that satisfy constraints (a feasible direction)?

No.
7. (2 min) Express the gradient of the objective function in terms of the design variables.

\[ F(\vec{X}) = X_1^2 + X_2^2 \]

\[ \nabla F(\vec{X}) = \begin{bmatrix} 2X_1 \\ 2X_2 \end{bmatrix} \]

8. (2 min) Express the gradient of the constraint in terms of the design variables.

\[ g(\vec{X}) = (X_2 - 1)^2 - X_1 + 1 \leq 0 \]

\[ \nabla g(\vec{X}) = \begin{bmatrix} -1 \\ 2(X_2 - 1) \end{bmatrix} \]

9. (5 min) The gradients of \( F \) and \( g \) are vectors. When an optimization method moves through a design space, it uses search directions that are vectors developed from local information and previous search information. At the point \((1.25, 1.5)\):

a) Does the search direction \( \{1,0\}^T \) improve the objective function (is it usable)?

No

b) Does the search direction \( \{1,0\}^T \) keep the constraints satisfied (is it feasible)?

Yes

c) For this problem at the point \((1.25, 1.5)\), describe (in words) what requirements a search direction would need to meet to be considered a usable direction?

The search direction would need to be opposite (more than 90 degrees and less than 270 degrees) of the gradient of the objective function.

d) For this problem at the point \((1.25, 1.5)\), describe (in words) what requirements a search direction would need to meet to be considered a feasible direction?

The search direction would need to be opposite (more than 90 degrees and less than 270 degrees) of the gradient of the constraint.

10. (10 min) Use calculus to express a general form of 6(c) and 6(d).

\[ \nabla F(\vec{X}) \cdot \vec{S} \leq 0 \text{ for usable directions} \]

\[ g(\vec{X}) \leq 0, \quad \nabla g(\vec{X}) \cdot \vec{S} \leq 0 \text{ for feasible directions} \]
Reflectors Report
1. Give one strength of the group and why is that an important characteristic of an effective group?

2. Give one area of improvement for the group and how that improvement can be made?