Quantification of Disaster Resilience of Health Care Facilities

by

Gian Paolo Cimellaro, Cristina Fumo, Andrei M. Reinhorn and Michel Bruneau

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Quantification of Disaster Resilience of Health Care Facilities

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Preface

The Multidisciplinary Center for Earthquake Engineering Research (MCEER) is a national center of excellence in advanced technology applications that is dedicated to the reduction of earthquake losses nationwide. Headquartered at the University at Buffalo, State University of New York, the Center was originally established by the National Science Foundation in 1986, as the National Center for Earthquake Engineering Research (NCEER).

Comprising a consortium of researchers from numerous disciplines and institutions throughout the United States, the Center’s mission is to reduce earthquake losses through research and the application of advanced technologies that improve engineering, pre-earthquake planning and post-earthquake recovery strategies. Toward this end, the Center coordinates a nationwide program of multidisciplinary team research, education and outreach activities.

MCEER’s research is conducted under the sponsorship of two major federal agencies: the National Science Foundation (NSF) and the Federal Highway Administration (FHWA), and the State of New York. Significant support is derived from the Federal Emergency Management Agency (FEMA), other state governments, academic institutions, foreign governments and private industry.

MCEER’s NSF-sponsored research objectives are twofold: to increase resilience by developing seismic evaluation and rehabilitation strategies for the post-disaster facilities and systems (hospitals, electrical and water lifelines, and bridges and highways) that society expects to be operational following an earthquake; and to further enhance resilience by developing improved emergency management capabilities to ensure an effective response and recovery following the earthquake (see the figure below).
A cross-program activity focuses on the establishment of an effective experimental and analytical network to facilitate the exchange of information between researchers located in various institutions across the country. These are complemented by, and integrated with, other MCEER activities in education, outreach, technology transfer, and industry partnerships.

This report presents concepts of disaster resilience of constructed infrastructure and proposes a methodology for its quantitative evaluation. A unified terminology framework is proposed and implemented for resilience evaluation of health care facilities subjected to earthquakes. The evaluation of disaster resilience is based on non-dimensional analytical functions describing variations of functionality that consider direct and indirect losses and the recovery path. The recovery path is estimated by using either simplified recovery functions or complex organizational and socio-political models. Due to the uncertain nature of structural behavior and functional limit states, hospital losses are described in terms of fragility functions. The framework for resilience quantification is formulated and exemplified for an existing medical facility and a hospital network. In addition, an organizational model describing the functionality of the emergency service of a hospital is developed and implemented. A hybrid simulation and analytical metamodel is developed to estimate, in real time, the hospital functional capacity and its dynamic response, accounting for the influence of structural and nonstructural physical damage on the hospital organization. The proposed metamodel covers a range of hospital configurations, taking into account hospital resources, operational efficiency and possible existence of an emergency plan, maximum capacity, and behavior in saturated and over-capacity conditions. The sensitivity of the metamodel to variations of these parameters is also investigated. Finally, a hospital network is modeled to study the effects on disaster resilience of collaborative operations of health care facilities. The damage to the network, the patients’ transportation time, and the distance among facilities are also considered in the model. The proposed resilience framework captures the effects of disasters, and the effects of preparedness and restoration, and therefore, constitutes a valuable tool for decision makers, designers and engineering practitioners.
Resilience, according to most dictionaries, is defined as the ability of systems to rebound after severe disturbances, or disasters. The definition applies to physical, spiritual, biological, engineering, social and political systems. In earlier work by the authors, resilience was defined including technical, organizational, economical and social aspects (Bruneau et al, 2004). In this report, concepts of disaster resilience and its quantitative evaluation are presented and a unified terminology for a common reference framework is proposed and implemented for evaluation of health care facilities subjected to earthquakes. The evaluation of disaster resilience is based on non-dimensional analytical functions related to the variations of functionality during a “period of interest”, including the losses in the disaster and the recovery path. This evolution in time including recovery differentiates the resilience approach from the other approaches addressing the loss estimation and their momentary effects.

The path to recovery usually depends on available resources and it may take different shapes, which can be estimated by simplified recovery functions or using more complex organizational and socio-political models. A loss estimation model including both direct and indirect losses that are uncertain in themselves due to the uncertain nature of the disaster (in the case of this report: earthquakes). Moreover the structural behavior as well as the functionality limit states are also uncertain. Therefore, losses are described as functions of fragility of systems and systems’ components. Fragility functions can be determined through use of multidimensional performance limit thresholds, which allow considering simultaneously different mechanical-physical variables such as forces, velocities, displacements and accelerations along with other functional limits, as well as considering different organizational and social thresholds and variables. A proposed framework for quantification of resilience is presented herein including the associated uncertainties.

The proposed framework is formulated and exemplified first for a typical California hospital building using a simplified recovery model, considering direct and indirect losses in its physical system and in the population served by the system. A hospital network is also analyzed to exemplify the resilience framework.
In the second part of the report, an organizational model describing the functionality of hospital’s emergency department is developed and implemented. A hybrid simulation/analytical model (called “metamodel”) was developed in order to estimate the hospital’s functional capacity and its dynamic response in real time, and incorporate the influence of the facility physical damage of structural and non-structural components on the organizational ones. The waiting time is defined as the main parameter of response and it is used to evaluate disaster resilience of health care facilities. The metamodel has been designated to cover a large range of hospital configurations and takes into account hospital resources, in terms of staff and infrastructures, operational efficiency and possible existence of an emergency plan, maximum capacity and behavior in both saturated and over-capacity conditions. The sensitivity of the model to different patient arrival rates, hospital configurations, and capacities and the technical and organizational policies applied during and before the strike of the disaster are investigated.

A network of multiple hospitals is also modeled to study the effects on disaster resilience of collaborative operations of health care facilities during a disaster. The damage to the network is also taken into account in the model as well as the transportation time of the patients and the relative distances separating the facilities. Uncertainties associated to the nature of the disaster (e.g. earthquakes, hurricane etc.), to the influence of the physical-structural damage on the organizational model and on the functionality limits are taken in account in the model. Numerical examples are presented for the typical Californian hospital and for a network of hospitals.

While the resilience function captures the effects of the disaster, it also captures the results of response and recovery, effects of restoration and of preparedness. Such function becomes an important tool in the decision process for both the policy makers and the engineering professionals.
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SECTION 1

INTRODUCTION

1.1 Research Motivation

Recent events have shown how systems (regions, communities, structures etc.) are vulnerable to natural disasters of every type like human errors, systems failures, pandemic diseases and malevolent acts, including those involving cyber systems and weapon of mass destruction (chemical, biological, radiological ). Hurricane Katrina (Mosqueda and Porter, 2007) clearly demonstrated the necessity to improve the local disaster management plans of different federal, state and private institutions. In order to reduce the losses in these systems the emphasis has shifted to mitigations and preventive actions to be taken before the extreme event happens. Mitigation actions can reduce the vulnerability of a system; however, also if there is insufficient mitigation, or the event exceeds expectations, recovery is necessary to have a resilient function to the community. Therefore, there is also need for cost-effective mitigation of potential and actual damage from disruptions, particularly those causing cascading effects capable of incapacitating a system or an entire region and of impeding rapid response and recovery.

To evaluate the capacity of a community to cope with and manage a catastrophic event, it is necessary to provide a measure of its capacity to respond.

The only way to decide which are the most effective ways to protect and prepare a community is by evaluating its ability to deal with and recovery from a disaster.

The main goal is to define an operationalized preparedness, in terms of a format dictated by scenario planning: a self – reliant, sustainable, resilient community needs to know what to do, how to handle with and what happens if a certain response is applied.

The communities are organized social units with flexibility to adapt, to change and accommodate their physical and social environment. Their disaster behavior depends on their dynamic social structure, their technical and economical resources and the capacity to restore order and normality.

“Organizing” is the most efficient and effective mean to survive: it is necessary to find out which are the weak elements of response chain and to invest on them.
1.2 Research Objectives

There is no explicit set of procedures in the existing literature that suggests how to quantify resilience of critical infrastructures in the context of various hazards. Considerable research has been accomplished to assess direct and indirect losses attributable to various hazards, and to estimate the reduction of these losses as a result of specific actions, policies, or scenarios. However, the notion of resilience suggests a much broader framework than the reduction of monetary losses alone. There is need to move beyond qualitative conceptualizations of disaster resistance and resilience to more quantitative measures, both to better understand factors contributing to resilience and to assess more systematically the potential contributions and benefits of various research activities. It is therefore necessary to clearly define resilience, identify its dimensions, and find ways of measuring and quantifying those dimensions.

This report will outline a conceptual framework and a set of measures that make it possible to empirically determine the extent to which different units of analysis and systems are resilient, discuss ways of quantifying system performance criteria, and illustrate how resilience can be improved through system assessment and modification in both pre-event and post-event contexts.

1.3 Report Organization

The report is organized in two main sections. In the first section, a general framework for analytical quantification of disaster resilience is presented, with emphasis on technical aspects of resilience. In the second section, a metamodel is presented to describe the organizational aspects of resilience, showing as case studies different types of health care facility network.
SECTION 2

FRAMEWORK FOR ANALYTICAL QUANTIFICATION OF DISASTER RESILIENCE

2.1 Introduction

Over the past years the natural and man made disasters with which the human society had to cope with had stressed the necessity to be prepared and to be able to recover in a short time from a sudden and unexpected change in the community technical, organizational, social and economical condition.

The concepts of ‘risk reduction’, ‘vulnerability’, ‘recovery’ and ‘resilience’ have become key words when dealing with hazardous events, but there is need to go beyond the intuitive definition and provide a quantitative evaluation of them.

When a disaster strikes, the community affected requires immediate help to survive, resources, and efforts to recover in a short time. In other words, the community needs to be “prepared” and less “vulnerable”, in order to achieve a high ‘resilience’.

2.2 Literature review

Resilience, according to the dictionary, means “the ability to recover from (or to resist being affected by) some shock, insult or disturbance”. The concept of resilience does not have a unique definition, because of its broad utilization in the field of ecology, social science, economy, and engineering with different meanings and implications. Various attempts have been made to provide a comprehensive definition, but recent literature review collected by Manyena (2006) point out that currently it is too vague a concept to be useful in informing the disaster risk reduction agenda. In his research, Manyena reviews the concept of resilience in terms of definitional issues, its relationship with the concept of vulnerability, its application in the field of disaster management and risk deduction.

As Klein et al. stated (2003), the root of the term has to be found in the Latin word ‘resilio’ that literary means ‘to jump back’. The field, in which it was originally used, first, is still contested, however, it has been claimed that the study of resilience evolved from the disciplines
of psychology and psychiatry in the 1940s, and it is mainly accredited to Norman Garmezy, Emmy Werner and Ruth Smith.

In physics and engineering the term resilience enters to describe the property of a material to absorb energy when it is deformed elastically and then, unloading, to have the energy recovered.

The outcomes of the 2005 World Conference on Disaster Reduction (WCDR) confirmed the importance of the entrance of the term resilience into disaster discourse and gave birth to a new culture of disaster response.

Among the experts in disasters, however, the definitions of resilience are diverse and sometimes contrasting. Resilience can be considered as a desired outcome or, in a broader way, as a process leading to a desired outcome. Reducing resilience to an outcome does not take into account the performance of the process itself, and the effort to reach a certain result.

On the other hand, viewing disaster resilience as a deliberate process (leading to desired outcomes) that comprises a series of events, actions, or changes to augment the capacity of the affected community, places emphasis on the human role in disasters.

Disaster resilience is seen as a quality, characteristic, or result that is generated or developed by the processes that foster or promote it.

Manyena (2006), evaluating all the possible definitions provided from the 90’ to nowadays, suggests that Resilience could be viewed as the “intrinsic capacity of a system, community or society predisposed to a shock or stress to adapt and survive by changing its non essential attributes and rebuilding itself”.

As regards its relationship with the concept of vulnerability, it can be accepted that the latter is closely associated to the level of resilience, but it is a complementary aspect of the community preparedness. Table 2-1 compares the elements of vulnerability and disaster resilience.

Both these groups of vulnerability and resilience aspects have consequences in planning the risk reduction and developing a practice approach.

Emphasizing the concept of resilience means to focus on the quality of life of the people at risk and to develop opportunities to enhance a better outcome. In contrast, the vulnerability approach places stress on the production of nature (Smith and O’Keefe, 1996) to resist the natural hazard. Engineers, guided by legislation, play a guiding role in the quantification of vulnerability.
Table 2-1 Difference between Vulnerability and Resilience (Manyena 2006)

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<td>10</td>
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Moreover, the concept of vulnerability has to be related with the definition of fragility.

In order to understand better the relationship between these two concepts, it is convenient to focus on the field of seismic engineering and provide two different methods of evaluation of vulnerability and fragility. Given a certain control parameter (for example the shaking intensity), vulnerability (and in particular a vulnerability function) defines the loss while fragility (more precisely a fragility function) gives the probability of some undesirable event (e.g. collapse). Thus fragility function may assess the probability that a building will collapse, as well as that a factory may release hazardous materials into the atmosphere, given a certain seismic intensity. On the other side, vulnerability functions would provide as a function of the same control parameter the damage factor for the building (e.g. valued as repair cost divided by replacement cost) or the quantity of hazardous materials released.

The necessity of complementary “map of resilience and vulnerability” has been highlighted in order to create and increase the aware role of the entire society in the restoration process. Furthermore, defining and mapping resilience has become an important tool in the decision process either for the engineering profession or for the policy makers.

In the last years, as the idea of the necessity of building disaster – resilient communities gains acceptance, new methods have been proposed to quantify resilience beyond estimating losses. Because of the vastness of the definition, resilience necessary has to take into account its entire complex and multiple dimensions, which includes technical, organizational, social, and
economic facets. Bruneau et al. (2003, 2007) offered a very broad definition of resilience to cover all actions that reduce losses from hazard, including effects of mitigation and rapid recovery. They defined the earthquake resilience of the community as “the ability of social units (e.g. organizations, communities) to mitigate hazards, contain the effects of disasters when they occur, and carry out recovery activities in ways to minimize social disruption and mitigate the effectors of future earthquakes”. The authors suggested that resilience could be conceptualized along four dimensions: technical, organizational, societal and economic (TOSE). The two components, technical and economic, are related to the resilience of physical systems, such as lifeline systems and essential facilities. The other two components, organizational and social, are more related to the community affected by the physical systems. However, Bruneau et al. (2003) defined a fundamental framework for evaluating community resilience without a detailed quantification and definition.

After the general framework provided by Bruneau et al. (2003) various studies have been carried out, with the goal of practically evaluate the concept of resilience and identify the main units of measurement of it.

Miles and Chang (2003) present a comprehensive conceptual model of recovery, which establishes the relationships among a community’s households business, lifeline networks, and neighborhoods. The primary aim is to discuss issues of community recovery and to attempt to operationalize it. A numerical simulation model has been created and discussed: it consists in 4 neighborhoods, each having 100 business and 100 households; the seismic hazard, the community characteristics, and demographics within the prototype are based on the city of Kobe (Japan) and the earthquake of 1995. The conceptual model considers the attributes and behaviors of socioeconomic agents (household and businesses). How the built environment, the policy decisions and the sociopolitical characteristics of the community affect these factors. Even if a measure of resilience is not provided, the paper points out the necessity to correlate the concept of recovery to real factors, such as the household object, whose attributes are the income, the year the building of residence was built, and the possible existence of any retrofit building.

Davidson and Cagnan, (2004) developed a model of the post earthquake restoration processes for an electric power system. A discrete event simulation model based on available data was built, with the goal of improving the quantitative estimates of restoration times that are required
to evaluate economic losses, and identify ways to improve the restoration processes in future earthquakes. The key advantages of this approach are: (i) to include the utility company’s decision variables explicitly, (ii) produce different restoration curves for each region within the service area and (iii) provide the uncertainty in the restoration curve.

The three models used to build the risk assessment method are:

(i) **seismic hazard model**, (Chang 2000) which is used to identify a limited set of deterministic earthquake scenarios and assign a “hazard consistent” probability of occurrence to each one. The software EPEDAT (Early Post – Earthquake Damage Assessment Tool (Eguchi 1997)) provides the estimation of the losses. In this way, 47 earthquake scenarios in the Los Angeles area were considered;

(ii) **damage estimation model**, (Dong 2002) which estimates the damage to the high voltage transmission substation of the LADWP (Los Angeles Department of Water and Power) electric power system under the 47 earthquake scenarios of the seismic hazard model;

(iii) **restoration model** (Cagnan et al. 2006) in which the statistical data are used to build a simulation model in a visual simulator software (ProModel, 1999).

The results are presented in terms of percentage of customers restored as a function of time elapsed after the earthquake (for the estimation of the duration of the power outages) and in terms of the so called power rapidity risk, or the full probability distribution for all possible combinations of number of customers and duration of the outage.

Chang and Shinozuka (2004) contribute to the literature on disaster resilience discussing a quantitative measure of resilience based on the case study of the Memphis water system. They explored the extent to which earthquake loss estimation models can be used to measure resilience. Two hundred Monte Carlo simulations for each of the three retrofit cases (1, 2 and no retrofit) and for two earthquake scenarios were run. The results were given in terms of percentage of simulations meeting performance criteria. The case study underlines the fact that resilience assessment goes beyond traditional loss estimations, introducing a way to relate these losses to standard of acceptable performance. It focuses the attention also on the speed of recovery: the pre-disaster mitigation and post-disaster response are placed in a common framework in which they can be evaluated and compared, addressing in a systematic fashion the multiple, interrelated dimensions of resilience.
Cimellaro et al. (2005), tempted to formulate the first framework to quantify resilience, however only the uncertainties of the intensity measure $I$ were considered, whereas in the framework proposed in this report all other uncertainties are involved (section 2.9).

Bruneau and Reinhorn (2007) for the first time relate probability functions, fragilities and resilience in a single integrated approach for acute care facilities. After having defined the main properties and concepts of resilience, two different options to quantify the disaster resilience of acute care facilities are exposed as the percentage of healthy population and as the number of patience/day that can receive service.

While this literature survey is by no mean comprehensive, it is presented here to highlight several distinct techniques, and set the stage for future developments in this work.

2.3 Definitions and formulations

To establish a common framework for resilience, a unified terminology is proposed, the fundamental concepts are analyzed, and two applications to health care facilities are presented in this report.

**Definition 1**: Resilience ($r$) is defined as a function indicating the capability to sustain a level of functionality or performance for a given building, bridge, lifeline networks, or community, over a period defined as the control time that is usually decided by owners, or society (usually is the life cycle, life span of the system etc.).

**Definition 2**: The recovery time ($T_{re}$) is the period necessary to restore the functionality of a structure, an infrastructure system (water supply, electric power, hospital building, etc., or a community), to a desired level that can operate or function the same, close to, or better than the original one.

The recovery time $T_{re}$ is a random variable with high uncertainties that includes the construction recovery time and the business interruption time and it is usually smaller than the control time. It typically depends on the earthquake intensities and on the location of the system with its given resources such as capital, materials and labor, following the major seismic event. For these reasons, this recovery time is the most difficult quantity to predict in the resilience
function. Porter et al. (2001) attempted to make a distinction between downtime and repair time, and tried to quantify the latter. In that work, damage states were combined with repair duration, and with probability distributions to estimate assembly repair durations. Other researchers calculate the recovery period in various ways as indicated further in this chapter.

While the previous definitions apply to structures, infrastructure, or societal organizations, a more general application of such definitions is for “disaster resilient communities”.

**Definition 3**: Disaster resilient community is a community that can withstand an extreme event, natural or man made, with a tolerable level of losses, and is able to take mitigation actions consistent with achieving that level of protection (Mileti, 1999).

In MCEER’s terminology the seismic performance of the system is measured through a unique decision variable (DV) defined as “Resilience” that combines other variables (economic losses, casualties, recovery time etc.) which are usually employed to judge seismic performance. This Resilience is defined graphically as the normalized shaded area underneath the functionality function of a system, defined as Q(t). Q(t) is a non stationary stochastic process and each ensemble is a piecewise continuous function as the one shown in Figure 2-1, where the functionality Q(t) is measured as a nondimensional (percentage) function of time. For a single event, Resilience is given by the following equation (Bruneau et al., 2005, 2007)

\[
R = \int_{t_{0E}}^{t_{tE} + T_{LC}} \frac{Q(t)}{T_{LC}} dt
\]  

(2-1)

where

\[
Q(t) = [1 - L(1, T_{RE})] \left[ H(t - t_{0E}) - H(t - (t_{0E} + T_{RE})) \right] f_{REC}(t, t_{0E}, T_{RE})
\]  

(2-2)

where \( L(I, T_{RE}) \) is the loss function; \( f_{REC}(t, t_{0E}, T_{RE}) \) is the recovery function; \( H(t_0) \) is the Heaviside step function, \( T_{LC} \) is the control time of the system, \( T_{RE} \) is the recovery time from event \( E \) and; \( t_{NE} \) is the time of occurrence of event \( E \).
2.4 The four dimensions of Resilience

While defining Resilience is clearly challenging, identifying the features of organizations and other social units that make them resilient is even more difficult. Resilience is an important concept for disaster managements of complex systems. Researchers at the MCEER (Bruneau, et al. 2003; Bruneau and Reinhorn 2007) have identified four dimensions along which resilience can be improved. These are robustness, resourcefulness, redundancy, and rapidity. These dimensions can better be understood by looking at the functionality curve shown in Figure 2-2.
2.4.1 Rapidity

Rapidity is the “capacity to meet priorities and achieve goals in a timely manner in order to contain losses and avoid future disruption” (Bruneau et al., 2003). Mathematically it represents the slope of the functionality curve (Figure 2-2a) during the recovery-time and it can be expressed by the following Equation (2-3)

$$\text{Rapidity} = \frac{dQ(t)}{dt}; \text{ for } t_{0E} \leq t \leq t_{0E} + T_{RE}$$

(2-3)

An average estimation of rapidity can be defined by knowing the total losses and the total recovery time to reach again 100% of functionality, as follows

$$\text{Rapidity} = \frac{L}{T_{RE}} \text{ (average recovery rate in percentage/time)}$$

(2-4)

where L is the loss, or drop of functionality, right after the extreme event.

2.4.2 Robustness

Robustness referring to engineering systems is, “the ability of elements, systems or other units of analysis to withstand a given level of stress, or demand without suffering degradation or loss of function” (Bruneau et al., 2003). It is therefore the residual functionality right after the extreme event (Figure 2-2b) and can be represented by the following relation

$$\text{Robustness} = 1 - \Phi(m_L, \sigma_L); \text{ (%)}$$

(2-5)

where \( \Phi \) is a random variable expressed as function of the mean \( m_L \) and the standard deviation \( \sigma_L \). A more explicit definition of robustness is obtained when the dispersion of the losses is expressed directly as follows

$$\text{Robustness} = 1 - \Phi(m_L + a\sigma_L); \text{ (%)}$$

(2-6)

where \( a \) is a multiplier of the standard deviation corresponding to a specific level of losses. A possible way to increase uncertainty in robustness of the system is to reduce the dispersion in the losses represented by \( \sigma_L \). In this definition, robustness reliability is therefore also the capacity of
keeping variability of losses within a narrow band, independently of the event itself (Figure 2-2b). Two examples of systems with and without robustness are respectively the Emergency Operation Center (EOC) and the Office of Emergency Management (OEM) organization during the World Trade Center disaster in 2001 (Kendra and Wachtendorf, 2003). The EOC facility, part of OEM, was not sufficiently robust to survive the September 11, attack (being located in the 23rd floor of the 7 World Trade Center). However, on the strength of its resourcefulness, OEM exhibited considerable robustness as an organization, demonstrating an ability to continue to function even after losing its WTC facility and a great part of its communications and information technology infrastructure. When the latter was restored, it contributed to the resilience of the OEM as a functional and effective organizational network.

2.4.3 Redundancy

According to the structural field, Redundancy is “the quality of having alternative paths in the structure by which the lateral forces can be transferred, which allows the structure to remain stable following the failure of any single element” (FEMA 356, 2000). In other words, it describes the availability of alternative resources in the recovery process of a system. Redundancy is “the extent to which elements, systems, or other units of analysis exist that are substitutable, i.e. capable [of] satisfying functional requirements in the event of disruption, degradation, or loss of functionality” (Bruneau et al., 2003). Simply, it describes the availability of alternative resources in the loss or recovery process.

Redundancy is a very important attribute of resilience, since it represents the capability to use alternative resources, when the principal ones are either insufficient or missing. If the system is resilient there will always be at least one scenario allowing recovery, irrespective of the extreme event. If this condition is not met by the system then changes to the system can be made, such as duplicating components to provide alternatives in case of failure.

An example of a system without redundancy is well illustrated in the World Trade Center terrorist attack mentioned above, where the EOC facility was destroyed and there was no other office, which could immediately, or instantaneously, replace the main facility. Redundancy should be developed in the system in advance and it should exist in a latent form as a set of possibilities to be enacted through the creative efforts of responders as indicated below.
2.4.4 Resourcefulness

Resourcefulness is “the capacity to identify problems, establish priorities, and mobilize resources when condition exist that threaten to disrupt some element, system, or other unit of analysis” (Bruneau et. al., 2003). This is a property difficult to quantify since it mainly depends on human skills and improvisation during the extreme event.

Resourcefulness and Redundancy are strongly interrelated. For example, resources, and resourcefulness, can create redundancies that did not exist previously. In fact, one of the major concerns with the increasingly intensive use of technology in emergency management is the tendency to over-rely on these tools, so that if technology fails, or it is destroyed, the response falters. To forestall this possibility, many planners advocate Redundancy. Changes in Resourcefulness and Redundancy will affect the shape and the slope of the recovery curve and the recovery time $T_{RE}$. It also affects Rapidity and Robustness. It is through Redundancy and Resourcefulness (as means of resilience) that the Rapidity and Robustness (the ends of resilience) of an entire system can be improved.

2.5 Loss Function

Loss estimation, and in particular the losses associated with extreme events, first requires damage descriptors that can be translated into monetary terms and other units that can be measured, or counted, e.g. the number of patients requiring hospitalization. The loss estimation procedure is by itself a source of uncertainty and this has been taken into account in section 2.9. One particular loss estimation procedure is adopted in this section; however, users can substitute their preferred methodology to estimate the losses, $L$ (NRC, 1992, Coburn et al., 2002, or Okuyama et al., 2004) in Equation(2-2). Earthquake losses are by their very nature highly uncertain, and are different for every specific scenario considered. However, some common parameters affecting those losses can be identified. In fact the loss function $L(I,T_{RE})$ is expressed as a function of earthquake intensity $I$ and recovery time $T_{RE}$. The total losses can be divided in two types: Structural losses ($L_S$) which occur “instantaneously” during the disaster, and Non-Structural losses ($L_{NS}$) which have also temporal dependencies

$$L(I,T_{RE}) = L_S(I) + L_{NS}(I,T_{RE})$$  \hspace{1cm} (2-7)
For simplicity $L_S$ and $L_{NS}$ are described with reference to a particular essential facility as a hospital, so that the physical structural losses can be expressed as ratios of building repair and replacement costs as follows

$$L_S(I) = \sum_{j=1}^{s} \left[ \frac{C_{S,j}}{I_S} \prod_{i=1}^{t} \left(1 + \delta_i \right) \right] \cdot P_j \left\{ \bigcup_{i=1}^{n} (R_i \geq r_{lim}) / I \right\} \quad (2-8)$$

where $P_j$ is the probability of exceeding a performance limit state $j$ conditional an extreme event of intensity $I$ occurs, also known as the fragility function; $C_{S,j}$ are the building repair costs associate with a $j$ damage state; $I_s$ are the replacement building costs; $r$ is the annual discount rate; $t_i$ is the time range in years between the initial investments and the occurrence time of the extreme event; $\delta_i$ is the annual depreciation rate. Equation (2-8) assumes that the initial value of the building is affected by the discount rate, but the value also decreases with time according to the depreciation rate $\delta_i$, which may vary with time. The nonstructural losses $L_{NS}$ consist of four contributions: (i) Direct economic losses $L_{NS,DE}$ (or Contents losses); (ii) Direct Causalities losses $L_{NS,DC}$; (iii) Indirect economic losses $L_{NS,IE}$ (or Business interruption losses); (iv) Indirect Causalities losses $L_{NS,IC}$ all function of recovery period $T_{RE}$. Nonstructural direct economic losses $L_{NS,DE}(I)$ are obtained for every non structural component $k$ used in the affected system using a formulation similar to Equation (2-8). In essential facilities like hospitals, research laboratories or some highly specialized manufacturing plants this term can be much larger than the structural losses. Then, the total non-structural direct economic losses are obtained using a weighted average expressed as

$$L_{NS,DE}(I) = \left( \sum_{k=1}^{N_{NS}} w_k \cdot L_{NS,DE,k}(I) \right) / N_{NS} \quad (2-9)$$

where $L_{NS,DE,k}(I)$ is the non-structural direct economic losses associated with component $k$, $N_{NS}$ is the total number of non-structural components in the buildings and $w_k$ is an importance weight factor associated with each non-structural component in the building. Non-structural components such as the ceilings, elevators, mechanical and electrical equipments, piping, partitions, glasses etc. are also considered. An important key factor in loss estimation is the determination of conversion factors for non-monetary values, like the value of human life, that
are used in equivalent cost analysis. According to FEMA 227 (1992), the suggested value of human life ranges from $1.1 million to $8 million. However, assigning a monetary value to human life might be somewhat controversial. In order to avoid this problem direct causalities losses $L_{NS,DC}$ are measured as a ratio of the number of injured or dead $N_{in}$ \(^1\) and the total number of occupants $N_{tot}$

$$L_{NS,DC}(I) = \frac{N_{in}}{N_{tot}}$$  \hspace{1cm} (2-10)

The number of injured patients $N_{in}$ depends on multiple factors such as, the time of day of earthquake occurrence, the age of the population and the number and proximity of available health care facilities. The time at which the earthquake occurs determines the number of patients exposed to injury, so the probability of having a large number of injured patients varies during the day. Moreover, the age of population is also very important as indicated by Peek-Asa et al. (1998) who found that during the 1994 Northridge earthquake the predominant number of injured patients were elderly.

\begin{table}[h]
\centering
\begin{tabular}{ll}
\hline
MMI Level & Casualties Rate per 100,000 people \\
\hline
(1) & (2) \\
<VI & 0.03 \\
VI & 0.16 \\
VII & 2.1 \\
VIII & 5.1 \\
IX & 44 \\
\hline
\end{tabular}
\caption{Casualty rate as a function of MMI (Peek-Asa et al. 2000)}
\end{table}

The number and proximity of available hospitals determine the proportion of fatalities among the seriously injured. In order to estimate risk by mean of resilience function it is necessary to make empirical predictions of casualties based on structural damage or ground motion intensity. Table 2-2 reports the four HAZUS (FEMA 2005) casualty severity levels as function of ground

\(^1\) the two groups can be considered separately, but in this formulation are grouped for simplicity
motion intensity. Peek-Asa found that for the 1994 Northridge earthquake the ground motion levels as measured by MMI were better predictor of casualty rates than building damage because the number of people injured in locations where structural damage occurred was only a small fraction of the total number of injured. For example, minor injuries resulted from being struck by objects and from falling, and not by structural damage. MMI allows a rough estimate of casualty rates, based on the population that is subjected to various intensities levels. Note that in Table 2-2 the construction type and the severity of injuries are not taken in account. In addition, the ratio in the table represents only the injuries treated at a hospital, while numerous minor injuries without hospitalization are not considered.

The indirect economic losses \( L_{NS,IE}(I, T_{RE}) \) are time dependent compared to all the previous losses considered. Among the post-earthquake losses these are the most difficult to quantify, because of the different forms they can take. They mainly consist of business interruptions, relocation expenses, rental income losses, etc. Losses of revenue, either permanent or temporary, can be caused by damage to structures and contents, and this is most important for manufacturing and retail facilities, and to lifelines. Damage to the former could mean less ability to deliver resources and services, like electricity, water, natural gas, or transportation. For example, structural damage such as collapse of a bridge span in a major highway generates direct losses, and indirect losses due to the loss of revenues from impact on the traffic to businesses served. In other cases, even if structural damage and loss of contents are minimal, there may be some indirect losses due to the disruption of services such as water and power. These losses can be more significant than the direct losses. Therefore, losses due to business interruption should be modeled considering both the structural losses \( L_s \), and the time necessary to repair the structure \( T_{RE} \) (Werner et al. 2003; Miles and Chang 2006). These two quantities are not independent; those are related because the recovery time \( T_{RE} \) increases with the extent of structural damage \( L_s(I) \). In addition, indirect causalities losses \( L_{IC} \) belong to this group. They describe the number of patients that are injured or die because of hospital dysfunction, for example. For a hospital, \( L_{IC} \) can also be expressed in a similar form of Equation (2-10) as the ratio of the number of injured persons \( N_{in} \) to the total population \( N_{tot} \) served

\[
L_{NS,IC}(I) = \frac{N_{in}}{N_{tot}}
\]

(2-11)
The total non-structural losses $L_{NS}$ can be expressed as a combination of the total direct losses $L_{NS,D}$ and the total indirect losses $L_{NS,I}$. Also direct losses $L_{NS,D}$ and indirect losses $L_{NS,I}$ are expressed as combination of economic ($L_{NS,IE}$, $L_{NS,DE}$) and casualties’ losses ($L_{NS,IC}$, $L_{NS,DC}$)

$$L_{NS} = \alpha_I(L_{NS,D} + \alpha_L L_{NS,I})$$

where

$$\begin{align*}
L_{NS,D} &= L_{NS,DE} \alpha_{DE} \cdot (1 + \alpha_{DC} L_{NS,DC}) \\
L_{NS,I} &= L_{NS,IE} \alpha_{IE} \cdot (1 + \alpha_{IC} L_{NS,IC})
\end{align*}$$

(2-12)

where $\alpha_I$ is the weighting factor related to indirect losses (i.e. importance of the facilities for the community, influence of the facilities versus other system, etc); $\alpha_{DE}$ is a weighting factor related to construction losses in economic terms; $\alpha_{IE}$ is a weighting factor related to business interruption, relocation expenses, rental income losses, etc.; $\alpha_{DC}$, $\alpha_{IC}$ are the weighting factors related to the nature of occupancy (i.e. schools, critical facilities, density of population). These weighting factors are determined based on socio-political criteria (cost benefit analyses, emergency functions, social factors, etc.). Engineers, economists, and social scientists usually address this subject jointly. It should be noted that the two casualties and life losses do not appear as loss function, but as penalty functions in the total loss picture. Finally, $L_S$ and $L_{NS}$ are summed together to obtain the total loss function $L(I, T_{RE})$, as shown in Equation (2-7).

### 2.6 Simplified Recovery Function models

Most of the models available in literature, including the PEER equation framework (Cornell and Krawinkler, 2000), are loss estimation models that focus on initial losses caused by disaster, where losses are measured relative to pre-disaster conditions. The temporal dimension of post-disaster loss recovery is not part of that formulation. As indicated in Figure 2-1 the recovery time $T_{re}$ and the recovery path are essential to evaluating resilience, so they should be estimated accurately. Unfortunately, most common loss models, such as HAZUS (Whitman et al., 1997) evaluate the recovery time in crude terms and assume that within one year, everything returns to normal. However, as shown in Figure 2-1 the system considered may not necessary return to the pre-disaster baseline performance. It may exceed the initial performance (Figure 2-1-curve C), when the recovery process ends, in particular when the system (e.g. community, essential facility, etc.) may use the opportunity to fix pre-existing problems inside the system itself. On
the other hand, the system may suffer permanent losses and equilibrate below the baseline performance (Figure 2-1-curve A).

These considerations show that the recovery process is complex and it is influenced by time dimensions, spatial dimensions (e.g., different neighborhood may have different recovery paths) and by interdependencies between different economic sectors that are interested in the recovery process. Therefore, different critical facilities (e.g. hospitals) that belong to the same community, but are located in different neighborhoods, have different recovery paths and in some areas (mainly poor areas), these essential facilities may experience long term or permanent losses (Chang, 2000). In summary, the recovery process shows disparities among different geographic regions in the same community, showing different rates and quality of recovery. Modelling recovery of a single critical facility or of an entire community is a complex subject. These two processes cannot be assumed independent.

![Functionality curves](image)

Figure 2-3 Functionality curves (a) Average prepared community, (b) not well prepared community, (c) well prepared community

Information on comprehensive models that describe the recovery process is very limited. Miles and Chang (2006) set out the foundations for developing models of community recovery presenting a comprehensive conceptual model and discussing some related issues. Once these complex recovery models are available, it is possible to describe relationship across different scales-socioeconomic agents, neighborhood and community, and to study the effects of different policies and management plans in an accurate way. In this chapter, the recovery process is oversimplified using recovery functions that can fit the more accurate results obtained with the Miles and Chang (2006) model or with the recovery model proposed in next section that is valid for health care facility systems.
Different types of recovery functions can be selected depending on the system and society preparedness response. Three possible recovery functions are shown in Equation (2-13) below:

(i) linear, (ii) exponential (Kafali and Grigoriu, 2005) and (iii) trigonometric (Chang and Shinozuka, 2004)

\[
\text{linear: } f_{\text{rec}}(t,T_{RE}) = \left( 1 - \frac{t-t_{0E}}{T_{RE}} \right);
\]

\[
\text{exponential: } f_{\text{rec}}(t) = \exp\left[ -(t-t_{0E}) \ast \left( \ln 200 \right) / T_{RE} \right];
\]

\[
\text{trigonometric: } f_{\text{rec}}(t) = 0.5 \ast \left\{ 1 + \cos \left[ \pi \left( t-t_{0E} \right) / T_{RE} \right] \right\};
\]

The simplest form is a linear recovery function that is generally used when there is no information regarding the preparedness, resources available and societal response (Figure 2-3a). The exponential recovery function may be used where the societal response is driven by an initial inflow of resources, but then the rapidity of recovery decreases as the process nears its end (Figure 2-3b). Trigonometric recovery function can be used when the societal response and the recovery are driven by lack or limited organization and/or resources. As soon as the community organizes itself, with the help of other communities (for example), then the recovery system starts operating and the rapidity of recovery increases (Figure 2-3c). For example, such recovery occurred after Nisqually Earthquake (Filiatrault et al., 2001).

2.7 Mechanical analogy

The functionality of a system (e.g. hospital, network of hospitals, etc.) can be described by nonlinear differential equations similar to the one that applies to the fundamental laws of mechanical systems. Using these mathematical identity organizational systems can be described using mechanical systems. The equation of motion for a damped harmonic oscillator would be equation (2-14) with \( F(t) = 0 \). Treated one dimensionally the equation becomes

\[
m\ddot{\xi}(t) + c\dot{\xi}(t) + k\xi(t) = 0
\]

The solution of this equation has three different outcomes, depending on the value of \( \zeta \), called the damping factor.
If $\zeta<1$, the system is **under-damped**, so it will oscillate around the mean value, with decreasing amplitude;

If $\zeta=1$, the system is **critically-damped**, meaning there will be no oscillation and the system will reach equilibrium quickly;

If $\zeta>1$, the system in **over-damped**, and will reach equilibrium slowly with no oscillation.

Because the recovery process after an extreme event usually tries to go back to its initial condition without oscillations, it will be taken in account only the last two cases.

For **over-damped** systems, $\zeta > 1$, the general solution is

$$Q(t) = 1 - e^{-\alpha t} \left( Ae^{\beta t} + Be^{-\beta t} \right)$$

with $\alpha = \omega \zeta$ and $\beta = \omega \sqrt{(\zeta^2 - 1)}$.

Placing the initial condition $Q(0) = 1 - L(I, T_{RE})$ and $Q(0) = 0$ where $L(I, T_{RE})$ are the total losses given in Equation (2-7), the solution will be

$$Q(t) = 1 - e^{-\alpha t} \left[ \frac{\alpha + \beta}{2\beta} e^{\beta t} + \frac{\beta - \alpha}{2\beta} e^{-\beta t} \right]$$

A **critically-damped** system has $\zeta = 1$, and the general solution is

$$Q(t) = 1 - A e^{-\alpha t}$$

The other linearly independent solution can be found by using the reduction of order method. This gives the solution

$$Q(t) = 1 - B t e^{-\alpha t}$$

Placing the same initial condition $Q(0) = 1 - L_0$ and $Q(0) = 0$, the solution will be

$$Q(t) = 1 - L_0 e^{-\alpha t} \left( 1 + \alpha t \right)$$
So the expression of functionality is described by three parameters $\omega$, $\zeta$ and $L_0$ for the case when the system is over-damped (Eq. (2-16)) and by a two parameters model ($\omega$ and $L_0$) for the case when the system is critically damped (Eq. (2-19)).

If in Equation (2-16) it is assumed that the initial losses $L$ are 70% and the parameter $\omega$ is maintained constant and equal to 0.2, then it is possible to understand the effect of the damping factor by increasing it. As the damping factor increases the speed of recovery reduces and it takes more time to reach the initial conditions as shown in Figure 2-4a. On the other hand if the system is critically damped and the circular frequency is increasing, then the system reduces its time to reach the initial condition as shown in Figure 2-4b. It is important to observe that in both cases above the system is assumed linear (Eq. (2-14)), therefore after the event it is possible to recover to the initial condition without permanent losses. In the case of permanent losses the system becomes nonlinear therefore Equation (2-14) becomes

$$m\ddot{Q}(t) + c\dot{Q}(t) + F_s(t) = 0$$  \hspace{1cm} (2-20)

where $F_s(t)$ is the equivalent of a restoring force that is able to recover the system partially.

![Figure 2-4](image.png)

Figure 2-4 Functionality curves (a) Three parameters model in Equation (2-16) ; (b) Two parameters model in Equation
2.8 Fragility Functions

The calculation of disaster resilience through functionality losses (see Equation (2-2)) makes use of the fragility functions, or the reliability of the system analyzed. Fragility curves are functions that represent the probability that the response \( R(x, I, t) = \{R_1, \ldots, R_n\} \) of a specific structure (or family of structures) exceeds a given performance threshold \( R_{LS}(x, I) = \{R_{LS1}, \ldots, R_{LSn}\} \), associated with a performance limit state, conditional on earthquake intensity parameter, \( I \) happens, such as the peak ground acceleration (pga), peak ground velocity (pgv), return period, spectral acceleration (\( S_a \)), spectral displacements (\( S_d \)), modified Mercalli Intensity (MMI), etc. The response \( R \) and the limit states, \( R_{LS} \), are expressions of the same variable (or measure) such as deformation, drift, acceleration, stresses, strains, (mechanical characteristics) or other functionality measures.

The response, \( R \), and response threshold, \( R_{LS} \), are functions of the structural properties of the system \( x \), the ground motion intensity \( I \) and the time \( t \). However, in the formulation it is assumed that the response threshold \( R_{LS}(x) \) does not depend on the ground motion history and so does not depend on time, while the demand \( R_i(x, I, t) \) of the generic \( i^{th} \) component is replaced by its maximum value over the duration of the response history \( R_i(x, I) \). The dependence of the response \( R(x, I) \) on \( x \) and \( I \), and the dependence of the response threshold \( R_{LS}(x) \) on \( x \) will be omitted in the following for sake of simplicity. With these assumptions, the general definition of fragility \( F_{LS} \) based on Earthquake Intensity \( I \) can be written as (Cimellaro et al. 2006a)

\[
F_{LS}(i^*) = P\left( R_i \geq R_{LSi} \left| I = i^* \right. \right) 
\]

(2-21)

where \( R_i \) is the response parameter related to a certain measure (deformation, force, velocity, etc.) and \( R_{LS,i} \) is the response threshold parameter correlated with the performance level; \( I \) is the Earthquake Intensity measure (\( Pga \), \( Pgv \), Modified Mercalli Intensity, etc.); and \( i^* \) is the earthquake intensity “level”. However, another definition of fragility based on earthquake hazard \( H \) can be given when considering all the events with a larger intensity than \( i^* \) which can be described by the seismic hazard curve,

\[
\lambda = H(I)_{1 yr} = P\left( I \geq i^* \right)_{1 yr} 
\]

(2-22)
where $\lambda$ is the average annual frequency of exceedance. The definition of fragility based on Earthquake Hazard $H$ is given by

$$
F_{R_{LS}} (h^*) = \int P (R_i \geq R_{LS(i)}| I = i^*) \frac{dH(I)}{di} \quad \text{or} \quad F_{R_{LS}} (h^*) = \int F_{R_{LS}} (i^*) \frac{dH(I)}{di}
$$

(2-23)

where the hazard $h^*$ corresponds to the multiple earthquake occurrences designated within a defined hazard, (also defined by the return period $T_r$, which can be expressed as $H(I)^{-1}$ within 1 year time range); $dH/di$ is the probability density function of earthquake intensity. The hazard related fragility is therefore an integral including the probability of event occurrence, $P(I=i^*)$.

![Image of Earthquake Intensity vs. Earthquake Hazard fragility curves](image)

Figure 2-5 Earthquake Intensity vs. Earthquake Hazard fragility curves

It is important to mention that there is not a one-to-one correspondence between Earthquake Intensity $I$ and Earthquake Hazard $H$ as shown in Figure 2-5. In fact, different values of earthquake intensities $I$ ($P_{ga}$, $P_{gv}$, $S_a$ etc.) can correspond to a unique earthquake hazard (e.g. $T_r$, the annual frequency of exceedance $\lambda$ etc.). However, seismic hazard curves that relate in average sense (assumed known the attenuation relationship) earthquake intensities and
earthquake hazard at various sites can be found using the USGS java application (USGS, 2008). The advantage of the second formulation in Equation (2-23) in respect to Equation (2-21) is that it takes into account directly the uncertainties of occurrence in estimating the Earthquake Intensity parameters $I$ at the site. Therefore, in professional practice, where buildings are designed according to a given return period $T_r$ (a measure of hazard), it is possible to use directly the expression of fragility curve given in Equation (2-23) for evaluating directly the probability of functionality, or damage, of the system. The details about the method to generate fragility curves according to Equation (2-23) are given in the following paragraphs. When the number of response parameters to be checked is $n$ the definition of fragility given in Equation (2-23) can be written in the following form

$$ F_{R_{ls}}(h^*) = \int P\left( \bigcup_{i=1}^{n} (R_i \geq R_{LSi}) | I = i^* \right) \frac{dH(I)}{di} \, di $$

(2-24)

where the first right term of Equation (2-24) is the conditional probability of the multi-component response exceeding multi-dimensional limit state which is explicitly written as

$$ F_{R_{ls}}(i^*) = P\left( \bigcup_{i=1}^{n} (R_i \geq R_{LSi}) | I = i^* \right) = $$

$$ = \left( \sum_{j=1}^{n} P\left( (R_j \geq R_{LSj}) | I = i^* \right) - \sum_{i=1}^{n} \sum_{j=2}^{n} P\left( (R_i \geq R_{LSi}) (R_j \geq R_{LSj}) | I = i^* \right) + $$

$$ + \sum_{i=1}^{n} \sum_{j=2}^{n} \sum_{k=3}^{n} P\left( (R_i \geq R_{LSi}) (R_j \geq R_{LSj}) (R_k \geq R_{LSk}) | I = i^* \right) + .... \right) $$

(2-25)

$$ + (-1)^n P\left( (R_1 \geq R_{LS1}) (R_2 \geq R_{LS2}) (R_n \geq R_{LSn}) \right) $$

where $(R_i \geq R_{LSi})$, for independent events $i=1,2,...,n$. The definition of fragility given in Equation (2-25) is based on the assumption that the structure can be simplified assuming it as a series system, with the weakest link system leading to failure. When the problem is reduced to a bi-dimensional case considering for instance, displacements and accelerations at a specific story of a building, the fragility curve in Equation (2-25) can be determined using the following expression
\[
F_{\text{rls}}(i^*; D_{LS}, A_{LS}) = P:\left(\frac{\Delta \geq D_{LS}}{Z \geq A_{LS}} \mid I = i^*\right) = P\left((\Delta \geq D_{LS}) \mid I = i^*\right) + \\
+ P\left((Z \geq A_{LS}) \mid I = i^*\right) - P\left((\Delta \geq D_{LS})(Z \geq A_{LS}) \mid I = i^*\right)
\]  \tag{2-26}

where \(\Delta\) is the random variable representing the displacement response, \(Z\) is the random variable representing the acceleration response, \(D_{LS}\) is the displacement threshold, \(A_{LS}\) is the acceleration threshold and \((\Delta \geq D_{LS})\) and \((Z \geq A_{LS})\) are assumed to be two independent events. The response of the structure can be visually represented for two variables by a “bell surface” (Bruneau and Reinhorn, 2007) where the x-axis is the spectral displacement \(S_d\), while the y-axis is the pseudo-spectral acceleration, designated here as \(S_a\), while z-axis is shows probability (Figure 2-6b). This surface is the joint probability density function of the response expressed in terms of the two variables, the maximum spectral displacement and the maximum spectral acceleration that are assumed to be lognormally distributed.

Fragility appears explicitly in the expression of the loss function in Equation (2-8) where normalized losses are multiplied by \(P_j\), the probability of exceeding a given performance level \(j\) conditional on an event of intensity \(I\). This value can be obtained from the fragility function when the intensity \(I\) of the event is known. The definition of fragility in Equation (2-21) requires implicitly the definition of the performance limit states, \(R_{LS}\), which are discussed in the following section.

\section*{2.8.1 Multidimensional performance limit state function}

The calculation of fragility is performed using a generalized formula describing the multidimensional performance limit state function (MLS), and it allows considering multiple limit states related to different quantities in the same formulation (Cimellaro \textit{et al.} 2006a).
The MLS function $g(R, R_{LS})$ for the $n$-dimensional case, when $n$ different types of limit states are considered simultaneously, can be given by

$$g(R, R_{LS}) = \sum_{i=1}^{n} \left( \frac{R_i}{R_{LS,i}} \right)^{N_i} - 1$$

(2-27)

where $R_i$ is the dependent response threshold parameter (deformation, force, velocity, etc.), that is correlated with damage; $R_{LS,i}$ is the independent capacity threshold parameter and $N_i$ are the interaction factors determining the shape of $n$-dimensional surface. The limit state corresponding to the boundary between desired and undesired performance, would be when $g=0$. when $g \leq 0$ the structure is safe, while when $g \geq 0$ the structure is not safe (undesired performance).

This model can be used to determine the fragility curve of a single nonstructural component, or to obtain the overall fragility curve for the entire building including its nonstructural components. Such function allows including different mechanical response parameters (force, displacement, velocity, accelerations etc.) and combining them together in a unique fragility curve. Different limit states can be modeled as deterministic, or random variables and they can be considered either linear, nonlinear dependent or independent using the desired choice of the parameters appearing in Equation(2-27). For example in a 3D-non-dimensional space, when the multidimensional performance threshold considers only three response parameters, Equation (2-27) assumes the shape as shown in Figure 2-6a.
In the two-dimensional case (Figure 2-6b) the response of the system can be visualized in a
space where on the x-axis can be the spectral displacements $S_d$, while on the y-axis can be the
pseudo spectral accelerations PSA and values on z-axis show the probability (shown by contour
lines). The shape of the response curve of the system in this space is similar to a “bell surface”
(Bruneau and Reinhorn 2004; 2007) while the multidimensional performance threshold (MPLT)
in this space is represented by a cylindrical nonlinear function that relates acceleration
performance threshold $A_{LS}$ to displacement performance threshold $D_{LS}$ (Figure 2-6b). The
probability that the response $R$ exceeds a specific performance threshold $R_{LS}$ can directly be
calculated from the volume under the surface distribution exceeding the specified limit
represented in Figure 2-6b by a dotted line.

Analytically equation (2-27) can be simplified in the following expression

$$g(R, R_{LS}) = \left( \frac{A}{A_{LS}} \right)^{N_a} + \left( \frac{D}{D_{LS}} \right)^{N_b} - 1 \tag{2-28}$$

where $A_{LS}$ is the independent acceleration limit state; $D_{LS}$ is the independent displacement limit
state; $A$ and $D$ are the peak acceleration and displacement response; $N_a$ and $N_b$ are interaction
factors determining the shape of limit state surface. The independent thresholds $A_{LS}$, $D_{LS}$ and the
interactions factors $N_a$ and $N_b$ are determined from either (i) field data after an earthquake or
from (ii) laboratory tests. The first procedure implies collecting past earthquake field data
(Shinozuka et al., 2000a and b). Damage data are related to drift as can be determined by field
observations, while acceleration thresholds can be determined in the field only when the building
is monitored with accelerometers. However, other types of threshold parameters can be obtained
from data in controlled experiments (e.g. number of tiles that fell out of a suspended ceiling)
(Badillo et al., 2006, Retamales et al., 2006). The advantage of the latter procedure is that for
the structure of interest, a range of earthquake intensities can be applied in a controlled fashion,
and interstory drifts, accelerations, or other parameters, can be monitored and measured more
accurately than in the field. However, both methods require multiple outcomes (structural
collapses), which are prohibitively expensive in costs and human lives (in real earthquakes).
Therefore, such limit thresholds would have to be derived by computations using basic
engineering principles.

When the MTLS function is calibrated, $A_{LS}$ and $D_{LS}$ can be assumed as either random
variables, or deterministic quantities, either dependent or independent. All cases can be
considered as particular realizations of the general Equation (2-27). The bidimensional MTLS function in Equation (2-28) is considered for illustrative purposes. For example, the most common and simplest form of performance function considers only drift (as unidimensional threshold) and it can be obtained assuming \( A_{LS} = \infty \); therefore Equation (2-28) becomes

\[
g \left( \mathbf{R}, R_{LS} \right) = \left( \frac{D}{D_{LS}} \right)^{N_a} - 1
\]

(2-29)

where \( D \) is the displacement response; \( D_{LS} \) can be either a deterministic or a random threshold variable (Figure 2-7a). In order to be safe \( g \leq 0 \) that implies \( D \leq D_{LS} \). Alternatively, if acceleration limit state is given, then this can be determined assuming \( D_{LS} = \infty \), therefore Equation (2-28) becomes

\[
g \left( \mathbf{R}, R_{LS} \right) = \left( \frac{A}{A_{LS}} \right)^{N_a} - 1
\]

(2-30)

as shown in Figure 2-7b where \( A \) is the acceleration response; \( A_{LS} \) can also be considered as either a deterministic or a random threshold variable. In order to be safe \( g \leq 0 \) that implies \( A \leq A_{LS} \). This shape of the performance function is important for nonstructural components such as sensitive equipment, used in the building functions (i.e. computers, scientific devices, lab equipment, etc.). Damage to this type of nonstructural components has gained significant attention following recent earthquakes, because in essential facilities like hospitals failure of such equipments may hinder emergency response immediately after an earthquake. Most of these components are short and rigid and are dominated by a sliding-dominated response (Chaudury and Hutchinson, 2006). The case when both accelerations and interstory drifts thresholds are considered as independent limit states can be determined from the generalized MTLS in Equation (2-27) by imposing \( N = N_a/N_b = \infty \) (Figure 2-7c)

\[
g \left( \mathbf{R}, R_{LS} \right) = \left( \frac{A}{A_{LS}} \right)^{\infty} + \left( \frac{D}{D_{LS}} \right)^{\infty} - 1
\]

(2-31)

In fact if \( A/A_{LS} < 1 \), then \( \left( A/A_{LS} \right)^{\infty} \Rightarrow 0 \), therefore Equation (2-31) becomes

\[
g \left( \mathbf{R}, R_{LS} \right) = \frac{D}{D_{LS}} - 1
\]

(2-32)
that corresponds to the interstory drift limit state. By imposing the safe condition \((g \leq 0)\) in Equation (2-31), then \((A/\Delta L_s) \leq (1 - D/\Delta D_s)^{1/2} \Rightarrow 1\), therefore Equation (2-31) becomes

\[
g \left( R, R_{LS} \right) = \frac{A}{\Delta L_s} - 1
\]  

that corresponds to the acceleration limit state. On the other hand, assuming a linear relationship between acceleration and interstory drift limit states for \(N = N_a/N_b = 1\), a velocity limit state is produced, as shown in Figure 2-7d.

\[
g \left( R, R_{LS} \right) = \left( \frac{A}{\Delta L_s} \right) + \left( \frac{D}{\Delta D_s} \right) - 1
\]  

Figure 2-7 Threshold Limit States: (a) Drift threshold limit state; (b) Acceleration threshold limit state; (c) Independent acceleration and interstory drift limit states; (d) Velocity limit state (Cimellaro et al., 2006)
2.8.2 Uncertainties of limit states

The performance limit states, PLS, represent the level of response for a certain functionality limit, or for a specific damage condition. The current practice in developing fragility curves is based on deterministic performance limit states, usually obtained from experimentation, design standards, engineering judgment, etc. The limits of functionality or of damage depend on mechanical properties, such as strength and deformability, which are in themselves uncertain. Therefore, a deterministic description of PLSs might be inappropriate, or limiting. Instead, PLSs should be modeled as random variables. Often, however, PLS are defined by deterministic quantities, because the uncertainty in the earthquake load is considerably larger than the uncertainty in the PLSs themselves. For fragility evaluations, however the uncertainties are significant. In this study, PLSs are considered as random variables, and are defined in terms both of interstory drifts and accelerations, for example, since the functionality and the failure modes in the case study (presented below) are governed by both.

Several cases are considered for the estimation of the fragility related to specific PLS, assuming that the limit thresholds are random variables. For simplicity of explanation, only two response parameters are considered: interstory drift and floor acceleration. It is assumed that the peak responses have a lognormal distribution. This assumption is made, since the maximum response is always positive. For each case different assumptions are made regarding the random variables considered. An analytical solution is formulated to calculate the probability of exceeding a certain performance limit state, given the probability distribution function of the response and of the limit states. The simplest case is where the interstory drift threshold $d^*$ is considered as a deterministic quantity, and is compared with the random variable $\Delta$ of the interstory drift response taking only positive values and assumed to be lognormally distributed:

$$ f_{\Delta}(\delta) = \begin{cases} \frac{1}{\delta \sigma_{\Delta} \sqrt{2\pi}} e^{-\frac{(\ln(\delta) - m_{\Delta})^2}{2\sigma_{\Delta}^2}} & \delta \geq 0 \\ 0 & \text{elsewhere} \end{cases} \quad (2-35) $$

The probabilities of exceeding the given performance limit state is
\[ P(\Delta \geq d) = 1 - F_\Delta(d) = 1 - \int_0^d f_\Delta(\delta) d\delta \] (2-36)

If also the interstory drift threshold \( D \) is a random variable taking only positive values (but assumed independent from \( \Delta \)) then the probability of exceeding the given performance limit state is

\[ P(\Delta \geq D) = P\left(\frac{\Delta}{D} \geq 1\right) = P(Y \geq 1) = 1 - F_Y(1) = 1 - \int_0^\infty f_D(u)F_\Delta(1) du; \] (2-37)

where

\[ F_Y(y) = \int_0^\infty f_D(u)d\delta du = \int_0^\infty f_D(u)\int_0^\infty f_\Delta(\delta)d\delta du = \int_0^\infty f_D(u)F_\Delta(yu) du; \] (2-38)

where \( u \) and \( \delta \) are auxiliaries variables. When the response is described by two lognormally distributed random variables corresponding to the interstory drift response \( \Delta \) and to the acceleration response \( Z \), then these two quantities are related. For example for linear SDOF systems the following relation holds

\[ \frac{Z}{\omega^2} = \Delta \] (2-39)

Because in general, for every type of structure, the displacements \( \Delta \) and accelerations \( Z \) are dependent random variables. Based on this relationship, the problem is reduced from two-dimensional to one-dimensional:

\[ P(\Delta \geq d \cup Z \geq a) = P(\Delta \geq d \cup \Delta \omega^2 \geq a) = P(\Delta \geq \min\left(\frac{d}{\omega^2}, a\right)) = \]

\[ = 1 - F_\Delta(\min(\frac{d}{\omega^2}, a)) = 1 - \int_0^{\min(\frac{d}{\omega^2}, a)} f_\Delta(\delta) d\delta \] (2-40)

The probability of exceeding the limit space can be evaluated when the probability density functions of interstory drift is known. Both parameters of the density functions can be calculated using the \textit{maximum likelihood method}. While in the cases shown above, the interstory drift performance limit state \( d \) and the acceleration performance limit state \( a \) were considered deterministic quantities, they can also be assumed as random variables lognormally distributed as
previously discussed. It is assumed that the acceleration response $Z$ and the displacement response $A$ are related in the elastic range, so that the relation $A = Z/\omega^2$ holds, while the performance limit state of interstory drift $D$ and acceleration $A$ are assumed independent random variables. This assumption is reasonable because, nonstructural components such as electronic devices (e.g. computers, etc.) for example, that are acceleration sensitive, cannot be related to the building PLSs that are typically displacement sensitive. In order to simplify the formulation, two new non-dimensional random variables $X = \Delta/D$ and $Y = Z/A$, are assumed. The probability of exceeding the given performance limits state can be expressed as

$$P(\Delta \geq D \cup Z \geq A) = P\left(\frac{\Delta}{D} \geq 1 \cup \frac{Z}{A} \geq 1\right) = P\left(\frac{\Delta}{D} \geq 1 \cup \frac{Z}{\omega^2 A} \geq 1\right)$$

$$= P\left(\frac{\Delta}{D} \geq 1 \cup A \frac{\omega^2 D}{A} \geq 1\right) = P\left(\frac{\Delta}{D} \geq 1 \cup \frac{A}{\omega^2 D} \geq 1\right) =$$

$$= P\left(X \geq \min\left(1, \frac{A}{\omega^2 D}\right)\right) = 1 - F_X\left(\min\left(1, \frac{A}{\omega^2 D}\right)\right) =$$

$$= 1 - \int_0^\infty f_D(u) F_A\left(\min\left(1, \frac{A}{\omega^2 D}\right) u\right) du;$$

Hence, for the evaluation of the exceedance probability in this case, only the probability density function of the interstory drift response and the probability density function of the interstory drift limit state are required.

In case that interstory drift performance limit state, “$D$”, and the acceleration performance limit state, “$A$”, are nonlinearly related through Equation (2-28), then the probability of exceedance the multidimensional performance limit state function is obtained by substituting Equation (2-28) into Equation (2-40):

The advantage of this formulation is that a limit state can be expressed as function of the other components, once the parameters of the model have been identified; hence, only a single parameter is needed to limit both displacement and acceleration. Once, the probability of exceeding the PLS is computed analytically, the procedure can be repeated again for different intensities measures, generating fragility curves using the procedure described in more detail in Cimellaro et al., (2006a).
2.9 Uncertainties in disaster resilience

When uncertainties are considered in Equation (2-1), then this unique decision variable (DV) called resilience becomes also a random variable, therefore its mean is defined by the following expression

\[ m_r = E(r) = \int \int \int \int R \cdot f_i, R, PM, L, T_e \, dT_e dL dPM dR dI \]  

(2-42)

where \( R \) is given in Equation (2-1) and \( f_{I,R,PM,L,T_e} \) is the joint probability density function (j.p.d.f.) of 5 r.v. that are: i) intensity measures \( I \); ii) Response parameters \( R \); iii) performance measures \( PM \); iv) losses \( L \); v) recovery time \( T_{RE} \). The random variables cannot be considered independent, therefore the j.p.d.f. can be determined as function of the conditional probability density functions as follow

\[ f_{i, R, PM, L, T_e} = f(T_{RE}/I, PM, R, L) \cdot f(L/PM, R, I) \cdot f(PM/R, I) \cdot f(R/I) \cdot f(I \geq i^*) \]  

(2-43)

The conditional probability density functions in Equation (2-43) is considering the various uncertainties that are: \( f(I_{TLC}>i^*) \), the probability of exceeding a given ground motion parameter \( i^* \) in a time period \( T_{LC} \); \( f(R/I) \) the conditional probability density function (c.p.d.f.) describing the uncertainties in the structural analysis parameters; \( f(PM/R) \) describes the uncertainties of the structural parameters and uncertainties of the model itself; \( f(L/PM) \) describes the uncertainties in the loss estimation model, while \( f(T_{RE}/L) \) describes the uncertainties in the time of recovery.

Finally, the probability that resilience is smaller than a specified value \( r_{crit} \) is defined as follow

\[ P(R \leq r_{crit}) = \int \int \int \int R \cdot f_{i, R, PM, L, T_e} \, dT_e dL dPM dR dI \]  

(2-44)

where \( f(I_{TLC}>i^*) \) in Equation (2-43) can be obtained from probabilistic seismic hazard analysis (PSHA). A common approach involves the development of seismic hazard curves, which indicate the average annual rate of exceedance \( \lambda_{i^*} \) of different values of the selected ground motion parameter \( i^* \). When combined with the Poisson model, the probability \( f(I_{TLC}>i^*) \) of exceeding the selected ground motion parameter \( i^* \) in a specified period of time \( T_{LC} \), takes the form (Kramer, 1996)
where the control time $T_{LC}$ for a decision analysis is based on the decision maker’s interest in evaluating the alternatives as is discussed subsequently in the case study presented herein.

The system diagram in Figure 2-8 identifies the key steps of the framework to quantify resilience. Note that closed form analytical solutions to evaluate resilience in Equation (2-42) cannot be always determined; therefore, integrals should be evaluated numerically. For simplicity, more than efficiency, the range of parameters considered as uncertain quantities have been divided in discrete ranges, so that the integrals can be substituted with summations. Indeed, Equation (2-42) becomes

$$m_r = E(r) = \sum_{I} \sum_{R} \sum_{PM} \sum_{L} \sum_{T_{RE}} R \cdot f_{I,R,PM,L,T_{RE}}$$

The loss function $L(I,T_{RE})$, the recovery function $f_{REC}(t,t_{OE}, T_{RE})$ and the fragility function are defined in the following sections, although they do not appear explicitly in Equation (2-42).

---

Figure 2-8 Performance assessment methodology (MCEER approach)

The methodology that is summarized in Equation (2-42) is more general than the one proposed by Cimellaro et al. (2005), because in that framework only the uncertainties of the
intensity measure \( I \) were considered, whereas in this framework all other uncertainties are involved. It should be noted that Equations (2-42) and (2-46) have similar forms as the “PEER integral” (Cornell and Krawinkler, 2000), which considers also uncertainties. However, the expressions suggested in this report consider the temporal kernel, which includes the recovery process and associated uncertainties, in addition to the immediate losses.

2.10 Probability of exceeding functionality thresholds

The variable resilience is not able always to describe the behavior of the system, because the same system can have different levels of functionalities through the time and it still will have the same values of resilience at the end of the control time as shown in Figure 2-9.

![Figure 2-9 Different functionality curves with the same values of resilience](image)

In these cases, in order to have a better description of the performance of the system is useful to introduce the functionality thresholds \( Q_{\text{crit}} \) and defining the probability of being below a given threshold as follow

\[
P(Q < Q_{\text{crit}}) = \frac{\Delta T_{Q_{\text{crit}}}}{T_{LC}}
\]

(2-47)

The functionality thresholds can describe the shape of the functionality curve while the parameter Resilience cannot, because it is an integral of a curve.
2.11 Case studies

Two case studies are illustrated in this section to show the implementation of the procedure for evaluating disaster resilience. The first case is a loss estimation study of a specific hospital; it is aimed to provide a more accurate evaluation of economic losses for buildings located at specific sites. In this case, an accurate analysis was performed using nonlinear dynamic analysis with an adequate description of limit state thresholds and their variability.

The second case is a regional loss estimation study aimed to evaluate the economic losses of a hospital network within a geographical region, such as a city (in this case Memphis, Tennessee). The responses of the buildings were estimated using an equivalent linearization spectral capacity method as presented by Reinhorn et al. (2001) similar to the procedure described in HAZUS. The limit states were expressed in terms of median and log-standard deviation chosen according to the building type and the design code (FEMA, 2005).
2.12 Example 1: Demonstration Hospital

The methodology described above has been applied to a hospital, an essential facility in the San Fernando Valley in Southern California, chosen as a typical case study for MCEER demonstration project. The hospital was constructed in the early 1970s to meet the seismic requirements of the 1970 Uniform Building Code (ICBO 1970, Yang et al., 2003).

It was selected since it is a complex structure with impact and implications related to various levels of functionality of services and structural safety (Reinhorn et al. 2005; Viti et al. 2006).

2.12.1.1 Description of the structural characteristic of the hospital

The structure is a four-story steel framed building with plan dimensions of $83.90 \times 17.25 \text{ m}$ ($275 \times 56.5 \text{ ft}$) (see Figure 2-11). It is rectangular in plan, with one small penthouse in the central part of the building. The height of the building, from grade level to the roof, is $15.54 \text{ m}$ ($51 \text{ ft}$).

The lateral force resisting system is comprised of four moment-resisting frames (three-bay) located on Lines B, F, J, and N in the North-South direction and two perimeter moment-resisting frames in the East-West direction on Lines 2 and 5 (Figure 2-11b).

The floor framing consists of 14 cm (5.5-inch) thick concrete slabs on metal decking that span east west to floor beams that span north south to steel girders that span east west to steel columns. A typical beam-girder connection and a typical beam-column moment-resisting connection with CJP groove welds of the beam flanges to the column flanges and a bolted web tab connection are shown in Figure 2-12. The gravity load resisting columns, defined herein simply as “columns” are not part of the lateral force resisting system.
Figure 2-11 Typical Longitudinal frame (a) and plan view (b) (Half view) of the model of the California hospital

Figure 2-12 Typical beam girder connection (a); b) Typical beam column moment resisting connection (b)
Structural steel moment frames are constructed with ASTM A572 and A588 Grade 50 steel. ASTM A36 steel was used for the remaining steel beams, girders, and for small size gravity columns. The foundation system beneath the moment-frames columns is composed of $1.37\text{m}$ ($54\text{-in}$) deep grade beams spanning to piles located on Lines 2, 5, B, F, J and N. The piles are $30.48\times30.48\text{cm}$ ($12\times12\text{inches}$) square and typically $15.24\text{m}$ ($50\text{ feet}$) long and they are embedded $15.24\text{cm}$ (6 inches) in the bottom of the pile cap. The pile caps are typically $1.67\times1.67\text{m}$ ($66\times66\text{ in}$) square and $1.52\text{m}$ ($60\text{ in}$) deep. The pile cap is reinforced with bottom rebar only. The typical pile cap reinforcement is #10 bars at $30.48\text{cm}$ (12 in) on center, each way. Typical pile caps are supported on 4 piles. The modal properties of the hospital are reported in Table 2-3. Further details of the hospital can be found in Cimellaro et al. (2006a).

<table>
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<th>Freq. (Hz)</th>
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<th>N-S direction</th>
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<td>6.55</td>
<td>98.6</td>
<td>98.6</td>
</tr>
</tbody>
</table>

### 2.12.1.2 Numerical models of the hospital

First, an equivalent SDOF system has been modeled as elastic-perfectly plastic model with a yield shear strength $V_Y=0.62W$ calculated by non linear monotonic static analysis (Yang et al., 2003). The mass, the stiffness and the damping coefficient used for the SDOF model are respectively $m=3675.63\text{ kN sec}^2/\text{m}$ ($20.98\text{ kips sec}^2/\text{in}$), $k=191712.7\text{ kN/m}$ ($1094.27\text{ kips/in}$) and $c=2\zeta\sqrt{k\cdot m}=2654.4\text{ kN sec/m}$ ($15.151\text{ kips sec/in}$).

A simple plastic analysis (“shake down”) in the East-West and North-South direction has been performed (Yang et al., 2003) using two mechanisms and two lateral load profiles: (1) a normalized first mode load profile and (2) a constant acceleration profile. The collapse loads, $V_m$
for the building frame were established to be 26878 kN (6040 kips) (0.62W) and 32881 kN (7389 kips) (0.75W) in the north-south and east-west directions, respectively.

A MDOF numerical model was developed in IDARC2D (Reinhorn et al., 2004) and used to perform the nonlinear time history analysis of the hospital. Simplified plans and sections have been assumed to represent the structural framing system. Secondary framing such as framing around the elevator shell and the longitudinal beams between axis line 3 and 4 (Figure 2-11) were omitted from the model. The number of column and beam sections was reduced and idealized to simplify the analytical model. The real building has approximately twenty different sections for beams and columns. This number has been reduced to seven different sections for the beams, and five different sections for the columns. Since the structure is symmetric, only one-half of the building has been modeled and the columns located on the symmetric axis (frame H) have been characterized by half size of their mechanical properties, to take into account the effect of symmetry (see Figure 2-11 b).

The element properties of each element are specified in terms of moment-curvature relationships for each of their sections. The three characteristic points of moment-curvature diagram (see Figure 2-13) are the cracking, the yield and the ultimate levels. They have been derived based on section analysis, according with the requirements described in this study. Namely, the model was simplified by eliminating the points PCP and PCN, while PYP/N was calculated as the product of the plastic section modulus and the nominal yield stress, $f_{sy}$. The ultimate curvature was set to 50 times the yield curvature, and the post-elastic stiffness was set equal to 1% to the elastic stiffness.

Figure 2-13 Tri-linear moment curvature relationship in IDARC2D (Reinhorn et al., 2004)
Depending the ultimate curvature on the yield one, and, consequently, on the yield strength, it is different for the two steel classes, resulting to be equal to 6% for the A36 steel (secondary frames) and 9% for the grade 50 one, constituting the moment-resistant frames. Such last value, together with the assumed hardening ratio, provides consistent values for the ultimate moment resistance of the elements. A spread plasticity model has been assumed for the inelastic strain distribution. The plasticized length is determined by the ratio between the maximum bending moment value in the element and the yield one of the element itself, and the inelastic stress has assumed to have a linear distribution inside the plastic regions. The assumed hysteretic model does not assume any degradation in stiffness or in strength (Figure 2-13), but only a reduction in the hysteretic energy dissipated in each cycle at the developing of the cyclic excitation of the system.

The building is modeled as a series of plane frames linked by a rigid horizontal diaphragm, where each frame is in the same vertical frame, and no torsional effects are considered.

![Figure 2-14 Hysteretic model](image)

It is a two dimensional model where all moment resisting frames are modeled with rigid beam-column connections and other beam-column connections of all the non moment resisting frames (MRF) were assumed to be pinned.

### 2.12.1.3 Fragility analysis

A series of 100 synthetic near fault ground motions, described as the “MCEER series” (Wanitkorkul et al., 2005) corresponding to different return periods (250, 500, 1000 and 2500 years) has been used to determine the fragility curves of the building (Viti et al., 2006) using the procedure described by Cimellaro et al. (2006a). Losses have been determined according to HAZUS (FEMA, 2005). The structural losses for this type of building have obtained as 0.2%,
1.4%, 7.0%, and 14.0% of the building replacement costs for the cases of slight, moderate, extensive, and complete damage, respectively.

If a temporal trade-off is considered in performing a decision analysis, future costs have to be converted to net present values. Discounting is usually considered for the future value because the future cost is usually less “painful” than the present cost. According to FEMA 227 (FEMA, 1992), several different approaches have been used to estimate the discount rate for public investment, and the resulting discount rate ranges from 3% to 10%. In this case study, a discount annual rate of 4% and a depreciation annual rate of 1% are assumed.

![Exponential recovery](image)

Based on HAZUS (FEMA, 2005) the nonstructural losses have been calculated as 1.8%, 8.6%, 32.8%, and 86% of the building replacement costs for the four damage states. The percentage of patients injured for the different damage states is 0.05%, 0.23%, 1.1%, 6.02%, 75% (FEMA 2005) of the 400 patients assumed in the hospital and 100 outside the hospital. Other losses such as relocation costs, rental income losses and loss of income have also been considered using the procedure described in HAZUS for this type of building (COM6). Figure 2-15 shows $Q(t)$ related to the four hazard levels considered for exponential recovery functions given in Equation (2-13). The values of resilience for the four different hazard levels represented by probability of exceedance $P$ in 50 years are reported in Table 2-4. The resilience of the building is better with a less severe hazard level (20%PE), but it is almost constant with the increase of earthquake intensity showing a good behavior of the building (see Table 2-4). Comparing the recovery curves $Q(t)$ it is noted that there is a drop with increasing earthquake magnitude due to the increasing losses, as expected, and consequentially on the effective
recovery time (Figure 2-15). Resilience was calculated from the control time $T_{LC}$ equal to the maximum recovery period $T_{RE}$, or 297 days in this example. Since the recovery period $T_{RE}$ for each hazard is different, resilience function has little changes, implying that the structure has consistent design for various levels of hazards. When combining resilience associated with different hazard levels, a final value of 83.1% is obtained.

Table 2-4 Resilience and time of recovery vs. different hazard levels for MRF

<table>
<thead>
<tr>
<th>Probability of exceedance in 50 yrs (%)</th>
<th>Time of recovery (days)</th>
<th>Resilience (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>71</td>
<td>96.6</td>
</tr>
<tr>
<td>10</td>
<td>94</td>
<td>94.0</td>
</tr>
<tr>
<td>5</td>
<td>228</td>
<td>79.9</td>
</tr>
<tr>
<td>2</td>
<td>297</td>
<td>57.3</td>
</tr>
</tbody>
</table>

Furthermore, four different seismic retrofit schemes to improve the disaster resilience of the hospital were considered for this case study: a) Moment resisting frames (MRF); b) Buckling restrained braces; c) Shear walls and d) Weakening and Damping (Viti et al., 2006). All retrofit strategies have been optimized with the procedure described in Viti et al. (2006) and Cimellaro et al. (2006c).

Table 2-5 Resilience vs. different hazard levels for different Retrofit strategies

<table>
<thead>
<tr>
<th>Probability of exceedance in 50 yrs (%)</th>
<th>Resilience (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Moment Resisting Frames</td>
</tr>
<tr>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>20</td>
<td>96.6</td>
</tr>
<tr>
<td>10</td>
<td>94.0</td>
</tr>
<tr>
<td>5</td>
<td>79.9</td>
</tr>
<tr>
<td>2</td>
<td>57.3</td>
</tr>
<tr>
<td>Total Hazard</td>
<td><strong>83.1</strong></td>
</tr>
<tr>
<td>Loss of Resilience</td>
<td><strong>16.9</strong></td>
</tr>
</tbody>
</table>

Table 2-5 shows the values of resilience for the four different retrofit techniques and for different probabilities of exceedance. The resilience values shown in the last row of Table 2-5
consider the uncertainties of the ground motion parameters. All values of resilience are normalized respect to the control period $T_{LC}$ assumed equal to the largest recovery $T_{RE}$ time among the different retrofit techniques. All values of resilience are comparable because all techniques are equally effective in improving the resilience of the hospital.

![Resilience graph](image)

**Figure 2-16 Comparison of different rehabilitation strategies in term of disaster resilience**

The same values of Resilience (y-axis) as function of the annual probability of exceedance (x-axis) are shown in Figure 2-16. This shows that the best improvement in terms of resilience is obtained using a retrofit strategy based on weakening and damping. Although in term of resilience the difference seems small the loss term (complementary to resilience) shows clearly the advantage of W&D scheme. This retrofit technique produces both a reduction of displacements and of accelerations (Viti et al., 2006). The reduction of accelerations is important for hospitals, because many of building contents (nonstructural components) are acceleration sensitive.

2.13 Example 2: Retrofit of a hospital network

An example based on a series of hospital buildings described by Park et al. (2004) is chosen to illustrate how to apply the proposed resilience framework to a group of structures. The six hospital buildings are selected and examined as candidates for retrofit. Table 2-6 shows the structural type of the hospitals.
The location information (Figure 2-17) is used to define the seismic hazard (USGS, 2002), and the structural types are used to define the seismic vulnerability (HAZUS, 2005). The first four hospitals are mid-rise buildings with concrete shear walls (C2M as per HAZUS classification), the fifth is a low-rise building with Unreinforced Masonry Bearing Walls (URML), and the sixth is a low-rise building with concrete shear walls (C2L). Alternative retrofit actions are selected as defined in FEMA 276 (FEMA, 1999) and directly correlated to the HAZUS code levels. Therefore, the HAZUS code levels are assigned as performance measures (PM) to the retrofit strategies mentioned above with following assumptions: (i) It is assumed that the “No Action” option, corresponds to the “low” code level; (ii) “Retrofit to life safety level” option is assumed to be a “moderate” code level; and (iii) “Retrofit to immediate occupancy level” option is assumed to be a “high” code level. For the “rebuild option”, a special “high” code level is assumed because hospitals are classified as essential facilities. It should be noted that the fragility curves for C2L are used in the evaluation of the seismic alternatives for URML type structure, because other specific fragility curves are not available in HAZUS.
Table 2-6 Description of the buildings of the hospital network in case study 2 (Park, 2004)

<table>
<thead>
<tr>
<th>Hospital</th>
<th>Stories</th>
<th>Structural Type</th>
<th>HAZUS model type</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>(Mid-rise)</td>
<td>Concrete shear wall</td>
<td>C2M</td>
</tr>
<tr>
<td>B</td>
<td>(Mid-rise)</td>
<td>Concrete shear wall</td>
<td>C2M</td>
</tr>
<tr>
<td>C</td>
<td>(Mid-rise)</td>
<td>Concrete shear wall</td>
<td>C2M</td>
</tr>
<tr>
<td>D</td>
<td>(Mid-rise)</td>
<td>Concrete shear wall</td>
<td>C2M</td>
</tr>
<tr>
<td>E</td>
<td>(Low-rise)</td>
<td>Unreinforced Masonry Bearing Walls</td>
<td>URML</td>
</tr>
<tr>
<td>F</td>
<td>(Low-rise)</td>
<td>Concrete shear wall</td>
<td>C2L</td>
</tr>
</tbody>
</table>

Some basic values involved in the description of the system such as the number of occupants, building replacement value etc., are defined using standard references that are commonly used in seismic building loss estimation (HAZUS, 2005). The number of occupants per 1000 ft² is assumed to be 5 in daytime and 2 at night time (FEMA, 1992). The total number of patients inside the hospital based on the above assumptions for the three different types of structure is given in Table 2-7.

Table 2-7 Number of Occupants according to FEMA (1992)

<table>
<thead>
<tr>
<th>Strut. Type</th>
<th>DAY occupants</th>
<th>NIGHT Occupants</th>
</tr>
</thead>
<tbody>
<tr>
<td>C2M</td>
<td>2000</td>
<td>800</td>
</tr>
<tr>
<td>C2L</td>
<td>200</td>
<td>80</td>
</tr>
<tr>
<td>URML</td>
<td>200</td>
<td>80</td>
</tr>
</tbody>
</table>

2.13.1 Intensity measures and associated uncertainty evaluation

Response spectra, used as intensity measure (I), were generated for each of the six hospitals using the information obtained from USGS (2002). The variation of the spectral accelerations (calculated independently) over the different hospital locations appears to be insignificant, as the structures are located close to each other. Four hazard levels are considered for generation of the loss-hazard curves taking into account a range of levels of earthquakes in the region. These levels include earthquakes with 2%, 5%, 10% and 20% probability of exceedance \( P \) in 50 years. Note that these probability levels are assigned based on a 50 year time span, and should be modified when a different time span \( T_{LC} \) is used, as follows.
\[ P_{T_{LC}} = 1 - (1 - P_{50})^{\frac{T_{LC}}{50}} \]  

(2-48)

where \( P_{T_{LC}} \) is the probability of exceedance in a period \( T_{LC} \) (in yrs) for a particular intensity \( i^* \) of earthquake, and \( P_{50} \) is the probability of exceedance in 50 years for the same earthquake level. Therefore, the probability \( P(T_{LC} > i^*) \) that an earthquake of a given intensity occurs in a given control period \( T_{LC} \) can be adjusted according to Equation (2-48) and substituted to evaluate resilience in Equation (2-42) and (2-46). The control period of the system \( T_{LC} \) is assumed to be 30 years and, a discount rate \( r \) of 6% is assumed. The control time for the decision analysis is usually based on the decision maker’s interest in evaluating the retrofit alternatives. A 50 years control period could be chosen for evaluating the hospital systems, which may be consistent with the period used for calculation of earthquake hazards (e.g. as in 2% probability of exceedance in 50 years). However, a decision maker in charge with financing the retrofit could be interested in a shorter period, more in line with the life span of new construction. Generally, seismic losses associated with seismic vulnerable structures increases if longer control periods are considered. For example, retrofit can hardly be justified for a one-year period because the probability of encountering a large earthquake within this period is very low, whereas the probability increases appreciably for a 50-year period, so the retrofit becomes more cost-effective in reducing losses. A decision maker siding with the users’ community could be interested therefore in a longer \( T_{LC} \). In this example, a control period of 30-years is assumed for \( T_{LC} \) as the baseline value in line with the lifespan of the structure as mentioned above.

2.13.2 Performance levels

As indicated before, four alternative actions related to retrofit are considered for each structural type: 1) no action; 2) rehabilitation to life safety level; 3) retrofit to the immediate occupancy level; 4) construction of a new building. The retrofit levels are, as defined in FEMA 276 (1999), the target performance expected for earthquake rehabilitation. The cost of seismic retrofit for building systems depends on numerous factors, such as building type, earthquake hazard level, desired performance level, occupancy or usage type. These costs generally increase as the target performance level becomes higher (e.g. rehabilitation to “immediate occupancy” level would obviously require more initial costs for retrofit than the retrofit to “life safety” level). On the contrary, with higher performance levels less seismic losses are expected. The initial
retrofit costs for the options considered here are obtained from FEMA 227 (1992) and FEMA 156 (1995), which provides typical costs for rehabilitation of existing structures taking into account above-mentioned factors.

2.13.3 Response evaluation

The maximum building response of these hospitals, which is used in the structural evaluation, is obtained from the intersection of the demand spectrum and the building capacity curve, which is determined from a nonlinear static pushover analysis (Reinhorn et al., 2001, HAZUS, 2005). The maximum building response is used in conjunction with the fragility curves to obtain the damage probability distributions (probability of being in or exceeding various damage states).

2.13.4 Fragility curves

Damage fragility curves are generated for both structural and nonstructural damage, using HAZUS assessment data. The nonstructural damage fragility curves consist of acceleration-sensitive components and drift sensitive components (HAZUS, 2005). In this way the structural, the nonstructural acceleration sensitive, and the drift-sensitive damage, can be assessed separately using their respective fragility curves. In this example both structural and nonstructural damage fragility curves for C2L, C2M and URML type structures for different code levels are generated.

Then, the multidimensional fragility curves are obtained by combining both structural and nonstructural fragility curves, following the procedure described by Equation(2-21) (Cimellaro et al., 2006a). Figure 2-18 shows the multidimensional fragility curves for C2M type structure related to the four different retrofit options and four different damage states. The other fragility curves related to other structural types are reported in the Appendix C.

The hazard level is shown along the x-axis as a function of the return period that takes to account the uncertainties in estimating the ground motion intensity at the site, which has been considered as a random variable, by performing a Probabilistic Seismic Hazard Analysis. As shown, different actions strategies lead to a move of the fragility curves to the right, indicating reduction of probability of “failure” for a specific seismic hazard.
Figure 2-18 Multidimensional fragility curves for C2M structure: (a) No Action; (b) Rehabilitation Life Safety (c) Rehabilitation Immediate Occupancy; (d) Rebuild
Figure 2-19 shows the structural performance (damage) probability distributions for C2M type structure for different retrofit strategies for a control period of 50 years.

Figure 2-19 Structural performance (damage) distribution for “No action” for C2M structures – ($T_{lc}=50$yrs)

Figure 2-20 shows the overall distributions for the C2M structures within a 30 years period, compared with a 50 years period. As expected, the probability of having no damage increases with the reduced control period. More details can be found in Cimellaro et al. (2009). The other performance distributions related to different actions and structural types are reported in the Appendix C.
2.13.5 Seismic losses

Among the large number of seismic losses described in the previous sections, several attributes that are typically considered to be crucial for hospital systems are selected for this study and are listed in Table 2-8 along with a brief explanation of each parameter. This list is valid for this case study and it can be different according to the decision maker’s choice. For example, “loss of income” is excluded because it is relatively less important in calculation of
monetary loss for the (hospital) system (less than 5% of the total monetary loss). In this case, it is assumed that the decision is taken by a public policy maker, who might be less concerned about hospital’s income, when compared to a hospital administrator. It is important to mention that losses in undamaged sectors of the hospital due to business interruption are not considered in this example.

Table 2-8 Losses considered in Example 2

<table>
<thead>
<tr>
<th>Category</th>
<th>Loss</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Structural Losses (LS)</td>
<td>Initial Cost</td>
<td>Cost of seismic rehabilitation or constructing a new building to improve structural performance</td>
</tr>
<tr>
<td></td>
<td>Structural Repair Cost</td>
<td>Cost for repairing damage to structural components such as beams, columns, joints, etc.</td>
</tr>
<tr>
<td>Direct Economic losses (LNS,DE)</td>
<td>Non Structural Repair costs</td>
<td>Cost for repairing damage to nonstructural components such as architectural, electrical and mechanical items.</td>
</tr>
<tr>
<td></td>
<td>Loss of Building contents</td>
<td>Cost equivalent to the loss of building contents such as furniture, equipment (not connected to the structure), computers, etc.</td>
</tr>
<tr>
<td>Indirect economic losses (LNS,IE)</td>
<td>Relocation Expenses</td>
<td>Disruption cost and rental cost for using temporary space in case the building must be shut down for repair</td>
</tr>
<tr>
<td>Indirect Casualties losses (LNS,IC)</td>
<td>Loss of functionality</td>
<td>Loss of function for an hospital may result in additional human life losses due to lack of medical activities and capability</td>
</tr>
<tr>
<td>Direct Casualties losses (LNS,DC)</td>
<td>Death</td>
<td>Number of deaths</td>
</tr>
<tr>
<td></td>
<td>Injury</td>
<td>Number of seriously injured</td>
</tr>
</tbody>
</table>

Using the performance (damage) probability distributions listed in the previous section, various seismic losses associated with the system are estimated. Table 2-9 shows the deterministic relationship between various damage states and the corresponding normalized seismic losses that are estimated from the fragility curves of the system for C2M type structure. Losses are estimated for the four earthquake levels and loss hazard curves are generated in order to calculate the overall expected losses (not shown). In Table 2-9 distinction is made between number of death and injuries.
Table 2-9 Normalized losses (ratios) for different damage states of C2M buildings (HAZUS, 2005)

<table>
<thead>
<tr>
<th></th>
<th>Slight</th>
<th>Moderate</th>
<th>Extensive</th>
<th>Complete</th>
<th>Complete ($/ft^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LS</td>
<td>0.0176</td>
<td>0.1000</td>
<td>0.500000</td>
<td>1.0000</td>
<td>17.0</td>
</tr>
<tr>
<td>LNS,DE</td>
<td>0.0190</td>
<td>0.1000</td>
<td>0.500000</td>
<td>1.000000</td>
<td>42.0</td>
</tr>
<tr>
<td>LNS,DC</td>
<td>0.0194</td>
<td>0.1000</td>
<td>0.300000</td>
<td>1.000000</td>
<td>62.0</td>
</tr>
<tr>
<td>Contents Loss</td>
<td>0.0200</td>
<td>0.1000</td>
<td>0.500000</td>
<td>1.000000</td>
<td>60.5</td>
</tr>
<tr>
<td>Death*</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.000015</td>
<td>0.125000</td>
<td></td>
</tr>
<tr>
<td>Injury*</td>
<td>0.0000</td>
<td>0.0003</td>
<td>0.001005</td>
<td>0.225000</td>
<td></td>
</tr>
<tr>
<td>Recovery Time</td>
<td>2</td>
<td>68</td>
<td>270</td>
<td>360</td>
<td></td>
</tr>
</tbody>
</table>

* From Park (2004)

As described in Table 2-8, the losses used in this case study should take into account the fact that loss of function in a hospital, may result in additional loss of life. Using the conversion factor of $C_F=$$100,000/day to recover 10,000ft$^2$, the normalized losses in Table 2-9 are determined. Then, losses are combined using the procedure described in the previous section 2.5.

Figure 2-21 Functionality curves: (a) C2M; (b) C2L and (c) URML type structure
2.13.6 Disaster resilience

The disaster resilience value is calculated according to Equation (2-1). The functionality curves for different structural type buildings related to different damage states are shown in Table 2-10, showing the dependence of $Q(t)$ on the performance measure expressed through damage state (PM) and on the seismic input $I$.

The expected equivalent earthquake losses for each rehabilitation scheme are shown in the third column of Table 2-11, which are obtained considering the probability of each level of the earthquake, along with the initial rehabilitation costs, followed by the total expected losses considering an observation period $T_{LC}$ of 30 years.

Table 2-10 Disaster resilience of individual buildings without any rehabilitation action for

$$T_{LC} = \max (T_{re,i}) = 360 \text{ days}$$

<table>
<thead>
<tr>
<th>Damage State</th>
<th>C2M Recovery Time - $T_{re}$ (days)</th>
<th>Normalized Resilience - $R_{es}$ (%)</th>
<th>C2L Recovery Time - $T_{re}$ (days)</th>
<th>Normalized Resilience - $R_{es}$ (%)</th>
<th>URML Recovery Time - $T_{re}$ (days)</th>
<th>Normalized Resilience - $R_{es}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slight</td>
<td>2</td>
<td>99.9 %</td>
<td>20</td>
<td>99.9 %</td>
<td>2</td>
<td>99.9 %</td>
</tr>
<tr>
<td>Moderate</td>
<td>68</td>
<td>98.2 %</td>
<td>68</td>
<td>98.2 %</td>
<td>68</td>
<td>98.3 %</td>
</tr>
<tr>
<td>Extensive</td>
<td>270</td>
<td>67.8 %</td>
<td>270</td>
<td>67.8 %</td>
<td>270</td>
<td>74.7 %</td>
</tr>
<tr>
<td>Complete collapse</td>
<td>360</td>
<td>0.0 %</td>
<td>360</td>
<td>0.0 %</td>
<td>360</td>
<td>0.0 %</td>
</tr>
</tbody>
</table>

If uncertainties in the seismic input are considered by using four different hazard levels, then resilience can be evaluated using Equation (2-1) for different rehabilitation strategies and compared as shown in Figure 2-22. The initial costs of rehabilitation for different rehabilitation strategies, the expected equivalent earthquake loss and the total costs (including the initial costs of the entire system that is estimated equal to 87.3 Million $) are all reported in Table 2-11.
The recovery time and resilience values are summarized in Table 2-11. For this case study it is shown that the Rebuild option has the largest disaster resilience of 98.7%, when compared with the other three strategies, but it is also the most expensive solution ($ 92.3 millions). However, if No Action is taken the disaster resilience is still reasonably high (65.0%). As shown in this case study, initial investments and resilience are not linearly related. When the functionality Q(t) is very high, improving it by a small amount requires investing a very large amount compared with the case when the function Q(t) of the system is low. Although this is an obviously expected engineering outcome, the procedure presented here provides a quantification, which may be used by decision makers.
Figure 2-22 Functionality curves: (a) No Action; (b) Life Safety Rehabilitation; (c) Immediate Occupancy Rehabilitation; (d) Rebuild for entire Hospital system
2.14 Summary and Remarks

The definition of disaster resilience combines information from technical and organizational fields, from seismology and earthquake engineering to social science and economics. Many assumptions and interpretations have to be made in the study of disaster resilience. However, the final goal is to integrate the information from these different fields into a unique function leading to results that are unbiased by uninformed intuition or preconceived notions of risk. The goal of this chapter has been to provide a framework for quantitative definition of resilience using an analytical function that may fit both technical and organizational issues. The fundamental concepts of disaster resilience discussed herein provide a common frame of reference and a unified terminology. Two applications of this methodology to health care facilities are presented in order to show the implementations issues. However, it is important to note that the assumptions made herein are only representative for the cases presented. For other problems, users can focus on those assumptions that are mostly affecting the problem at hand, while using the case study as guidance.
SECTION 3

ORGANIZATIONAL RESILIENCE: APPLICATION TO A HEALTH CARE SYSTEM

3.1 Introduction

Health care facilities have been recognized as strategic buildings in hazardous events and play a key role in the disaster rescues; however, no attempt to practically relate the structural damage on the organizational aspects has been proposed so far. As shown in section 2, there is an extensive literature review in the definition of the main parameters of disaster resilience for health care systems and in the definition of the general framework, but no references have been found regarding the modeling and the measure of the organizational aspects of resilience.

Indeed, an organizational resilience is needed, to be able to valuate the response of the community to hazardous events, and evaluate the real loss in terms of healthy population and quality of care provided.

In this section, an organizational model describing the response of the hospital emergency Department has been implemented. The model wants to offer a more comprehensive valuation of the multidimensional aspects of resilience.

This hybrid simulation/analytical model is able to estimate the hospital capacity and dynamic response in real time (‘on line’) and incorporate the influence of the facility damage of structural and non-structural components on the organizational ones. The waiting time is the main parameter of response and it is used to evaluate disaster resilience of health care facilities. A double exponential function is formulated and its parameters are calibrated ‘off line’ based on simulated data. The metamodel has been designated to cover a large range of hospitals configurations and takes into account hospital resources, in terms of staff and infrastructures, operational efficiency. The sensitivity of the model to different arrival rates, hospital configurations, and capacities and the technical and organizational policies applied during and before the strike of the disaster has been investigated.

The hospital metamodel has been used to an hospital network of two hospitals to analyze the variation of resilience in the case of collaboration of two health care facilities in coping with a disaster. The model takes into account also the damage of the network, the transport time of the
patient, the relative distance in between and the presence of an operative center, coordinating the dispatch of the injured.

Uncertainties associated to the uncertain nature of the disaster (e.g. earthquakes, hurricane etc.) and to the influence of the structural damage on the organizational model, are included in the formulation.

3.2 Technical and Organizational resilience

The main purpose of this study is to relate the technical and organizational aspects of health care facilities, to obtain a measure of organizational resilience that has not been attempted so far. The goal is to relate the measure of resilience to the quality of care provided and the eventual loss of healthy population, caused by the performance of the health care facility during the disaster.

Technical resilience as described in section 2 in Equation (2-1) is defined as the integral of the normalized function \( Q(t) \) indicating capability to sustain a level of functionality, or performance over a control period of time \( T_{LC} \). In other words, it describes the ability to recover from a disastrous event.

In this section technical aspects are combined with organizational aspects and the formulation of organizational resilience for a hospital facility is provided using a hybrid simulation analytical model (metamodel) that is able to describe the response of the Emergency Department during an hazardous event (section 3.6).

The system diagram in Figure 3-1 identifies the key steps of the framework to quantify resilience. The left part of the diagram mainly describes the steps to quantify technical aspects of resilience while the right side describes the organizational aspects to quantify resilience. The penalty factors \( PF \) that appear in the diagram describe the interaction between the technical and the organizational aspects and their evaluation is discussed in section 3.7.
3.3 Functionality of a hospital

In order to define resilience it is necessary to define first the functionality $Q$ of the hospital facility.

Three different definitions of functionality and its components are discussed below:

- a qualitative functionality (section 3.3.1) related to the quality of service;
- a quantitative functionality (section 3.3.2) related to the losses in healthy population;
- a combined generalized functionality (section 3.3.3) related to both qualitative, (i.e. the quality of service) and quantitative services.
3.3.1 Qualitative Functionality

As it will be discussed in the following sections (section 3.4), the quality of service can be defined using the waiting time (WT) spent by patients in the emergency room before receiving care. The WT is the main parameter to valuate the response of the hospital during normal and hazardous event operating conditions.

Common sense, but also a relevant literature review reported in various references (Maxwell, 1984; McCarthy et al. 2000; Vieth and Rhodes, 2006) indicates that functionality of a hospital is definitely related to the quality of service (QS). Therefore if a measure of QS is found then it is possible to measure the functionality Q of the health care facility.

Maxwell (1984) identified several multidimensional aspects of the quality of service. In particular six dimensions were suggested, such as the access to care, the relevance of need, the effectiveness of care, the equity of treatment, the social acceptability and the efficiency and economy. Each dimension needs to be recognized and requires different measures and different assessment skills.

In this study, the access to services is considered as the most important dimension to measure the QS in emergency conditions and it should be assessed in terms of ambulance response time and waiting time in the Emergency Department. Moreover, other researchers (McCarthy et al., 2000) pointed out the choice of the waiting time as an indicator of quality of service. Therefore, based on the references above, the qualitative functionality has been defined as

\[ Q_{QS}(t) = (1-\alpha)Q_{QS,1}(t) + \alpha Q_{QS,2}(t) \] (3-1)

Eq. (3-1) is a linear combination of two functions, \( Q_{QS,1}(t) \) and \( Q_{QS,2}(t) \), expressed in equation (3-2) and equation (3-3) respectively, while \( \alpha \) is a weight factor that combine the two functions describing the behavior in saturated and non saturated conditions. In particular, in non saturated condition, when \( \lambda \leq \lambda_U \), the quality of care is expressed by the function \( Q_{QS,1}(t) \), equal to

\[ Q_{QS,1}(t) = \max \left( \left( \frac{WT_{crit} - WT(t)}{WT_{crit}}, 0 \right) \right) \text{ if } \lambda \leq \lambda_u \] (3-2)

where all the following quantities are defined analytically in section 3.6.3.4

\( WT_{crit} \) = critical waiting time of the hospital in saturated conditions, when \( \lambda = \lambda_U \);
\[ WT(t) = \text{waiting time when } \lambda = \lambda(t) \]

The loss of healthy population is related to the patients that are not treated, so in saturated condition when \( \lambda > \lambda_U \), the function \( Q_{QS,2}(t) \) can be written as

\[
Q_{QS,2}(t) = \frac{WT_{\text{crit}}}{\max \left( WT_{\text{crit}}, WT(t) \right)} \quad \lambda > \lambda_U \tag{3-3}
\]

\( Q_{QS,1}(t) \) and \( Q_{QS,2}(t) \) are showed in Figure 3-2, while their combination is showed in Figure 3-3. It is important to mention that QS is a good indicator of functionality only in pre-saturated conditions. The saturated condition is the subject of the next section.
Figure 3-2 Functionality related to waiting time for a hospital with 500 beds, 15 Operating Room, 600 operations per operating room per year, for the arrival rate of Northridge.
Functionality in post saturated condition

\[ Q(t) = 1 - L(t) \]  \hspace{1cm} (3-4)

where the loss function is defined by the normalized patients not treated

\[ L(t) = \frac{N_{NTR}(t)}{N_{tot}(t)} \]  \hspace{1cm} (3-5)

3.3.2 Quantitative Functionality

The literature review does not consider the evaluation of the performance of the hospital in saturated condition, when the maximum capacity of the hospital is reached. In this last condition the hospital is not able to guarantee a normal level of \( QS \), because the main goal now is to provide treatment to the most number of patients. Therefore, in this case the number of patients treated \( N_{TR} \) is a good indicator of functionality \( Q \).

The quantitative functionality \( Q_{LS}(t) \) is then defined as a function of the losses \( L(t) \), which are defined as the total number of patients not treated \( N_{NTR} \) versus the total number of patients receiving treatment \( N_{tot} \). In this case the loss is given by the number of patients that are not treated, as follows

Figure 3-3 Influence of the weighting factor on the valuation of the total functionality related to waiting time
The total number of patients requiring care \( N_{tot} \) is given by the following formula

\[
N_{tot}(t) = \int_{t_0}^{t+t_0} \lambda(\tau) \cdot d\tau 
\]

while the total number of patients that do not receive treatment \( N_{NTR} \) are

\[
N_{NTR}(t) = N_{tot} - N_{TR}(t) = N_{tot} - \int_{t_0}^{t+t_0} \min(\lambda(\tau), \lambda_u) \cdot d\tau 
\]

The quantitative functionality thus can be defined as

\[
Q_{LS}(t) = 1 - L(t) = 1 - \frac{N_{NTR}(t)}{N_{tot}(t)} = \frac{N_{TR}(t)}{N_{tot}(t)} 
\]

where the expression of functionality is similar as Equation (2-2) given in previous chapter. The number of patients waiting for care depends on the queue already present in the emergency room. Previous patients control the delay between their arrival and the treatment. Hospital is fully functional when is able to absorb with a minimum delay all the patients requiring care. When the number of patients waiting is higher than the number of patients treated the functionality decreases.

The time variation of \( N_{TR}(t) \) and \( N_{tot}(t) \) for a big size hospital (500 beds), with high surge capacity (15 operating rooms) and the highest class of efficiency (1200 operation per operating room per year) under the Northridge arrival rate (Table 3-3) are shown in Figure 3-4.
If the number of patients that can be treated is larger to the number of patients that arrive then the quantitative functionality, $Q_{LS}$, is equal to one, because the capacity of absorbing the flow is higher than the actual arrival rate. If the treatment rate capacity is smaller than the arrival rate the quantitative functionality takes a value smaller than one. The quantitative functionality $Q_{LS}$ for a big size hospital (500 beds), high surge capacity (15 operating rooms) and highest class of efficiency (1200 operation per operating room per year) is presented for example is shown in Figure 3-5.

Figure 3-4 Number of patients treated during Northridge arrival rate

Figure 3-5 Quantitative functionality for a hospital with 500 beds, 15 OR and 1200 class of efficiency
3.3.3 Combined generalized functionality related for qualitative and quantitative services

The total functionality $Q(t)$ of the hospital is given by

$$Q(t) = Q_{OS}(t) \cdot Q_{LS}(t)$$

(3-9)

where

$Q_{OS}(t) = \text{qualitative functionality related to the quality of service (section 3.3.1);}$

$Q_{LS}(t) = \text{quantitative functionality related to the losses in terms of healthy population (section 3.3.2).}$

Equation (3-9) is the first approximated combination of these two correlated quantities (higher terms should be considered for complete analysis). The combined functionality and the sensitivity to the weighting factor of the qualitative part $Q_{OS}(t)$ is shown in Figure 3-6. It is important to mention that both $Q_{OS}$ and quantitative $Q_{LS}$ functionality require the estimation of the waiting time. The importance of this parameter is described in section 3.4, while its evaluation is given in section 3.6.

![Figure 3-6 Influence of the weighting factor on the valuation of the combined functionality](image)

Figure 3-6 Influence of the weighting factor on the valuation of the combined functionality
3.4 Waiting time as measure of quality of service

The first issue to solve when approaching the problem of modelling of a health care system is defining the main parameter of response that can be used to measure the functionality of a hospital.

A well-acknowledged study (Maxwell, 1984) has demonstrated that the waiting time in an emergency department may be used as a key parameter in the quantification of the quality of service in health care settings. WT is defined as the time elapsed between the received request of care by the hospital and the provision of the care to the patient.

Maxwell (1984) reports that the waiting time and non-attendance are important indicators of the quality of service. They can be used as a measure of the accessibility, efficiency, and relevance of the outpatient service.

Thompson and Yarnold (1995) proved the validity of the disconfirmation paradigm, according to which satisfaction is a function of the magnitude and direction of the difference between perceived service and expected service. The study demonstrates the validity of this paradigm in relating patient satisfaction to waiting time perceptions and expectations. Furthermore, it emphasizes that achieving satisfaction in a service encounter necessitates that perceived performance meet or exceed patient expectations.

Thompson et al. (1996) recognize that the waiting time is considered an important determinant of patient satisfaction, which results from meeting or exceeding patient expectations, but providing information, projecting expressive quality, and managing waiting time perceptions and expectations may be a more effective strategy to achieve improved patient satisfaction in the ED than decreasing actual waiting time.

McCarthy et al. (2000) studied outpatient-waiting times, their association with perceptions of quality of service and if concern about waiting times influences attendance. Patient satisfaction can be considered with the framework of Donabedian’s three markers of quality (McCarthy, 2000):

1. Structure (resources and facilities);
2. Process (longer and more informative medical consultations);
3. Outcomes of care (higher levels of patient adherence to health recommendations and higher levels of health and well being);

McCarthy’s research (2000) demonstrates that outpatient satisfaction with clinic treatment was not associated with waiting times, but lengthy waiting times in outpatient clinics are recognized to be a challenge to the quality of care.

Waiting time is related to the hospital resources, in particular to those of the emergency department, such as stuff on duty, number of labs and operating rooms (OR) grade of utilization of the OR, but also to the degree of crowding (Vieth and Rhodes, 2006) of the Emergency Department.

Richards et al. (2006) point out that the main factors, which may influence the waiting time are:

1. the arrival mode, which is a statistically significant predictor (e.g. those who arrive by ambulance had the shortest waiting time);
2. the hospital staffing characteristics (patient/physician ratio and patient/triage nurse staffing ratio), the race, ethnicity, payer source;
3. the metropolitan location of the hospital, triage category, gender, age, arrival time.

On the other side the waiting time influences patient satisfaction, the decision to utilize an emergency department or to leave without been seen (WBS). They also provide a classification of immediacy with which the patient should be seen or triage category, reported in Table 3-1.

Table 3-1 Maximum waiting time for different patient types (Richard et al., 2006)

<table>
<thead>
<tr>
<th>Kind of patient</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>EMERGENT</td>
<td>&lt; 15 min</td>
</tr>
<tr>
<td>URGENT</td>
<td>15 – 60 min</td>
</tr>
<tr>
<td>SEMIURGENT</td>
<td>1 – 2 h</td>
</tr>
<tr>
<td>NON URGENT</td>
<td>2 – 24 h</td>
</tr>
</tbody>
</table>

The waiting time influences also the quality and the time devoted by the staff to each patient (even if not always this time has to be related to the quality of the service provided), the access and the availability of pharmaceutics and, in the most critical case, may affect the state of care of the patients already inside the hospital. When a disaster happens, in order to provide a higher availability of beds and staff to new patients, the emergency strategy of the hospital may involve
the premature discharge of those inpatients whose conditions are considered stable, but who would remained hospitalized in normal operational conditions.

Di Bartolomeo et al. (2007) present an attempt to validate process indicators (PI) for trauma care in the settings they are applied for quality assurance (QA) and quality improvement (QI). The pre – hospital time (PT) and the Emergency Department disposition time (EDt) have been chosen as possible PI. This choice is based on the generally acknowledged principle that time to receive care is an essential component of the survival chain. Short waiting times improve outcomes, imply patients at higher risk where care is expedited all along the line and patients at lower risk where care runs naturally and easily fast. On the other side, long waiting times worsen outcomes, imply patients at lower risk where care is slowed down, imply high-risk patients whose complexity inevitably prolongs time.

The authors point out the existence of the complex relationship between the time to care and the patients’ outcome: a higher risk of unfavorable patient outcome was observed in hospitals with longer intervals to definitive care. However, this direct proportionality was no longer present after adjusting the statistical data for confounding factors. Even if the utility of the waiting time as a process indicator is in doubt, no other indicator is proposed.

3.5 Modelling health care facilities

Modelling, especially scientific modelling, refers to the process of generating a model as a conceptual representation of some phenomenon or a system. Health care systems are inherently complicated (Taylor and Lane, 1998), in terms of details, dynamic and organizational aspects, because of the existence of multiple variables, which potentially can produce an enormous number of connections and effects. The presence of relationships not obvious over time, the difficulties (or impossibility) to quantify some variables (e.g. the quality and value of treatment, the waiting time and patient expectation on emergency admission), are only few factors that affect the error in the valuation of the actual response. Furthermore, in a disaster, the emergency adds more complexity to the health care system. The increase of the patient flow, the consequent crowding of the emergency department, the chaos and disorganization that may result from the resuscitation of a patient in extremis are the most stressful conditions in a hospital.
3.5.1 Crisis vs. disaster

In order to understand the emergency condition, it’s necessary to clarify the difference between crisis and disaster.

A sudden change of the normal flow and composition of patients that arrive at the emergency department, caused by a hazardous event, may result in an internal crisis or, in the worst case, in a complete disaster. Stenberg (2003) makes a clear distinction between the two.

When a “sudden-onset” event strikes (smoke spreads through the corridor or tremors shake the building etc.), the hospital enters a period of crisis: normal operations are thus disrupted. This condition can last a period of time, which varies between zero and few days.

The hospital must respond with actions that have to minimize the losses in terms of human treats and functionality, reduce danger to occupants and maintain patient care.

When these actions are not effective or not sufficient or there is not enough time to make them work, the crisis turns into a disaster.

The disaster is defined as a crisis that has gone out of control.

In the United States, the Joint Commission on Accreditation of Healthcare Organizations (JCAHO) defines two kinds of disasters: an internal and an external one. In the former the hospital facility itself is affected, disrupting the normal operations while in the latter an external event that brings an unusual flow of patients requiring care, does not affect the hospital facility directly but imposes a sudden demand for emergency services. An earthquake is one of the rare kinds of disaster that is simultaneously internal and external (Reitherman, 1986).

There are two ways to cope with disasters: avoiding the crisis, i.e. making the facility less vulnerable in order to “mitigate” the possible effects of hazard (correct seismic design, an optimal choice of the location of the hospital, organization of a “network” response with other adjacent facilities etc.) or “preparing” by providing resources and capabilities to control the crisis and to prevent it from turning into disaster.

Both mitigation and preparedness have necessary to take place in the normal, pre – crisis condition (Stenberg, 2003). Although mitigation is an effective tool in the emergency response, it cannot be used extensively in disaster planning. On the other side, preparedness offers a larger field of intervention.

Before crisis, the hospital should plan and carry out actions to be able to improve the absorbing and buffering capacity and minimizing the losses during the disrupting event.
Though most crisis events are in some extent unknowable ahead of time, they can be partially predicted.

In similar types of emergencies, some recurrent patterns and problems can offer a good starting point in the foreknowledge of the crisis. On the other side, various kinds of uncertainties must be considered. Sternberg (2003) categorizes them as follows:

1. **incidental uncertainties**, such as initiating cause, time of the day, and location within the hospital;
2. **sequential uncertainties**, related to the fact that crisis also are subjected to unpredictable chains of occurrences;
3. **informational uncertainties** related to the chain in the transmission of information;
4. **organizational uncertainties**, which may arise from personnel characteristics, administrative features, multi agency and multi – organizational capacities;
5. **cascade uncertainties** which may arise from a series of events that become even more unpredictable, because failure in one system subvert other systems (e.g. cascade failures in utilities and equipment can throw the facility into turmoil).

Auf der Heide’s (in Stenberg, 2003) introduced the *bipartite strategy*, moving from the observations that there are some recurrent patterns and problems and on the other side disaster invariability produce unexpected challenges that call for flexibility and that require innovation. This strategy is based on the assumption that planners must prepare through *anticipation*, which “provides capabilities by which to manage crisis and prevent it from turning into disaster”, and *resilience* which is “the capacity to adapt existing resources and skills to new situations and operating conditions” (Bruneau *et al.*, 2003).

Planning the emergency preparedness is a way to “anticipate” and it is meant as a holistic process that includes activities aimed at improving emergency response (Adini *et al.* 2006).

Any disaster/emergency could result in a loss of human lives as well as a huge waste of resources. However, well-designed disaster preparedness with a well-planned effective response can help to reduce such loss.

Response action has to minimize the disruption, counter human threats, reduce damages to occupants, eventually move patients to safer location, and/or take other actions that maintain patient care.
Effective response is based on a pre–designed contingency plan, which maps the various activities that will operate during an emergency (Adini et al., 2006).

The valuation of the hospital capability to handle with the crisis is given by an objective assessment of its actual level of readiness. Only being able to predict the behavior during the crisis, it is possible to figure out which the weak rings in the response chain are and to start the disaster planning. As Adini et al. (2006) point out, comprehensive efforts to develop key indicators for the assessment of the emergency preparedness are underway. However, the assessment of emergency preparedness should include elements of disaster planning and emergency coordination, a well-organized internal and external communication, the capacity of expansion of the surge laboratory capacity, appropriate preparation of staffing and personnel, availability of equipment and stockpiles of pharmaceutics.

To evaluate the effectiveness of all these factors, a hospital model able to estimate the response of the health care system in real time and capable to take into account human and technical factors during a disaster situation is needed. Disaster response is time sensitive and decision makers need to be updated on the latest emergency. Thus, a model to obtain dynamic capacity in quantitative analysis of hospital operations is necessary.

3.5.2 Literature review of hospital operation modelling

A variety of modelling methods are available in literature to represent hospital operations that are summarized in Table 3.2 and divided by model type. For each group the best field area of application of the model together with advantages and disadvantages are indicated. Eight main groups of hospitals models can be found and they are herein briefly presented and discussed.

Among all the models described, the metamodel is the simplest because it requires less computational efforts, it is reliable and it is able to describe the dynamic behavior during the transient.
### Table 3-2 Summary of hospital models

<table>
<thead>
<tr>
<th>No</th>
<th>Model Type</th>
<th>Areas of utilization</th>
<th>Advantages</th>
<th>Disadvantages</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Mental Models (MM)</td>
<td>Likelihood of cause – effect relationships Interactions between humans and system</td>
<td>Comparison of management policies</td>
<td>High sensitive Dependent on their usage rate Lack of defined boundaries</td>
<td>Sinreich et al., (2004)</td>
</tr>
<tr>
<td>8</td>
<td>Metamodels</td>
<td>Waiting time modelling Efficiency evaluation</td>
<td>Suitable for a set of similar system</td>
<td>Based on other modelling tool such as simulation</td>
<td>Gonzalez et al. (1997) Barton (1998)</td>
</tr>
</tbody>
</table>
1. Conceptual models

These models are constructed to enable reasoning within an idealized logical framework. They describe the input parameters and the objectives of the analysis; therefore, they offer a good theoretical guide for further developments of more detailed models. A conceptual model has an ontology that is the set of expressions in the model, which are intended to denote some aspects of the modeled object. They draw a high-level sketch of hospital operations: qualitative relationships between the components of the model are presented and checks of different control actions of the management system are possible.

![Figure 3-7 Example of a Hospital conceptual model](image)

Although they great capability of covering difficult problems and their power of generalization, they are not suitable for the detailed description of the system in emergency conditions.

2. Mental models (MM)

A Mental Model is a combination of the individual’s subjective perceptions, concepts, ideas and perceived system status (Sinreich et al., 2004). It provides a subjective internal representation of the system, able to assess the likelihood of cause – effect events and to forecast the system behavior based on the current system state. In the field of health care system analysis, they are employed as Management’s Operation Policy (MOP) models: they are a valuable tool to describe the way management perceives the system, uses resources, including its view of different role workers. As Sinreich et al. (2004) state from their literature review, potential errors in the management can be defined only by comparison between the actual
behavior and a desired one: this implies that the insight provided by the MM offers the possibility to detect faulty system performance. In the field of healthcare system modelling, a MM can be employed for mapping and restating the properties of the working environment of an Emergency Department in terms of ‘resources’ and ‘transitions’ (Sinreich et al., 2005).

3. Determinist models

They are defined as “mathematical models in which the parameters and variables are not subject to random fluctuations, so that the system is at any time entirely defined by the initial conditions chosen”. Deterministic models use mathematical representations of the underlying regularities that are produced by the entities being modeled and generate theoretically perfect data. These types of models are useful to make predictions and try "What If?" scenarios. For this reason they are usually employed, for example, for the evaluation of resource allocation in healthcare systems.

4. Input - Output (I-O) models

The conceptual basis for an I-O analysis is a system consisting of several interdependent internal components open to its external environment (Correa and Parker, 2005). The outputs of certain components are used as inputs to others. The system is “open”: elements of the external environment receive inputs from the internal ones, but generate no output. The most common use of this kind of modes is to investigate the system – wide impact of exogenous changes in external components on independent ones, and on primary inputs. An input-output model is widely used in economic forecasting to predict flows between sectors. They are also used in local urban economics. In an health care system I-O analysis can be applied in the study of efficiency and effectiveness in the allocation of human, financial and physical resources (Correa and Parker, 2005).

5. Queuing theory models

Queuing theory is the mathematical study of queues. Queue theory was created by Agner Krarup Erlang in 1909, whilst he worked at the Copenhagen Telephone Exchange. He tried to understand the flow of visitor traffic, and why queues build up when they do. The theory enables mathematical analysis of several related processes, including arriving at the (back of the) queue,
waiting in the queue (essentially a storage process), and being served by the server(s) at the front of the queue. The theory permits the derivation and calculation of several performance measures including the average waiting time in the queue or the system, the expected number waiting or receiving service and the probability of encountering the system in certain states, such as empty, full, having an available server or having to wait a certain time to be served. These types of models are used to capture the stochastic nature of arrivals. Queuing theory is not always suitable for all systems, due to the characteristics of the input or service organization, complexity of the dynamic system configuration, nature of the queue rule. Often is not possible to develop analytical models for queuing system.

Some mathematical solution of the problem for the transient behavior of non-empty – M/M/1\(^2\) queue have been recently proposed. Tarabia (2002) studied the general model of the Markovian queues (M/M/1/\(\infty\)\(^3\)) with any arbitrary number “\(i\)” of customers being present initially in the system. For this model, he proposed a series form for the transient state probability.

These types of models are usually employed to study the waiting time or the length of a queue in an emergency department in normal (not critical) operative conditions. In fact, the hospital can be treated as a single service channel, with Unlimited Queue Length, M/M/1

Some additional information regarding the queue theory can be found in Appendix A.

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\(^2\) It’s the Kendall's notation of this queuing model: the first part represents the input process, the second the service distribution, and the third the number of servers. The M represents an exponentially distributed interarrival or service time, specifically M is an abbreviation for Markovian. The M/M/1 Waiting line system has a single channel, single phase, Poisson arrival rate, exponential service time, unlimited population, and First-in First-out queue discipline

\(^3\) It is a M/M/1 queue with an unlimited queue length.
6. Control theory and System dynamic models

Coyle (1977) defined system dynamic models as methods “of analyzing problems in which time is an important factor, and which involve the study of how a system can be defended against, or made to benefit from, the shocks which fall upon it from the outside world”. One of the advantage of the System Dynamics models (SD) is the option of including nonlinear behavior of some of the variables involved and the interaction between different service areas (emergency room, intensive care unit, wards, operating rooms) in the operation of the health care facility (Arboleda et al., 2007). Therefore, they are used to model ‘complex’ systems, which mean high – order, multiple loops, and non – linear feedback structure. Feedback refers to a procedure by which a certain behavior returns to affect subsequent behavior. An example of feedback is shown in Figure 3-9. Control theory and system dynamics models are used to describe both steady state (in normal operative conditions) and transient behavior (in disaster situation) of health care facilities.

![Figure 3-9 Example of feedback loop in a System Dynamics Simulation (Arboleda et al., 2007)](image)

One of the models that need attention, within this category, is the *complex adaptive evolutionary framework* developed by Dargush and Hu (2006), which integrates artificial earthquake models, normative organizational behavioral models, and economic loss estimation models. The *dynamic behavioral model* of health care facilities is represented by a set of ordinary differential equations (ODEs), where four key variables are considered: patient (P), employee (E), building...
and equipment (B), monetary asset (M). However, the model of Dargush and Hu (2006) has limitations because it considers routine operation of a hospital only. Some abrupt decisions, such as a major investment on building and equipment, a significant hiring or lay-off of employees, extreme events such as earthquakes, epidemics etc. are not taken into account. The same model has been trying to be extended to hospital networks (Hu and Dargush, 2006).

7. Discrete event simulation models

Discrete event simulation models (DES) are useful method capable of modelling complex systems, such as hospital operations and obtaining capacity estimates in a dynamic environment. In DES, the operation of a system is represented as a chronological sequence of events that occur in an instant in time and mark a change of state in the system.

DES models are described as "the technique of imitating the behaviors of some situation or system (Economic, Mechanical etc.) by means of an analogous model, situation, or apparatus, either to gain information more conveniently or to train personnel." DES concerns the modelling of a system as it evolves over time by representing the changes as separate events.

A number of mechanisms have been proposed for carrying out discrete event simulation. They are (Pidd, 1998):

1. Event-based
2. Activity-based
3. Process-based

The three-phase approach is used by a number of commercial simulation software packages, the specifics of the underlying simulation method are generally hidden (Promodel, 1999).

DES models due to their flexibility are capable of modelling the transient behavior of systems. In any simulation model, which starts from an empty and idle state, the system passes through a transient stage before reaching a steady state. This transient modelling capability of simulation is demonstrated by many applications in the literature. The diagram of Figure 3-10 shows the key stages in using Discrete Event Simulation.
In particular, simulation optimization provides a structured approach to determine optimal input parameter values, where optimal is measured by a function of the output variables associated with the simulation model (Swisher et al., 2001). A discrete event simulation model usually deals with $p$ deterministic input parameters, defined over a feasible region $\mathcal{P}$, and $q$ stochastic output variables such as

$$\psi = (\psi_1, \psi_2, ..., \psi_p) \overset{Y = \psi}{\rightarrow} Y = (Y_1, Y_2, ..., Y_q) \quad (3-10)$$

In the single response optimization it is necessary to define a real function of $Y$, for example $C = C(Y)$, that combines all the $q$ output variables into a single stochastic one. The goal is to find out which set of $\psi$ variables optimizes the simulation response function $F(\psi)$, such as

$$\bar{\psi} = (\bar{\psi}_1, \bar{\psi}_2, ..., \bar{\psi}_q) : F(\bar{\psi}) = E[C(Y(\bar{\psi}))] \quad (3-11)$$

The problem is that $F(\psi)$ cannot be observed directly, but rather must be estimated. This may require multiple simulations run replications or long simulation runs. The stochastic nature of the output from the simulation run complicates the optimization problem as shown in section 3.6.1.2. This kind of models are used for the description of detailed functioning of a specific section of the hospital, to forecast the impact of changes in the patient flow, to optimize the utilization of existing resources and examine current needs, to assess the efficiency of existing health care delivery system. Moreover, this operational research technique may help designing new systems or redesign existing ones. Simulation modelling requires multiple replications for...
the results to be acceptable statistically. Since real-time capacity estimation is required for disaster relief, multiple replications of all of the hospitals in the disaster region require prohibitively considerable computing effort. Therefore, the direct use of simulation in real-time applications for disaster relief is impractical.

8. Metamodels

They are also called “hybrid simulation/analytical modelling” (Buzacott and Yao, 1986) and their conceptual formulation is presented in Yu and Popplewell (1994). The procedure is summarized and described in the flow chart in Figure 3-11 (Yu and Popplewell, 1994). As it can be seen, metamodels are constructed in several stages. Initially a properly designed set of simulation experiments must be conducted. The relationship between the response variable \( Y \) and the inputs \( X_j \) of the system can be represented as

\[
Y = f_1(X_1, X_2, \ldots, X_s)
\]  
(3-12)

where \( s \) is the total number of variables in the system. A simulation model is an abstraction of the real system, in which only a selected subset of \( r \) input variables is considered. The response of the simulation is defined as a function \( f_2 \) of this subset.

\[
Y' = f_2(X_1, X_2, \ldots, X_r, v) \quad \text{with } r < s
\]  
(3-13)

where \( v \) is a vector of random numbers representing the effect of the excluding input.

As a second step, regression analysis is typically used to identify the most significant input variables and the shape of the function \( f_2 \), by determining correlation between independent and dependent variables. The metamodel is a further abstraction, in which a subset of the initial simulation variables is selected and the system is now described by

\[
Y'' = f_3(X_1, X_2, \ldots, X_m) + \varepsilon \quad \text{with } m < r < s
\]  
(3-14)

Where \( \varepsilon \) is a vector, which gives the fitting error and \( m \) is the total number of independent variables among the \( r \) variable selected.
Discrete Event Simulation Model vs. Metamodel

Discrete event simulation models are valuable tools for modelling the dynamic operation of a complex system (Table 3-2), because the emergency nature of a disaster can be easily incorporated in discrete event simulation, for different hospitals configurations (Lowery, 1993). However, simulation metamodel is easier to manage and provides more insights than discrete events simulation models. Further, the relative computational simplicity offers the prospect of practically modelling systems that are more extensive. Simulations are time consuming and the necessary various simulation runs tend to produce extensive tables of results of not always easy interpretation. Metamodel provides one approach to statistical summarization of simulation results, allowing some extrapolation from the simulated range of system conditions and therefore potentially offering some assistance in optimization.

A metamodel is a generalized model for a set of similar systems. Therefore, it is possible to create a metamodel to represent a large range of hospitals using proper parameters. Furthermore, since a metamodel is simply a set of equations, it does not require a long execution time as in the case of discrete event simulation models. Due to the generic and real time features, the
metamodel becomes a good candidate for modelling operations for any general hospital in disaster condition.

Since all simulations are run off line and the metamodel is developed in advance, system dynamic behavior can be obtained virtually instantaneously in real – time (Cochran and Lin, 1993). Therefore the use of a continuous metamodel seems to be the best candidate for the description of a generic hospital because it satisfies all the following requirements:

1. it represents the operations for any generic hospital;
2. it can be used in real time, with a great reduction in time consuming;
3. it describes the complexity of the hospital operations;
4. it captures the transient dynamic behavior of the hospital;
5. it predicts the long – term steady state behavior of the hospital;
6. it uses the continuously changing dynamic arrival rate as input.

3.6.1 Assumptions and limits of the metamodel

The metamodel is based on some assumptions that are organized and reported in the following sections:

- Assumptions on the input data and hospital type (e.g. the input patient flow and injury composition during an earthquake, the hospital type, etc.) (section 3.6.1.1);
- Assumptions done during the discrete event simulation model used to calibrate all the parameters of the metamodel (section 3.6.1.2);
- Assumptions on the hypothesis of the construction of the analytical solution (section 3.6.3);

3.6.1.1 Assumptions of the input data

The hospitals of interest in disaster are those that treat all general types of injury and have emergency room (ER) and operating rooms (OR). Specialty hospitals (e.g. cancer and cardiac centers) are not considered to contribute significantly to the treatment of injuries resulting from a disaster. Therefore, only non-specialty hospitals are included in the formulation. In details, the hospitals considered are those corresponding to the general acute care license type of California Hospital Statistics. Furthermore, in this research endeavor, only the initial time for treatment of
patients was modeled, even if after the initial treatment is over, there still could be bed surge capacity issues.

This research focuses on dealing with the initial surge of trauma injuries and other patients in the early stages of a sudden-onset disaster. The subsequent services, such as intensive care and impatient care, are not within the scope of this report. The main goal is to describe the types of injuries that should be anticipated after a major earthquake and evaluate the ability of the hospital to absorb them with a waiting time that assures to everybody to survive and receive the care required. The lack of data is the first problem to handle with when dealing with disasters. This deficiency (Stratton et al. 1996) is related to the difficulties in collecting data during a disaster, because the emergency activity is the first aim, and the registration of the patient is, of course, not done with the usual procedure.

Usually, after a natural disaster, the emergency department in the hospital can expect an increment of 3 to 5 times the number of patients that walk in normal operative conditions. The arrival rate gradually decreases over the next 3 days and return to usual pre-disaster values within 4 to 5 days.

As pointed out by previous research, medical disaster teams usually provide primary care and very little acute trauma resuscitation. In order to define the composition of the injuries, some factors have to be considered (Peek-Asa et al., 2003). Deaths and injuries from earthquakes vary dramatically based on characteristics of the earthquake, the environment, and the population where the earthquake strikes. Prepared communities with high-level seismic building code design have fewer losses than developing countries with no seismic experience or protection.

Previous research (Peek-Asa et al., 2003) has shown that factors such as average population age and gender (which affects the reaction and the ability of movement during the emergency), building characteristics (high, medium, low or no seismic code), and shaking intensity are related to the likelihood of being killed in an earthquake. At the same time seismic characteristics including peak ground acceleration, shaking intensity, and distance to the earthquake rupture plane, individual characteristics including age and gender, and building characteristics including building occupancy type and damage were independently related to earthquake injury.

Peek – Asa et al. (1998) describes the injuries occurring in the Northridge earthquake which resulted in death or hospital admission. They identified entrapment from building collapse as the biggest risk factor. Moreover, the survival after the building collapses was unlikely. However,
neither entrapment nor structural collapse was a major predictor of hospitalized injury: the primary risk factor for serious injuries was falls or being hit by objects. Motor vehicles injuries and burns were also common causes of injury, head and chest injuries were common fatalities, and extremity injuries were the most common among those admitted to the hospital. Secondary disasters following earthquakes, including fires, landslides, or floods, have been identified as increasing the lethality of an earthquake.

The only data available regarding the patient inflow to an emergency department during an earthquake are those collected during the Northridge Earthquake that are the one that will be used in this report. On 17th of January 1994, the disaster stroke, with an epicenter located in the northern area of Los Angeles County (California). The magnitude of 6.7 on the Richter scale resulted in the strongest ground motions ever instrumentally recorded in an urban area in North America. Although moderate in size, the earthquake had immense impact on people and structures because it was centered directly beneath a heavily populated and built-up urban region. Seventy-two people were killed: thirty three deaths occurred as a direct result of the earthquake, over 11,000 people were injured, 7,757 were treated and managed as outpatients by local hospital emergency departments; 1,496 individuals with earthquake related illness or injury required hospitalization.

Salinas et al. (1998) provided data on the arrival rate, visit distributions by problem category and comparisons with the data of pre-disaster situation for the earthquake experience of the Emergency department of the Northridge hospital that is one of the 14 facilities located in the S. Fernando Valley that treated injuries related to or caused by the earthquake.

Stratton et al. (1996) report data collected from multiple independent sources and described the earthquake experience of the local emergency medical services (EMS) agency. In particular, the pattern per hour of persons coming to health care facilities is provided, together with the pattern of acute care hospital beds available throughout Los Angeles County during the earthquake response period.

The input arrival rate and the injury distribution assumed (in terms of percentage of patients needing an operating room) regard a western community with a medium level of preparedness and a medium seismic code design level, hit by a magnitude 6.7 earthquake.

The choice of this earthquake intensity as a representative case study is not only dictated by the available data, but also with the fact that an average of 120 earthquakes per year worldwide
in the magnitude range of 6.0 - 6.9 (like the Northridge and Kobe – 17 January 1995 -events) have occurred since 1900. For the past decade, the annual number of M = 6.0 - 6.9 shocks worldwide has ranged from 79 in 1989 to 141 in 1993. These numbers confirm that events like those affecting the urban areas of Northridge and Kobe are typical, and that we should be prepared for such shocks wherever cities and towns are located in seismically active areas (USGS, 2002). The pattern of the arrival rare used for the Northridge earthquake case study was elaborated by Yi (2005) on the basis of the available data. The number of patients per hour trying to enter the emergency department in the 4 days following the earthquake is reported in Table 3-3. The main assumption made to obtain this pattern is that the dynamic arrivals can be assumed to follow a non–homogeneous Poisson process, where the inter–arrival times can be considered by following an exponential distribution.

### Table 3-3 Northridge arrival rate (Yi, 2005)

<table>
<thead>
<tr>
<th>Time</th>
<th>Duration (hours)</th>
<th>Start time (min)</th>
<th>N of patients</th>
<th>Arrival rate (patients/min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Day 0</td>
<td>24</td>
<td>0</td>
<td>125</td>
<td>0.087</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>1440</td>
<td>48</td>
<td>0.200</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>1680</td>
<td>77</td>
<td>0.321</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>1920</td>
<td>58</td>
<td>0.242</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>2160</td>
<td>57</td>
<td>0.238</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>2400</td>
<td>52</td>
<td>0.217</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>2640</td>
<td>51</td>
<td>0.213</td>
</tr>
<tr>
<td>Day 1 (Eq. happens)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>343</td>
<td></td>
</tr>
<tr>
<td>Day 2</td>
<td>4</td>
<td>2880</td>
<td>36</td>
<td>0.150</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>3120</td>
<td>36</td>
<td>0.150</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>3360</td>
<td>34</td>
<td>0.142</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>3600</td>
<td>34</td>
<td>0.142</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>3840</td>
<td>32</td>
<td>0.133</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>4080</td>
<td>31</td>
<td>0.129</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>203</td>
<td></td>
</tr>
<tr>
<td>Day 3</td>
<td>4</td>
<td>4320</td>
<td>31</td>
<td>0.129</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>4560</td>
<td>31</td>
<td>0.129</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>4800</td>
<td>29</td>
<td>0.121</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>5040</td>
<td>29</td>
<td>0.121</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>5280</td>
<td>27</td>
<td>0.113</td>
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<td></td>
<td>4</td>
<td>5520</td>
<td>26</td>
<td>0.108</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>173</td>
<td></td>
</tr>
<tr>
<td>Day 4</td>
<td>24</td>
<td>5760</td>
<td>125</td>
<td>0.087</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>125</td>
<td></td>
</tr>
</tbody>
</table>
3.6.1.2 Assumptions in discrete event simulation model

All the parameters of the metamodel depend on the calibration done on the data generated by more complicated simulation models such as the DES model. The results are strongly dependent on the assumption made at the simulation level. The parameters of the metamodel are obtained from the statistical analysis on a set of simulation runs performed by Yi (2005) and Paul et al. (2006), using the DES model. The software used is developed by Promodel Corporation, (Promodel v 4.2, 1999). Two types of simulations are available using Promodel that are able to describe respectively the transient response (called terminating simulation) and the steady state response (called non-terminating simulation) of the system. In detail, a terminating simulation starts at a defined state condition and ends when it reaches some other defined state or time: the final production of the system and the changing patterns of behavior are of interest, rather than the statistic on the overall average response. On the contrary non-terminating simulations are used when it is necessary to investigate the steady state behavior: the simulation could theoretically run for an indefinite interval of time, without recording any statistical change in the parameters of response.

During an earthquake, a hospital has to handle an internal crisis: a sudden increase of the arrival rate is recorded and the normal operative conditions are disrupted. The main goal is to explore the short-term performance as affected by a given perturbation event. As reported by Stratton et al. (1996) during the Northridge earthquake, the acute phase of the disaster ended within 48 to 72 hours. Thus the terminating simulation is the most suitable to describe the transient during the crisis.

The main assumptions done in this phase are related to the definition of the simulation model itself. Therefore, it is necessary to understand the way in which entities, locations, resources, and path processing have been defined.

The entities are the items processed by the simulator, that in the case of an ED are the patients requiring care. Patients are grouped in various sets listed in Table 3-4, according to their injury type and the consequent similar medical needs. The components of each entity, or group of patient, go through the same treatment procedure, and they require for a given statistically probable time the hospital resources (physicians, nurses, OR operating rooms, medical equipment). The types of injuries of Northridge Earthquake, divided in percentage are reported in Table 3-4.
Table 3-4 Distribution of injuries types, also called patient mix α (Yi, 2005)

<table>
<thead>
<tr>
<th>Groups</th>
<th>Injuries types</th>
<th>Percentage [Northridge earthquake]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cuts, wounds, laceration, contusion, sprain (not OR: they are released after treatment in ER)</td>
<td>Type 1</td>
<td>46.5%</td>
</tr>
<tr>
<td>Fracture</td>
<td>Type 2</td>
<td>9.8%</td>
</tr>
<tr>
<td>Burn, head injuries</td>
<td>Type 3</td>
<td>3.7%</td>
</tr>
<tr>
<td>Neuron/Psychiatric, respiratory, gastrointestinal</td>
<td>Type 4</td>
<td>21.9%</td>
</tr>
<tr>
<td>Cardiovascular</td>
<td>Type 5</td>
<td>13.5%</td>
</tr>
<tr>
<td>OB/GYN (obstetrics and gynecological)</td>
<td>Type 6</td>
<td>4.6%</td>
</tr>
</tbody>
</table>

HAZUS (FEMA 1999), the reference FEMA’s Software Program for Estimating Potential Losses from Disasters divides the injuries in four classes, according to the level of severity (from 1= minor to 4=maximum). The correspondence between patient types used in the hospital simulation model and HAZUS, is reported in Table 3-5.

Table 3-5 Correlation between HAZUS and patient types and the one used in the hospital simulation model (Yi, 2005)

<table>
<thead>
<tr>
<th>HAZUS</th>
<th>Hospital Simulation model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Severity 1: injuries requiring basic medical aid without requiring hospitalization</td>
<td>Type 1</td>
</tr>
<tr>
<td>Severity 2: injuries requiring a greater degree of medical care and hospitalization, but not expected to progress to a life threatening status</td>
<td>Type 2 who do not need surgery Type 4 Type 5 who do not need surgery</td>
</tr>
<tr>
<td>Severity 3: injuries that pose an immediate life threatening condition if not treated adequately and expeditiously. The majority of these injuries are a result of structural collapse and subsequent collapse of the occupants</td>
<td>Type 3                     Type 6</td>
</tr>
<tr>
<td>Severity 4: Instantaneously killed or mortally injured</td>
<td>No access to the emergency department</td>
</tr>
</tbody>
</table>

Each patient is assigned to a certain survivability time corresponding to his/her severity of injury, according to HAZUS definition. The survivability time (WT_{crit}) is the maximum time that a patient can wait before receiving treatment in an OR or ED. The survivability time for each level of severity is herein reported (Paul et al. 2006):

Table 3-6 Critical waiting time corresponding to different severity levels

<table>
<thead>
<tr>
<th>Severity level</th>
<th>WT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inpatients:</td>
<td>∞ minutes</td>
</tr>
<tr>
<td>Severity 1</td>
<td>390 minutes</td>
</tr>
<tr>
<td>Severity 2</td>
<td>270 minutes</td>
</tr>
<tr>
<td>Severity 3</td>
<td>80 minutes</td>
</tr>
<tr>
<td>Severity 4</td>
<td>0 access</td>
</tr>
</tbody>
</table>
The survivability time gives the priority to accede resources but when two patients of the same severity level attempt to enter the same unit, a First In, First Out (FIFO) rule is used. According to the types of patient and the health care required, each entity experiences different paths and queue within the hospital. The entities trying to enter a location\(^4\) have to wait until the previous entity finishes to be processed (Figure 3-12, Figure 3-13). Furthermore, it is assumed that each patient retains its group classification attribute throughout his/her stay in the ED.

The structure of the Emergency Department of the Mercy Hospital (Figure 3-12) located in Buffalo and the human resources has been modeled (Yi, 2005) using Promodel and the DES model is shown in Figure 3-12

\(\text{\footnotesize Figure 3-12 Mercy Hospital (Yi, 2005)}\)

---

\(^{4}\text{ The Locations are fixed places in the system where entities are routed for processing, queuing, or making some decisions about further routing. Locations are used to model waiting areas and queues, which are related to the time necessary to carry out the process of care required.}\)
The acronyms in Figure 3-12 are detailed below:

- **GENL** = general medicine;
- **NEUR** = neurology;
- **OBGY** = obstetrics / gynecology;
- **ORTH** = orthopedic;
- **OTHER** = other department (e.g. geriatrics,…)
- **EMERG** = emergency department (e.g. geriatrics,…)
- **OPEN HEARTH** = cardiologic and cardiovascular surgeries
- **UROL – OP** = urological operations

The probability density functions used in the model of Mercy Hospital to describe the time of utilization of hospital resources is shown in Table 3-7 (Yi, 2005). The data used to calibrate the distributions (Paul et al., 2006) has been provided by the Mercy Hospital of Buffalo (GEM, 2002) and the EIRE county medical Center of Buffalo (GEM, 2001).

<table>
<thead>
<tr>
<th>Procedure</th>
<th>Surgery Time distribution</th>
<th>Recovery time distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>GENL</td>
<td>Empirical distribution</td>
<td>Empirical distribution</td>
</tr>
<tr>
<td>NEUR</td>
<td>25 + Log – normal (4.48, 0.549)</td>
<td>Triangular(0, 70.665)</td>
</tr>
<tr>
<td>OBGY</td>
<td>Empirical distribution</td>
<td>Parson5(3.62, 218)</td>
</tr>
<tr>
<td>ORTH</td>
<td>15 + Weibull (2.41, 123)</td>
<td>Parson5(5.31, 435)</td>
</tr>
<tr>
<td>OTHER</td>
<td>10 + Erlang (117, 58.7)</td>
<td>20 + Erlang (117, 58.7)</td>
</tr>
<tr>
<td>UROL</td>
<td>5 + Log – normal (3.8, 0.705)</td>
<td>Empirical distribution</td>
</tr>
<tr>
<td>EMERG</td>
<td>20 + Exponential (61.6)</td>
<td>Parson5(4.37, 435)</td>
</tr>
<tr>
<td>OPEN - HEART</td>
<td>225 + Weibull (1.53, 135)</td>
<td>Empirical distribution</td>
</tr>
<tr>
<td>UROL - OP</td>
<td>Empirical distribution</td>
<td>Not applicable</td>
</tr>
</tbody>
</table>

The main Locations in the discrete simulation model shown in Figure 3-13 are reported in Table 3-8:

<table>
<thead>
<tr>
<th>ER emergency room;</th>
<th>Lab laboratory;</th>
<th>OR operating room</th>
</tr>
</thead>
<tbody>
<tr>
<td>ICU intensive care unit;</td>
<td>HR Holding Room;</td>
<td>discharge;</td>
</tr>
<tr>
<td>Normal arrivals;</td>
<td>EQ arrivals;</td>
<td>second ER;</td>
</tr>
<tr>
<td>queue ER;</td>
<td>queue Lab;</td>
<td>queue ER;</td>
</tr>
<tr>
<td>Inpatient;</td>
<td>fake Arrivals;</td>
<td></td>
</tr>
</tbody>
</table>

It is assumed that all the operating rooms have the same capability.
A path processing assigns to each entity the routing through the system and defines which operation take place at each location. Patients coming through ED are treated in different units of the ED and after treatment they are discharged and either they leave the ED, or admitted for longer treatment to one of the other units of the facility. Once entities have entered the system, the path processing describes what it will happen to them until they are either discharged or admitted to other department. According to the severity class of injury, each patient can utilize the laboratories for screening, the operating rooms or the intense care unite, and it can exit the care process after an eventual hospitalization.

The model in Figure 3-12 is representative of a specific hospital; however, a more general model can be obtained using off-line simulation runs, which can represent hospitals of various sizes and capabilities as the one shown in Figure 3-13, which describes statistically one ensemble of hospitals located in California.

![Diagram of simulation model](image)

Figure 3-13 Structure of the simulation model (Yi, 2005)

The resources are the patients used to transport entities and perform operations (e.g. doctors, nurses, technicians, anesthetists). The key human resources in a clinical environment as reported
by Swisher et al. (2001) are physicians, physician’s assistants, nurses, medical assistants, lab
technicians and clerical staff. For the purpose of the study, they have been grouped in four sets
(doctors, nurses, technicians and anesthetist) and they are considered present in a high number,
because the model does not consider the influence of human resources on performance.
Furthermore, it is assumed that all medical personnel of the same type have the same skill levels.

The numerical simulations of the model described in Figure 3-13 provided the data for the
calibration of the parameters of the metamodel. The parameters are therefore dependent on:

1. Patient mix $\alpha$ (Table 3-4), assumed as representative of an earthquake flow;
2. Routing of patients inside the hospital (Figure 3-13);
3. Statistical processing time for the health care required (Table 3-7);
4. FIFO rule for patient of the same severity that attempt to enter the same location: no
priority rule has been considered;
5. Number of replications of the same simulation, which affects the statistical analysis of the
results.

### 3.6.2 Variables of the metamodel

Waiting time is the main parameter of response of the hospital under a sudden increase in the
patient flow. It depends by internal organizational factors and external ones. As shown in Table
3-9 the number of beds (B), the number of operating rooms (OR), the resources and staff
productivity belong to the first group; the arrival rate $\lambda$ and the patient mix $\alpha$ are the external
input that can be defined as the percentage of patients who need the operating room, which is the
most critical resource in disaster condition.

Hospitals are divided in three classes (small size=100B, medium size=300B, large
size=500B), according to the number of beds (Table 3-10). The equipments/instruments are not
modeled explicitly, but with the factor efficiency $E$ which provides the number of surgeries per
operating room per year.

<table>
<thead>
<tr>
<th>Internal factors</th>
<th>External factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of beds B</td>
<td>arrival rate $\lambda(t)$</td>
</tr>
<tr>
<td>Number of operating rooms OR</td>
<td>patient mix $\alpha$</td>
</tr>
<tr>
<td>Efficiency $E$</td>
<td></td>
</tr>
</tbody>
</table>

Table 3-9 Input of the metamodel
The 21 combinations of B – OR – E considered are those proposed by Paul et al. (2006) and reported in Table 3-10. Small hospitals with a high number of operating rooms, as well as large ones with low surgery capacity (only five OR) are considered unfeasible combinations and were not taken in account.

Table 3-10 Hospital configurations

<table>
<thead>
<tr>
<th>n</th>
<th>B</th>
<th>OR</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>5</td>
<td>600</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>5</td>
<td>900</td>
</tr>
<tr>
<td>3</td>
<td>100</td>
<td>5</td>
<td>1200</td>
</tr>
<tr>
<td>4</td>
<td>100</td>
<td>10</td>
<td>600</td>
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<tr>
<td>5</td>
<td>100</td>
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<td>6</td>
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<td>7</td>
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<td>1200</td>
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<tr>
<td>19</td>
<td>500</td>
<td>15</td>
<td>600</td>
</tr>
<tr>
<td>20</td>
<td>500</td>
<td>15</td>
<td>900</td>
</tr>
<tr>
<td>21</td>
<td>500</td>
<td>15</td>
<td>1200</td>
</tr>
</tbody>
</table>

If we consider the National California Statistics (http://www.oshpd.ca.gov/oshpdKEY/FindData.htm), the composition of health care facilities according to the size is reported in the following Figure 3-14.

Figure 3-14 Composition of Californian hospitals
The medium size hospital (300 beds) is the highest representative type. This data partly justify the selection of the configurations given in Table 3-10.

As regards the capacities of emergency room and Labs are assumed to be proportional to the size of the hospital (i.e. the number of beds), and are not direct input of the simulation model. The resources and staff productivity are expressed by the operating efficiency E.

3.6.2.1 Severity of injuries and priority assignments

Three different severities are considered in the metamodel:

- **Severity 1** class patients are patients with minor injuries, e.g. lacerations, cuts, wounds, minor respiratory problems, fractures, who do not require surgery.
- **Severity 2** patients are patients who arrive with comparatively more severe problems, and might not require surgery.
- **Severity 3** patients are the highest severity patients arriving with major issues, e.g., burns, head injuries, fractures, which require surgery.

Since severity 1 patients do not require surgery, only severity 2 and 3 patients are considered in the queue, before entering the OR. In addition, inpatients also utilize the operating room. Their surgery can be postponed due to large number of emergency patients requiring surgery and they have the lowest priority for using the ORs.

To each class of severity is assigned a survivability time, defined as the maximum time that a patient can wait before the treatment in an OR or ED. In Table 3-6 are reported the survivability times obtained from interviews with hospital staff. Priority assignment is not given based on severity of injuries, but on remaining allowable waiting time, which is defined as the difference between patient’s survivability time \( WT_{\text{crit}} \) and the current waiting time \( WT \). The highest priority is given to those patients with the least remaining allowable waiting time.

3.6.3 Construction of the Metamodel

In this report the hospital functionality during a disaster is indicated by how quickly it can treat the injured patients, therefore it is directly correlated to the patient waiting time \( WT \) that is the response variable of the metamodel and it indicates how busy the hospital is.
The mathematical formulation for the evaluation of waiting time is taken in analogy with the model of a manufacturing production line system in Yi (2005), Lin and Cochran (1990) and in Cochran and Lin (1993). In fact, the transient behavior of the hospital during a disaster resembles that of a machine breakdown in a manufacturing production line (Cochran and Lin 1993, Lin and Cochran 1990). When a disaster occurs, there is a perturbation of the initial conditions and the system shifts from the original steady state waiting time $WT_1$ to new steady state $WT_2$. Common characteristic of the production line system and the hospital is that in the case of the hospital response both waiting times $WT$ and number of patients gradually approach their new steady state levels, without exceeding them. There is no overshoot with respect to the upper bound steady state condition, characterized by the waiting time $WT_2$.

By combining the potentiality of off–line simulation runs and the capability of meta modelling to describe the transient behavior of the system, a generic hospital model is built and calibrated, according to the ED patient volume, hospital size and operating efficiency considered.

The metamodel obtained in this way is built in two steps:

1. Off line simulations (normal condition). In this phase, the calibration of the parameters of the model is based on hospital steady state behavior under constant arrival rate. Simulation and national statistics usually provide data to calibrate the initial parameters of the model.

   In particular, three steady state conditions are considered:
   a. Normal operative condition, (section 3.6.3.1) in which the hospital copes with the normal arrival rate expected in a facility of that size, efficiency and normal duties of nurses, doctors, physicians and anesthetists.
   b. Base case condition (section 3.6.3.2): it is the instant in which, after the disaster stroke, the hospital activates the emergency plan, calls all the physicians and the nurses on duties and accedes to the emergency resources. It is assumed that there is a delay between activation of the emergency plan (EP) and the highest flow of patients in the hospital, so it can be assumed that the arrival rate in the base condition is equal to the normal arrival rate, as well.
   c. Critical case condition (section 3.6.3.3): it’s the steady state reached by the hospital in saturated condition, in which all the resources are used and any further patient cannot be accepted. The hospital works at full capacity and it would be over capacitated with any additional input.
2. **Online simulations** during the disaster condition. In this phase, the results of the off line simulation are used to build the response of the hospital in real time, when the disaster patient flow reaches the emergency department.

In the following sections are explained in detail the three steady state conditions of the offline simulations while in the following paragraphs the steps to build the hospital metamodel are described.

### 3.6.3.1 Normal operating condition

The **normal operating condition** also called **pre-disaster steady–state condition** is characterized by the following parameters:

- \( \lambda_0 \) = pre–disaster average daily patient arrival rate under normal hospital operations (which is obtained from national statistics);
- \( WT_0 \) = pre–disaster average waiting time (obtained from simulation data).

The parametric form of quadratic non-linear regression used for the arrival rate in normal operative condition is

\[
\lambda_0 = \text{const}_0 + a_0B + b_0OR + c_0E + d_0\alpha + e_0B^2 + f_0OR^2 + g_0E^2 + h_0\alpha^2 + i_0B \text{ OR } + j_0BE + k_0B\alpha + l_0ORE + m_0OR\alpha + n_0E\alpha
\]

(3-15)

where \( B \) is the number of beds, \( OR \) is the number of operating rooms, \( E \) is the efficiency of the hospital; \( \alpha \) is the patient mix and \( \text{const}_0, a_0, b_0, c_0, d_0, e_0, f_0, g_0, h_0, i_0, j_0, k_0, l_0, m_0, n_0 \) are nonlinear regression coefficients obtained by the statistical analysis of national CA data ([http://www.oshpd.ca.gov/oshpdKEY/FindData.htm](http://www.oshpd.ca.gov/oshpdKEY/FindData.htm)). The values of the coefficients used in equation (3-15) are shown in Table 3-11 where it has been included the influence of the patient mix \( \alpha \). The statistical analysis can be found in Appendix B. The value of the normal arrival rate depends on all the parameters considered in the metamodel. In the table is also indicated the fitting of the regression to the initial data, indicated by the parameter \( R^2 \).
The parametric form of $\lambda_0$ in equation (3-15) can be simplified using linear regressions, where the linear regression coefficients are evaluated on the basis of U.S. national statistical data (AHA, 2001) on emergency department visits prior to the disaster. The expression (Yi, 2005) is given in equation (3-16), and it depends only on the class size of the hospital

$$\lambda_0 = 6.1204 + 0.2520 \cdot B$$  

(3-16)

The pre–disaster average waiting time $WT_0$, can be evaluated by the general parametric form of nonlinear regression, shown in equation (3-17)

$$WT_0 = \text{const}_0 + a_0 \cdot B + b_0 \cdot OR + c_0 \cdot E + d_0 \cdot \alpha + e_0 \cdot B^2 + f_0 \cdot OR^2 + g_0 \cdot E^2 + h_0 \cdot \alpha^2 + i_0 \cdot B \cdot OR + j_0 \cdot B \cdot E + k_0 \cdot B \cdot \alpha + l_0 \cdot OR \cdot E + m_0 \cdot OR \cdot \alpha + n_0 \cdot E \cdot \alpha$$  

(3-17)

If a linear regression of equation (3-17) (Yi 2005) is assumed, without considering the influence of the partial mix, the linear regression coefficients are given in Table 3-12. The quality of fitting is indicated by the $R^2$ coefficient reported in the same table.

| Table 3-11 Coefficients of the nonlinear regression for $\lambda_0$ |
|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
| $\text{const}$ | $a_0$ | $b_0$ | $c_0$ | $d_0$ | $e_0$ | $f_0$ |
| -13.254 | -0.0539 | 4.5457 | 0.0815 | 39486.8581 | 0.0002 | -0.0353 |
| $g_0$ | $h_0$ | $i_0$ | $j_0$ | $k_0$ | $l_0$ | $m_0$ |
| -0.0006 | -127433266.4197 | 0.0002 | 0.0014 | -141959 | 0.0037 | 1365.7695 |
| $n_0$ | $R^2$ |
| -125.7832 | 0.85 |

| Table 3-12 Coefficients of the linear regression for $WT_0$ |
|------------------|------------------|------------------|------------------|------------------|
| $\text{const}$ | $a_0$ | $b_0$ | $c_0$ | $R^2$ |
| 13.0659 | 0.0067 | -3.4069 | 0.0435 | 0.82 |
3.6.3.2 Base Case condition

The base case condition, also called lower case, corresponds to the instant in which, after the disaster stroke, the hospital activates the emergency plan. The base case is characterized by the following quantities:

- $\lambda_L$: lower arrival rate during the disaster condition, equal to the normal operative arrival rate $\lambda_0$;
- $WT_L$: steady-state mean value of waiting time in the system after a variation of initial conditions (e.g. calling all the staff on duty, applying an emergency plan, reducing the quality of care and the time of care, etc.), under a given arrival rate equal to $\lambda_L = \lambda_0$.

The parametric form of quadratic non-linear regression used for the arrival rate in base case condition $\lambda_L$ is the same as in normal operative condition that is shown in equation (3-15).

The $WT_L$ is calculated for each hospital configuration, and it is given by the general parametric form of nonlinear regression, shown in equation (3-17), but the coefficients of the nonlinear regression are obtained by statistical analysis of the data provided by the numerical simulations in Promodel, under a constant arrival rate equal to $\lambda_0$, but in disaster condition.

Three different cases of nonlinear regressions are considered:

1. linear regression coefficients without the influence of patient mix (Yi, 2005) and results of the constants are given in Table 3-13.
2. Nonlinear regression with the influence of the patient mix (Yi, 2005), whose coefficients are given in Table 3-14.
3. Nonlinear regressions with the influence of the patient mix (Paul et al., 2006) in Table 3-15 for different severity levels where the severity level refers to Table 3-6.

Table 3-13 Coefficients of the Linear regression for $WT_L$ without the influence of the patient mix

<table>
<thead>
<tr>
<th>const</th>
<th>$a_0$</th>
<th>$b_0$</th>
<th>$c_0$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.3000</td>
<td>0.0033</td>
<td>-0.4470</td>
<td>0.0010</td>
<td>0.72</td>
</tr>
</tbody>
</table>
### Table 3-14 Coefficients of the Non linear regression for $WT_L$ with the influence of the patient mix

<table>
<thead>
<tr>
<th>const</th>
<th>$a_0$</th>
<th>$b_0$</th>
<th>$c_0$</th>
<th>$e_0$</th>
<th>$f_0$</th>
<th>$g_0$</th>
<th>$i_0$</th>
<th>$j_0$</th>
<th>$l_0$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.930287</td>
<td>0.078105</td>
<td>-5.575928</td>
<td>0.016679</td>
<td>0.000012</td>
<td>0.336647</td>
<td>-0.000007</td>
<td>-0.007313</td>
<td>0.0000003</td>
<td>-0.000418</td>
<td>0.90</td>
</tr>
</tbody>
</table>

### Table 3-15 Coefficients of the Non linear regression for $WT_L$ with the influence of the patient mix

<table>
<thead>
<tr>
<th>Coeff</th>
<th>Severity 2</th>
<th>Severity 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>const</td>
<td>12.4</td>
<td>5.4</td>
</tr>
<tr>
<td>a</td>
<td>-0.0542</td>
<td>-0.0373</td>
</tr>
<tr>
<td>b</td>
<td>-1.86</td>
<td>-0.957</td>
</tr>
<tr>
<td>c</td>
<td>0.012</td>
<td>0.0059</td>
</tr>
<tr>
<td>d</td>
<td>82.7</td>
<td>0.000048</td>
</tr>
<tr>
<td>e</td>
<td>0.00005</td>
<td>0.208</td>
</tr>
<tr>
<td>f</td>
<td>0.125</td>
<td>0.000005</td>
</tr>
<tr>
<td>g</td>
<td>0.00006</td>
<td>-0.00338</td>
</tr>
<tr>
<td>h</td>
<td>43</td>
<td>0.000008</td>
</tr>
<tr>
<td>i</td>
<td>0.00007</td>
<td>-0.0012</td>
</tr>
<tr>
<td>j</td>
<td>0.0000008</td>
<td>279</td>
</tr>
<tr>
<td>k</td>
<td>0.176</td>
<td>0.271</td>
</tr>
<tr>
<td>l</td>
<td>0.000378</td>
<td>-15.9</td>
</tr>
<tr>
<td>m</td>
<td>-9.86</td>
<td>0.029</td>
</tr>
<tr>
<td>n</td>
<td>0.0134</td>
<td>30.2</td>
</tr>
<tr>
<td>$R^2$ [%]</td>
<td>87.8</td>
<td>89.5</td>
</tr>
</tbody>
</table>

#### 3.6.3.3 Critical Case condition

The Critical Case, also called Upper Case, corresponds to the case when the system will become over-capacitated with any additional volume and it is characterized by the following quantities:

- $\lambda_U$: maximum arrival rate that the hospital is able to handle: it corresponds to the maximum number of patients that the hospital can treat;
- $T_{U}$: steady-state mean value of time in the system after a variation of initial conditions, under a given arrival rate $= \lambda_U$;

These values are calculated for each hospital configuration, and they can be evaluated with the formulas of nonlinear regression given in equation (3-15) and (3-17). The three sets of
values of the regression coefficients and the value of $R^2$ are reported in Table 3-16, Table 3-17 and Table 3-18. Three different cases of nonlinear regressions are considered:

1. Linear regression coefficients without the influence of patient mix (Yi, 2005) and results of the constants are given in Table 3-16.
2. Linear regression without the influence of the patient mix (Yi, 2005), whose coefficients are given in Table 3-17.
3. Nonlinear regression with the influence of the patient mix (Yi, 2005), whose coefficients are given in Table 3-18.

Table 3-16 Coefficients of the regression for $WT_U$ and $\lambda_U$ without the influence of the patient mix

<table>
<thead>
<tr>
<th>$\lambda$ or $\lambda_U$</th>
<th>const</th>
<th>$r \times B$</th>
<th>$s \times OR$</th>
<th>$t \times E$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_U$</td>
<td>-0.0699</td>
<td>0.0001</td>
<td>0.0124</td>
<td>0.0001</td>
<td>0.95</td>
</tr>
<tr>
<td>$T_U$</td>
<td>136.4097</td>
<td>0.0533</td>
<td>-9.7421</td>
<td>0.0743</td>
<td>0.61</td>
</tr>
</tbody>
</table>

Table 3-17 Coefficients of the Linear regression for $WT_U$ without the influence of the patient mix

<table>
<thead>
<tr>
<th>const</th>
<th>a</th>
<th>B</th>
<th>c</th>
<th>e</th>
<th>f</th>
<th>g</th>
<th>L</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>96.999055</td>
<td>0.164427</td>
<td>-27.633186</td>
<td>0.267850</td>
<td>0.000613</td>
<td>2.031526</td>
<td>-0.000062</td>
<td>-0.049692</td>
<td>-0.000020</td>
</tr>
</tbody>
</table>

Table 3-18 Coefficients of Non linear regression for $WT_U$ and $\lambda_U$ with the influence of the patient mix $\alpha$

<table>
<thead>
<tr>
<th>Coeff</th>
<th>Severity 2</th>
<th>Severity 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$T_U$</td>
<td>Vol</td>
</tr>
<tr>
<td>const</td>
<td>365</td>
<td>-137</td>
</tr>
<tr>
<td>a</td>
<td>-0.317</td>
<td>0.537</td>
</tr>
<tr>
<td>b</td>
<td>45.6</td>
<td>-8.61</td>
</tr>
<tr>
<td>c</td>
<td>-0.283</td>
<td>0.248</td>
</tr>
<tr>
<td>d</td>
<td>-2147</td>
<td>-0.000654</td>
</tr>
<tr>
<td>e</td>
<td>-0.000636</td>
<td>-0.704</td>
</tr>
<tr>
<td>f</td>
<td>0.36</td>
<td>-0.000067</td>
</tr>
<tr>
<td>g</td>
<td>-0.000054</td>
<td>0.0488</td>
</tr>
<tr>
<td>h</td>
<td>6727</td>
<td>-0.000142</td>
</tr>
<tr>
<td>i</td>
<td>0.026</td>
<td>0.0291</td>
</tr>
<tr>
<td>j</td>
<td>0.00016</td>
<td>-1084</td>
</tr>
<tr>
<td>k</td>
<td>2.18</td>
<td>-1.63</td>
</tr>
<tr>
<td>l</td>
<td>-0.0107</td>
<td>4.4</td>
</tr>
<tr>
<td>m</td>
<td>-290</td>
<td>-1.03</td>
</tr>
<tr>
<td>n</td>
<td>1.87</td>
<td>1049</td>
</tr>
<tr>
<td>$R^2$ [%]</td>
<td>70.2</td>
<td>98.3</td>
</tr>
</tbody>
</table>
3.6.3.4 Continuous Metamodel

As observed in the numerical simulations runs of the discrete event simulation model, the higher the arrival rate, the longer it takes for the hospital to reach a steady state after the earthquake. Therefore, under base case condition, the hospital will take the shortest time to reach a steady state condition while the opposite happens under the critical case condition. In the transient, when the system is shifting from a base case condition to a critical one, the system will take a time in between the boundary steady state conditions (lower base case and upper critical case). This assumption is true in non saturated condition, under the hypothesis that the arrival rate doesn’t exceed the upper bound imposed by the critical case and therefore it is assumed that the system is able to reach a new equilibrium, working at fully capacity.

The dynamic hospital model described in this section is valid for those systems with grade of utilization $\rho \leq 1$, where $\rho$ is defined as ratio $\lambda/\lambda_U$.

3.6.3.4.1 Single Exponential Function

The simulation results of Paul et al. (2006) show that the waiting time grows nearly exponentially with the increase in arrival rate, therefore an exponential function is the most appropriate to describe the transient behavior. Using a single exponential function, the waiting time in the transient is given by

$$WT = e^{A + Bl}$$

(3-18)

where the constant $A$ and $B$ are provided by the boundary conditions. In the base case (or lower bound) equation (3-18) becomes

$$WT_L = e^{A + Bl_L}$$

(3-19)

While in the critical case, equation (3-19) becomes

$$WT_U = e^{A + Bl_U}$$

(3-20)
After some mathematical manipulations of equation (3-19) and (3-20) the coefficients A and B are determined

\[
\begin{align*}
A &= \frac{\lambda_U \ln(WT_U) - \lambda_L \ln(WT_L)}{\lambda_U - \lambda_L} \\
B &= \frac{\ln(WT_U) - \ln(WT_L)}{\lambda_U - \lambda_L}
\end{align*}
\]  
(3-21)

Back substituting the parameters A and B of Equation (3-21) in Equation (3-18) the following expression is obtained

\[
WT = e^{\frac{\lambda_U \log(WT_U) - \lambda_L \log(WT_L) + (\log(WT_U) - \log(WT_L))\lambda}{\lambda_U - \lambda_L}}
\]  
(3-22)

that describes the exponential relationship between waiting time and arrival rate for any given hospital configuration where the parameters are obtained from regression analysis based on the two boundary conditions.

Based on the output of the DES model an exponential function appears to be appropriate to describe the transient behavior of the waiting time.

Using an exponential function, the path of the waiting time over time during the transient is described by the following function, which holds for the system running between the base and the critical case

\[
WT(t) = WT_L + (WT_U - WT_L) \cdot \left(1 - e^{-\frac{t-t_0}{\tau}}\right)
\]  
(3-23)

where \(t_0\) is the instant in which the disaster strikes while \(\tau\) is a time constant which depends on the time that the system takes to reach the steady state condition. The constant \(\tau\) depends on the time it takes for the system to reach the steady state. It is function of the arrival rate and it needs to be calibrated on a given hospital configuration and arrival rate. Although the single exponential form can represent the waiting time for a given hospital adequately, no common, underlying function that can represent a relationship between arrival rate and the time constant \(\tau\)
3.6.3.4.2 Double Exponential Function

The constant $\tau$ is different according to the arrival rate considered ($\lambda_L$ and $\lambda_U$), and it is determined with the least square method of estimation. In particular, for the base case $\tau_1$ is determined such that

$$\text{minimize } F(\tau) = \sum_{i=1}^{n} \left[ W_{t,i} - WT_{t,i}(\tau) \right]^2$$

subjected to constraint $\tau_1 \geq 0$  

(3-24)

where $W_{t,i}$ is the waiting time obtained from simulations with $\lambda = \lambda_L$, $WT_{t,i}(t)$ is the waiting time obtained from the analytical model in equation (3-23) and $n$ is the total number of points considered. Therefore, equation (3-23) becomes

$$WT_{t,i}(\tau) = WT_L + \left( WT_U - WT_L \right) \cdot \left( 1 - e^{-\frac{t_{n-i}}{\tau_1}} \right)$$

(3-25)

For the critical case $\tau_2$ is determined such that

$$\text{minimize } F(\tau) = \sum_{i=1}^{n} \left[ W_{t,2i} - WT_{2i}(\tau) \right]^2$$

subjected to constraint $\tau_2 \geq 0$  

(3-26)

where $W_{t,2i}$ is the waiting time obtained from simulation with $\lambda = \lambda_U$, while $WT_{2i}(t)$ is the waiting time obtained from the analytical model in equation (3-23). Therefore, equation (3-23) becomes

$$WT_{2i}(\tau) = WT_L + \left( WT_U - WT_L \right) \cdot \left( 1 - e^{-\frac{t_{n-i}}{\tau_2}} \right)$$

(3-27)
Paul et al. (2006) found a relationship between the patient arrival rate $\lambda$ and $p$, testing different hospitals configurations that are given by the following logarithmic function

$$\ln(p) = C + D\lambda$$

(3-31)

where $C$ and $D$ are constant that need to be determined for a particular hospital. These coefficients can be determined using the “base case” ($p=0$ and $\lambda=\lambda_L$) and the critical case values ($p=1$ and $\lambda=\lambda_U$), for any specific hospital. When $p=0$, $\ln(p)$ does not exist, however Yi (2005) showed that as $p$ approaches zero, $\ln(p)$ approaches -3.2, therefore $\ln(0)=-3.2$ is used as satisfactory approximation in the calculation. Therefore imposing the boundary conditions for the base case and the critical case in Equation (3-31), the following coefficients are determined

**base case** $\lambda = \lambda_L; \quad p = 0; \quad C + D\lambda_L \approx -3.2$

(3-32)

**critical case** $\lambda = \lambda_U; \quad p = 1; \quad C + D\lambda_U = 0$

(3-33)

Therefore

$$C = \frac{3.2\lambda_U}{\lambda_U - \lambda_L}$$

$$D = \frac{3.2}{\lambda_U - \lambda_L}$$

(3-34)
\[
\ln(p) = C + D\lambda 
\]  

(3-31)

where \(C\) and \(D\) are constant that need to be determined for a particular hospital. These coefficients can be determined using the “base case” \((p=0\) and \(\lambda=\lambda_L\)) and the critical case values \((p=1\) and \(\lambda=\lambda_U\)), for any specific hospital. When \(p=0\), \(\ln(p)\) does not exist, however Yi (2005) showed that as \(p\) approaches zero, \(\ln(p)\) approaches -3.2, therefore \(\ln(0)=-3.2\) is used as satisfactory approximation in the calculation. Therefore imposing the boundary conditions for the base case and the critical case in Equation (3-31), the following coefficients are determined

- **base case**: \(\lambda = \lambda_L\); \(p = 0\); \(C + D\lambda_L \approx -3.2\)

- **critical case**: \(\lambda = \lambda_U\); \(p = 1\); \(C + D\lambda_L = 0\)

Therefore

\[
C = \frac{3.2\lambda_L}{\lambda_L - \lambda_U} \\
D = \frac{3.2}{\lambda_U - \lambda_L}
\]

(3-34)

Paul et al. (2006) also assumed that if the arrival rate is a continuous function of time namely \(\lambda(t)\), then the transient waiting time of equation (3-29) is given by the following equation expressed in discrete form

\[
WT(t_i) = WT(t_{i-1}) + \left(1 - e^{C+D\lambda(t)}\right) \left(e^{A+B\lambda(t)} - WT(t_{i-1})\right) \left(1 - e^{-\frac{A}{\tau_1}}\right) + e^{C+D\lambda(t)} \left(e^{A+B\lambda(t)} - WT(t_{i-1})\right) \left(1 - e^{-\frac{C}{\tau_1}}\right)
\]

(3-35)

Considering a small time interval \(\Delta t\), and writing the derivative in discrete form the following expression is obtained after some simple mathematical calculations

\[
\lim_{\Delta t \to 0} \frac{WT(t_i + \Delta t) - WT(t_i)}{\Delta t} = e^{A+B\lambda(t)} - WT(t) \left\{ \frac{1}{\tau_1} + \frac{1}{\tau_2} e^{C+D\lambda(t)} - \frac{1}{\tau_1} e^{C+D\lambda(t)} \right\}
\]

(3-36)

By integrating equation (3-36) the following corrected continuous expression of waiting time (Paul et al. 2006) is obtained

106
\[
WT(t) = \left[ WT(0) + \frac{1}{\tau_1} \left( \frac{\tau_1 - \tau_2}{\tau_1 \tau_2} \right) \exp \left\{ \frac{1}{\tau_1 \tau_2} \left[ \int_0^t \left( \begin{array}{c} A \tau_1 \tau_2 + B \lambda(t) \tau_1 \tau_2 - \frac{1}{\tau_1} \left( -\tau_1 e^{C_1(t)} - \tau_2 + \frac{C_1(t) \tau_1}{\tau_1} \right) \\ \frac{1}{\tau_2} \left( -\tau_1 e^{C_1(t)} - \tau_2 + \frac{C_1(t) \tau_1}{\tau_1} \right) \end{array} \right) dt + C \lambda(t) \tau_1 \tau_2 + D \tau_1 \tau_2 \right] \right\} \right] + \tau_2 \exp \left\{ \frac{1}{\tau_1 \tau_2} \left[ A \tau_1 \tau_2 + B \lambda(t) \tau_1 \tau_2 - \frac{1}{\tau_1} \left( -\tau_1 e^{C_1(t)} - \tau_2 + \frac{C_1(t) \tau_1}{\tau_1} \right) \right] \right\} \right] \right] \] (3-37)

where \( WT(0) \) is the initial waiting time at time 0 and the coefficients A, B, C, D, \( \tau_1 \) and \( \tau_2 \) are all functions of the number of beds B, number of operating rooms OR and the efficiency E. In saturated condition (\( \rho = 1 \), with \( \lambda = \lambda_U \)), the solution is equal to \( WT_{crit} \), which is the max allowable waiting time under the maximum arrival rate feasible

\[
WT_{crit} = WT_l + WT_u \left( 1 - e^{-\frac{\tau_{lc}}{\tau_1}} \right)
\] (3-38)

where \( T_{lc} \) is the observation time. In particular for an observation time \( T_{lc} = \infty \), we have

\[
WT_{crit} = \lim_{\tau_{lc} \to \infty} \left[ WT_l + WT_u \left( 1 - e^{-\frac{\tau_{lc}}{\tau_1}} \right) \right] = WT_l + WT_u
\] (3-39)

The error made in the simulation using a single exponential function or a double exponential function is comparable, however, the double exponential model offer distinct advantage with an improved functionality, since the time constants are obtained without the need for simulation runs for any patient arrival rate. Simulation is only needed for the base case and the critical case.

### 3.6.3.5 Modified Continuous Metamodel (MCM)

The dynamic hospital model proposed in this report is a modification of the model of Paul et al. (2006) given in Equation (3-37) that is valid for the systems that are never over capacitated, with \( \rho \leq 1 \). When \( \rho > 1 \) the equilibrium is not satisfied and the system is overcapacitated.

The treatment rate \( \lambda_e \) (Figure 3-16) and the value of the coefficient of utilization \( \rho \) (Figure 3-17) can be plotted, knowing the waiting time from equation (3-37) and the external arrival rate given in Table 3-3.
Figure 3-16 Treatment rate for a hospital of 500 beds, 15 OR and class of efficiency 1200 operation per operating room per year.

Figure 3-17 Coefficient of utilization $\rho$ for a hospital of 500 beds, 15 OR and class of efficiency 1200 operation per operating room per year.

As can be seen from Figure 3-17, during a disaster the hospital has to cope with a long period of over capacitated condition ($\rho>1$), in which the assumption of steady state condition is not valid. The emergency department can be compared with a production line chain and the overflow of patients in saturated condition can be associated to the effect of a sudden machine...
breakdown. This dynamic event in a multi server machine assembly line will result in an increase in production time as well as the number of assemblies in the system. Similarly, it is assumed that when the hospital saturates it starts to work in steady state condition and can process only a volume of assemblies ("patients") given by the critical arrival rate \( \lambda_U \). The processing or waiting time for the compound assembly is then given by the sum of the steady state time \( WT(t, \lambda = \lambda_U) \), and the disturbance \( \Delta WT(t) \), which represents the impact of the overflow \( (\lambda - \lambda_U) \) on the equilibrated condition \( \lambda = \lambda_U \), and can be expressed by the following equation

\[
WT(t) = WT(t, \lambda = \lambda_U) + \Delta WT(t); \quad \text{if } \lambda \geq \lambda_U
\]  

(3-40)

The first term can be calculated with equation (3-37) with \( \lambda = \lambda_U \), while the second term describes the over capacitated condition, in which the inflow is greater than the outflow. In extensive form equation (3-40) can be written as follow

\[
WT(t) = WT(t, \lambda = \lambda_U) + \left( \frac{\lambda(t) - \lambda_U}{\lambda_U} \right) \cdot (t - t_{\lambda_U}); \quad \text{if } \lambda \geq \lambda_U
\]  

(3-41)

The mathematical expression of the second term of equation (3-41) has not been proved mathematically yet, but simulation results for the production chain lines, confirms its validity (Lin and Cochran, 1990). Therefore, the final expression of the waiting time for a hospital facility is given by

\[
WT(t) = \begin{cases} 
\text{eq.(3-37)} & \text{if } \lambda \leq \lambda_U \\
WT(t, \lambda = \lambda_U) + \left( \frac{\lambda(t) - \lambda_U}{\lambda_U} \right) \cdot (t - t_{\lambda_U}); & \text{if } \lambda > \lambda_U
\end{cases}
\]  

(3-42)

The MCM in equation (3-42) has been tested with different shapes of arrival rates in order to supply the reader about its dynamic behavior. If the system is pushed with a constant arrival rate equal to the critical arrival rate, the solution tends to the critical waiting time, \( WT_{\text{crit}} \) (Figure 3-18).
Case 1: $\lambda \leq \lambda_U$ for every $t$. The hospital is below the critical condition and it is able to follow the pattern of patient flow with a waiting time proportional to the distance of $\lambda$ from the upper critical value $\lambda_U$. In this case the response of the hospital model is always given by the continuous metamodel in equation (3-37). The waiting time $WT$ doesn’t reach the critical waiting time $WT_{crit}$ and it is able to absorb the arrival rate without entering in critical condition regardless the shape of the arrival rate (Figure 3-20 to Figure 3-23).
Triangular Arrival rate ($\lambda$) 

Waiting Time (WT)

Figure 3-20 Sensitivity to a triangular arrival rate

Exponential Arrival rate ($\lambda$) 

Waiting Time (WT)

Figure 3-21 Sensitivity to an exponential arrival rate

Trigonometric Arrival rate ($\lambda$) 

Waiting Time (WT)

Figure 3-22 Sensitivity to a sinusoidal arrival rate
Case 2: $\lambda > \lambda_U$ for every $t$. The hospital is all the time in saturated condition. Figure 3-24 clearly shows that the waiting time is the superposition of the waiting time obtained with $\lambda = \lambda_U$ that tends to the waiting time critical plus the waiting time that is proportional to the level of crowding of the emergency department that is obtained when $\lambda > \lambda_U$.

Case 3: The hospital cross the saturated condition at a certain time $t$. When the hospital has to cope with an arrival rate beyond the one is able to cope with in saturated conditions, the waiting time increases proportionally to the difference between the actual arrival rate and the critical arrival rate (Figure 3-19 to Figure 3-26). The hospital is not able to cope with the increased volume and the hospital needs additional resources to be able to absorb all the patients.
Figure 3-25 Sensitivity to a sinusoidal arrival rate, shifting between saturated and non saturated condition

Figure 3-26 Sensitivity to Northridge arrival rate

3.7 Interaction between technical and organizational resilience

Structural and non–structural damage cause reduction of functionality of the hospital at the organizational level. However, the hospital is more affected by nonstructural damage than structural damage, because if power water and medical resources are damaged, they can render the hospital useless.

MCM is able to take incorporate the effect of structural and non-structural damage on the organizational model by incorporating a penalty factor that is used to update the available emergency rooms, operating room and bed capacity of the hospital (Figure 3-1). Its value is determined by the fragility curve of each structural and nonstructural component inside the
hospital. Fragility curves are functions that represent the conditional probability that a given structure’s response to various seismic excitations exceeds given performance limit state (Cimellaro et al. 2006). The compact form assumed is given by

\[ F_y(y) = \Phi \left[ \frac{1}{\beta} \ln \left( \frac{y}{\theta_y} \right) \right] \quad y \geq 0 \] (3-43)

where

\( \Phi \) is the standardized cumulative normal distribution function,
\( \theta_y \) is the median of \( y \), and
\( \beta \) is the standard deviation of the natural logarithm of \( y \) (Soong, 2004).

From fragility curves it is possible to evaluate penalty factors that are applied to all the internal parameters of the hospital (i.e. \( B \), \( OR \), \( E \)).

### 3.7.1 Construction of the penalty factors

The penalty factors \( PF_i \) for each structural or non-structural component are given by the linear combination of the conditional probabilities of having certain levels of damage. Four levels of damage are traditionally considered: \( P_1 \) slight, \( P_2 \) moderate, \( P_3 \) extensive and \( P_4 \) complete. These probabilities can be read for a given EDP on the fragility curves provided for each structural and non-structural component. The total penalty factor affecting each component analyzed is given by

\[ PF_i = a \cdot (P_1 - P_2) + b \cdot (P_2 - P_3) + c \cdot (P_3 - P_4) + d \cdot P_4 \] (3-44)

where the coefficient \( a \), \( b \), \( c \), and \( d \) are obtained by normalized response parameters (e.g. drifts, accelerations, etc.) that define the thresholds of Slight, Moderate, Extensive and Complete damage states. For example if a drift sensitive nonstructural component is considered, the coefficient \( a \) is defined as \( a = \text{drift}_{\text{slight}}/\text{drift}_{\text{complete}} \). A complete list of damage states drift ratios for all building types and heights are provided in HAZUS (FEMA, 2005).
The total penalty factor $PF_{tot}$ affecting all the organizational parameters of the hospital is given by linear combination of the individual penalty factors using weight factors obtained as ratio between the cost of each component and the overall cost of the building

$$PF_{tot} = w_1 PF_{str} + \sum_{i=2}^{n} \frac{(1-w_i)}{n} PF_i \leq 1$$

(3-45)

where

- $w_i$ weighting factor of the structural component of the building;
- $PF_{str}$ penalty factor of the structural component of the hospital;
- $PF_i$ penalty factors of the non structural components considered;
- $n$ number of non structural components.

The proposed model incorporating facility damage can be used to identify the critical facilities, which would need increased capacities based on the casualties; this can be used to plan for any future expansion and reduction.

### 3.8 Uncertainties in the organizational resilience for a hospital facility

The uncertainties in the resilience framework can be grouped as shown in Table 3-19. These are uncertainties related to the intensity measures, the performance measures and the internal structural parameters.

<table>
<thead>
<tr>
<th>Table 3-19 Uncertainties parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Intensity measures</strong></td>
</tr>
<tr>
<td>Arrival rate ($\lambda$)</td>
</tr>
<tr>
<td>Patient mix ($\alpha$)</td>
</tr>
<tr>
<td>Peak ground intensity measures ($l, pga, pgv$ etc.)</td>
</tr>
</tbody>
</table>

When only uncertainties related to the intensity measures are considered in the framework, then the decision variable DV called Resilience becomes also a random variable and its expectation is defined as
\[ m_r = E(r) = \iiint r(\text{WT}_{crit}, \alpha, \lambda, I) f_{\text{WT}_{crit}, \alpha, \lambda, I} d\text{WT}_{crit} d\alpha d\lambda dI \]  

(3-46)

where \( f_{\text{WT}_{crit}, \alpha, \lambda, I} \) is the joint probability density function (jpdf) of the 4 r.v. defined above, that are not independent, so in this case it is simpler to determine the jpdf as function of the conditional probability density functions as follow

\[ f_{\text{WT}_{crit}, \alpha, \lambda, I} = f(\text{WT}_{crit} | \alpha, \lambda, I) f(\alpha | \lambda, I) f(\lambda | I) f(I) \]  

(3-47)

Besides the mean, the most important moment is the variance, which measures the dispersion of the r.v. \( r \) about its mean and it is defined as follows

\[ \mu_2 = \sigma_r^2 = E\left\{ (r - m_r)^2 \right\} = \iiint (r(\text{WT}_{crit}, \alpha, \lambda, I) - m_r)^2 f_{\text{WT}_{crit}, \alpha, \lambda, I} d\text{WT}_{crit} d\alpha d\lambda dI \]  

(3-48)

A dimensionless number defined coefficient of variation \( \nu_r \) is used to characterize the dispersion respect to the mean

\[ \nu_r = \frac{\sigma_r}{m_r} \]  

(3-49)

The formulation of resilience in equation (3-46) includes only the uncertainties due to the intensity measures that are also called from Stenberg (2003) incidental uncertainties, assuming that the model describing the organizational behavior of the hospital is not affected by the eventual structural damages that may happen inside the hospital itself. If also sequential, informational, organizational and cascade uncertainties want to be considered (Stenberg, 2003) then three more random variables describing the organizational system of the hospital should be taken into account: the number of operating rooms \( OR \), the number of bed \( B \) and the efficiency \( E \). Therefore, the jpdf becomes

\[ f_{\text{WT}_{crit}, \alpha, \lambda, I, OR, B, E} = f(OR | D, PM, R, I) f(B | D, PM, R, I) f(E | D, PM, R, I) \cdot f(D | PM, R, I) f(OR | PM, R, I) f(R | I) f(\text{WT}_{crit} | \alpha, \lambda, I) f(\alpha | \lambda, I) f(\lambda | I) f(I) \]  

(3-50)

Therefore the mean and the variance is given by the following expressions
while the probability that resilience is smaller than a specified value $r_{\text{crit}}$ for the case when structural damage is included in the model is defined as follow

$$P(R \leq r_{\text{crit}}) = \int \int \int \int \int f_{W_{\text{crit}}, B_{\text{crit}}, \alpha, \lambda, I, OR, B, E} d\alpha d\lambda dI \quad (3-53)$$

### 3.9 Resilience of hospital network

The disaster response of a community depends directly on the healthcare response, but also on the organization at the regional level. Transportation systems, including such facilities as highways, railroads, airports and harbors represent a critical component of the societal infrastructure systems. In fact, if a natural disaster strikes, it is necessary to have the transportation system to remain operational in order to ensure its reliable and safe serviceability. The disaster mitigation efforts could be severely affected by the damage that a natural disaster could cause to the roads. Furthermore, the extent of these impacts will not only depend on the seismic response of the individual road components, but also on the characteristics of the roadway system, that contains these elements. These considerations lead to the conclusions that the road damage needs to be taken into account to obtain more accurate estimates of the time that the casualty would take to reach the hospital.

#### 3.9.1 Review of network models

When considering a hospital network the different models available in literature can be grouped in conceptual and simulation models. Table 3-20 reports a summary of the literature review and a list of the related advantages and disadvantages of each set considered.

The conceptual models provide a clear definition of the variables considered and the iterations among them, but no numerical model is proposed. For example, recently Mathew (2006) proposed a conceptual model for the Public Health Management of Disasters that visualizes the use of IT in the public health management of disasters by setting up the Health and Disaster
Information Network and Internet Community Centers, which will facilitate cooperation among all those in the areas of disaster and emergency medicine.

<table>
<thead>
<tr>
<th>No</th>
<th>Model</th>
<th>Areas</th>
<th>Advantages</th>
<th>Disadvantages</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Conceptual</td>
<td>Definition of the variables and interactions</td>
<td>Clear view</td>
<td>Rough</td>
<td>MCEER Mathew (2006)</td>
</tr>
</tbody>
</table>

Simulation models for complex integrated systems like hospital networks are very few because of the extensive data requirements that are needed to support such studies. In particular, Lowery (1993) describes the design and the validation of a general simulation model of a hospital’s care unit that can be easily extended to a multiple hospital system. Each hospital has to provide data regarding the lognormal distribution of average patient inter-arrival times (IAT) and exponential length of stay (LOS), the variance of LOS, the number of beds and the unit configuration (organization of the emergency department). Fawcett & Oliveira (2000) present a simulation model, which describes a new approach based on a mathematical formulation of how a regional system of health care facilities responds to an earthquake event. The main purpose is to investigate planning and policies options applied on a regional system of hospitals through a model, which simulates the movement of casualties from the stricken area to the hospitals.

However, in such models there is no information regarding the evaluation of resilience of a hospital network where the roadway system and the consequences due to damage are included. This report describes a model to quantify resilience of hospital networks that include both technical and organizational aspects as well as the impact of the damage of the roadway system. Each hospital in the network is modeled using a metamodel (Cimellaro et al. 2008b) that is able to estimate the hospital resilience and incorporate the influence of the structural damage in the organizational model. The damage of the road network is evaluated in increments of the travel time (Werner et al., 2006).
3.9.2 Resilience of the roadway system

In order to include the roadway system in the hospital network it is necessary first to evaluate the disaster resilience of the road network, which can be defined as the ability of the system to recover rapidly from an earthquake event. Let us define the recovery time $T_{re}$ of a roadway system, as the “time after the earthquake that would be required for the system wide-travel times to attain their pre-earthquake levels”. The recovery time will vary over the range of earthquake events that could occur within the surrounding region. The recovery time $T_{re}$ can be computed as function of the return period using programs as REDARS (Welner et al., 2006). Therefore, resilience and recovery time are directly related in a roadway system. An acceptable level of resilience should be determined by balancing the costs that would be required to upgrade the system to achieve a given recovery time against the socio-economic impacts to society that would result if that recovery time is not achieved.

The resilience of the roadway system depends on such factors as:

1. the seismic performance characteristics of the individual components within the system;
2. the rate at which damage to the components can be repaired;
3. the roadway links along which the damaged components are located;
4. the redundancy and traffic carrying capacity of the roadway links;
5. the trip demands on these system which will vary according to the post-earthquake traffic carrying capacity of the system’s roadway links;

All these factors are considered in REDARS and therefore the program can be used to evaluate the disaster resilience of the roadway system. Then the disaster resilience of the hospital network can be evaluated by adding at the waiting time $WT$ of each single hospital the travel time to reach the hospital through the damaged roadway system. The total time is then compared with the critical waiting time of each single patient. The more you are far from the $WT_{crit}$ the better is the functionality of the hospital network and the higher are the values of resilience.
3.9.3 REDARS methodology
This session describes the methodology for seismic risk analysis (SRA) of roadway systems that has been implemented in the software REDARDS (Werner at al. 2006) (Risks from Earthquake Damage to the Roadway System). It estimates the economic impact of earthquake on roadway networks by evaluating damage to bridges and modelling the subsequent impacts on traffic flow. This software can also be used to assess a roadway system’s disaster resilience and support seismic-risk-reduction decision making.

The REDARDS methodology is shown in Figure 1. It includes input-data development and analysis set up (Step 1), seismic analysis of the roadway system for multiple simulations (Step 2 and 3) and aggregation of the results from each simulation (Step 4).

The heart of the methodology is a series of modules that contains:

1. input data;
2. roadway system and its post-earthquake travel times;
3. traffic flows;
4. trip demands (system module);
5. seismic hazards (hazard module);
6. component damage state at various times after the earthquake (e.g. how this damage will be repaired and whether it will be partially or fully closed to traffic during the repairs) (component module);
7. The economic losses due to repair costs and travel disruption (economic module).
Figure 3-27 REDARS methodology for SRA of roadway systems (Werner et al., 2006)
REDARS uses a walkthrough process that considers earthquake occurrence over a specified time duration (typically thousand years). Then the SRA steps are carried out to develop a simulation for each earthquake occurrence during each year of the walkthrough:

Seismic hazards models form the **hazard module** are used to estimate the site specific ground-shaking and ground deformation hazards at each components’ site. Then the values of all uncertain parameters are randomly selected. Finally the fragility models from the **component module** are used to estimate each component’s damage state due to these hazards, and its repair cost, downtime, and traffic state at various post-earthquake times as the repairs proceeds. The components traffic states are used to develop post-earthquake “system states” (e.g. roadway closure throughout the system at various post-earthquake times). The network analysis procedure in the **system module** is applied to each system state at each post-earthquake time, to estimate travel times, traffic flows, and trip demands. The above results are used to estimate various types of losses due to earthquake damage to roadway system, such as economic losses, increased travel times to/from key locations and along key routes, and reduced trip demands.

After each simulation is completed, a variance-reduction statistical-analysis procedure computes and displays confidence intervals (CIs) in the average annual economic-loss results. At any time,
the user can stop the RSA to examine these CIs and other results obtained thus far. If the CIs are deemed acceptable, the RSA can be ended; otherwise, the SRA can be restarted and continued in order to develop additional simulations.

In detail, the extent to which travel times to and from key locations in a region could be impacted by a certain earthquake is estimated through (Werner et al., 2003):

1. a rapid pushover approach to assess median damage states for each bridge in the highway – roadway network;
2. Werner’s model (2000) to evaluate bridge traffic states as a function of bridge damage states, number of spans and number of lanes;
3. a user equilibrium algorithm to compute traffic flows and travel times for each post earthquake system state (Moore et al., 1997);

In particular the user equilibrium algorithm quantifies for each zone of a given network the traffic demand and it evaluates its state of damage in terms of traffic flow.

The software is able to estimate the shortest path between two points by equilibrating traffic volume and time based on competitions between routes.

For this purpose a cost function $C_a(w)$ in equation (3-54) is minimized under a series of constrains:

- on the traffic flow on the single link $a$ (equation (3-55));
- on the traffic flow on a certain path $r$ (equation (3-56) and (3-57));
- on the travel demand from zone $i$ to zone $j$, $T_{ij}$ and the total travel demand (equation (3-58)).

$$\min Z = \sum_{a} \int_{0}^{\chi_a} C_a(w) dw \tag{3-54}$$

so that

$$x_a = \sum_{r \in R} \delta_{ar} h_r \quad \forall a \in A \tag{3-55}$$
\[ \sum_{r \in R} h_r = T_{ij} \quad \forall i \in I, \ j \in J \quad (3-56) \]

\[ h_r \geq 0 \quad \forall r \in R \quad (3-57) \]

\[ T = \sum_i O_i = \sum_j D_j = \sum_{ij} T_{ij} \quad \forall i \in I, \ j \in J \quad (3-58) \]

where \( \delta_{ir} \) is the link – path incidence variable, equal to 1 if link belongs to path \( r \); \( r, r \in R \) denotes a network path \( i, i \in I \ j, j \in J \) an origin and destination zone respectively.

Link volumes are adjusted throughout an iterative solution process. The algorithm builds all paths in each iteration, and loads the demand in the regional origin destination module to provide the optimal distribution of the traffic.

### 3.10 Case studies

#### 3.10.1 Example 1: MCEER hospital (W70)

As an application of the methodology proposed herein an existing California hospital building located in the San Fernando Valley, which was damaged during the Northridge Earthquake (1994), was selected. Details about the description of the hospital characteristic are already given in section 2.12.

Four types of retrofit have been compared in terms of hospital performance and community resilience. Each type of retrofit has been associated to a different HAZUS structural type, as reported in Table 3-21, in which also the value of the structural penalty factors, calculated with Equation (3-45) are shown.

<table>
<thead>
<tr>
<th>Retrofit</th>
<th>Structural TYPE</th>
<th>( PF_{str} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>MRF</td>
<td>S1M</td>
<td>0.556</td>
</tr>
<tr>
<td>Unbound</td>
<td>S2M</td>
<td>0.514</td>
</tr>
<tr>
<td>Panels</td>
<td>S4M</td>
<td>0.526</td>
</tr>
<tr>
<td>W+D</td>
<td>S1M</td>
<td>0.556</td>
</tr>
</tbody>
</table>
For the valuation of the penalty factors of the drift sensitive and acceleration sensitive non-structural components, the fragility curves calculated by Viti et al. (Viti et al., 2006) have been used (Table 3-28, Table 3-29).

<table>
<thead>
<tr>
<th>Retrofit</th>
<th>Drift Sensitive Components</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>None</td>
</tr>
<tr>
<td>MRF</td>
<td>0</td>
</tr>
<tr>
<td>Unbound</td>
<td>0.4</td>
</tr>
<tr>
<td>Panels</td>
<td>0</td>
</tr>
<tr>
<td>W+D</td>
<td>0.3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Retrofit</th>
<th>Acceleration Sensitive Components</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>None</td>
</tr>
<tr>
<td>MRF</td>
<td>0</td>
</tr>
<tr>
<td>Unbound</td>
<td>0</td>
</tr>
<tr>
<td>Panels</td>
<td>0</td>
</tr>
<tr>
<td>W+D</td>
<td>0</td>
</tr>
</tbody>
</table>

Three structural configurations have been taken into account to study the influence on the hospital and community resilience:

- The structure without nonstructural components (Table 3-25);
- The structure has *drift sensitive* nonstructural components (Table 3-26);
- The structure has *drift sensitive* plus *acceleration sensitive* nonstructural components (Table 3-27).

The weighting factors used for the evaluation of the total PF in Equation (3-45) are shown in Table 3-24.

<table>
<thead>
<tr>
<th>case</th>
<th>Weighting Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>structural drift acc</td>
</tr>
<tr>
<td>1 (only structural)</td>
<td>1.00 0.00 0.00</td>
</tr>
<tr>
<td>2 (structural + drift)</td>
<td>0.143 0.857 0.00</td>
</tr>
<tr>
<td>3 (structural + drift + acc)</td>
<td>0.143 0.4285 0.4285</td>
</tr>
</tbody>
</table>
Then qualitative functionality $Q_{f}$ is calculated according to equation (3-1) and plotted in Figure 3-29 where different retrofit solutions are compared. The MRF, the buckling restrained braces (BRB) and the shear walls report comparable shapes of qualitative functionality. Moreover, the retrofit solution based on weakening plus damping provides a shortest drop and a quicker recovery to the normal operative condition.

![Functionality curves for different types of retrofit](image)

All the four types of retrofit are effective in terms of increment of hospital performance and resilience, but when considering the influence of drift sensitive and acceleration sensitive nonstructural components, weakening and damping (W+D) gives the best results as shown in Table 3-27. The resilience of the hospital increases up to the 64% if compared to the non retrofitted structure (Moment resisting frame), and up to the 54% with respect to the unbound braced frame.

<table>
<thead>
<tr>
<th>Retrofit</th>
<th>PF</th>
<th>$R_{\text{hosp}}$ [%]</th>
<th>$N_{\text{Patients not treated}}$</th>
<th>R [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>MRF</td>
<td>0.556</td>
<td>95.45</td>
<td>105</td>
<td>88.85</td>
</tr>
<tr>
<td>Unbound</td>
<td>0.514</td>
<td>94.63</td>
<td>136</td>
<td>85.53</td>
</tr>
<tr>
<td>Panels S4M</td>
<td>0.526</td>
<td>95.12</td>
<td>126</td>
<td>86.56</td>
</tr>
<tr>
<td>W+D</td>
<td>0.556</td>
<td>95.45</td>
<td>105</td>
<td>88.85</td>
</tr>
</tbody>
</table>
Table 3-26 Case 2: Resilience for different retrofit options when considering drift sensitive nonstructural components

<table>
<thead>
<tr>
<th>Retrofit</th>
<th>PF</th>
<th>$R_{hosp}$ [%]</th>
<th>$N_{Patients}$ not treated</th>
<th>$R_{comm}$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>MRF</td>
<td>0.419</td>
<td>85.83</td>
<td>283</td>
<td>69.78</td>
</tr>
<tr>
<td>Unbound</td>
<td>0.523</td>
<td>95.03</td>
<td>128</td>
<td>86.36</td>
</tr>
<tr>
<td>Panels S4M</td>
<td>0.444</td>
<td>89.15</td>
<td>232</td>
<td>75.25</td>
</tr>
<tr>
<td>W+D</td>
<td>0.574</td>
<td>96.72</td>
<td>92</td>
<td>90.24</td>
</tr>
</tbody>
</table>

Table 3-27 Case 3: Resilience for different retrofit options when considering drift sensitive and acceleration sensitive nonstructural components

<table>
<thead>
<tr>
<th>Retrofit</th>
<th>PF</th>
<th>$R_{hosp}$ [%]</th>
<th>$N_{Patients}$ not treated</th>
<th>$R_{comm}$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>MRF</td>
<td>0.266</td>
<td>48.02</td>
<td>655</td>
<td>30.16</td>
</tr>
<tr>
<td>Unbound</td>
<td>0.371</td>
<td>78.06</td>
<td>390</td>
<td>58.45</td>
</tr>
<tr>
<td>Panels S4M</td>
<td>0.319</td>
<td>66.35</td>
<td>521</td>
<td>44.38</td>
</tr>
<tr>
<td>W+D</td>
<td>0.531</td>
<td>95.45</td>
<td>119</td>
<td>87.28</td>
</tr>
</tbody>
</table>

3.10.2 Example 2: Statistical hospital model of California hospital

The example showed is a statistical hospital model, representative of a typical configuration of a Californian hospital (Figure 3-14). Three levels for each of the following parameters, the number of beds (B), the number of operating rooms (OR) and the efficiency (E) are used. They are:

- Number of Beds: 100, 300 and 500;
- Number of operating rooms: 5, 10 and 15;
- Operating room efficiency index: 600, 900 and 1200;

In total 27 combinations are possible, but some non-feasible combinations were removed, so in total 21 combinations were considered. The scope of this design is sufficient to support the development of a generic hospital model, representative of a typical configuration of a Californian hospital. The parameters of the metamodel are calibrated from the statistical analysis of data obtained on a set of simulation runs performed by Yi (2005) and Paul et al. (2006), using a DES model during the post earthquake event. Regression equations are obtained for both pre-
earthquake and post-earthquake waiting times using average daily patient arrival rates calculated from national statistics. For the case of patient inflow to an ED during an earthquake the only data available are those collected during the Northridge Earthquake that are the one that will be used in this example.

3.10.2.1 Sensitivity of resilience to B, OR and E

In the following figures the sensitivity of resilience to the main parameters that characterize the organizational metamodel is investigated. In Figure 3-30 is keep constant the efficiency (E=900) and the number of operating room (OR=10), while the number of beds is increased. Plot shows that the number of beds does not have a relevant effect on improvement of resilience for this type of configuration. By comparing Figure 3-31 with Figure 3-32 it can be concluded that increments of efficiency E has a certain impact on resilience for small hospital configurations with a small number of operating rooms (OR=5), respect to bigger hospitals (OR=15). Finally Figure 3-34 shows that for a medium size hospital (B=300) the best way to improve the organizational resilience of the hospital is to increase the number of operating rooms OR. The weighting factor $\alpha$ used for the qualitative functionality (Equation (3-1)) is equal to 0.8.

Figure 3-30 Effect of the damage on the configuration with 10 OR and $E = 900$ operation per year, for different size of hospitals
Figure 3-31 Effect of the damage on a small size hospital with 15 OR and for different classes of efficiency

Figure 3-32 Effect of the damage on a small size hospital with 5 OR and for different classes of efficiency
Figure 3-33 Effect of the damage on a medium size hospital with 10 OR and for different classes of efficiency $t$.

Figure 3-34 Effect of the damage on a medium size hospital with medium class of efficiency and for surgery capacities.
3.10.2.2 Sensitivity to the presence of an emergency plan

The metamodel is also able taking into account also the capabilities of the staff and the existence of an emergency plan during the disaster. During a disaster, a facility may elect a tiered response, which provides for different actions to be taken, according to the number of casualties expected. The hospital can apply the so called ‘surge in place response’: it can increase its capability with a premature discharge of the inpatients already present, adapt the existing surge capacity, organizing temporary external shelters (Hick et al, 2004). A large portion of in – patient can be discharged within 24 – 72 hours in the event of mass casualty accident. The discharge function is not an exact science, and there is no mathematical formulation. Usually 10 – 20 % of operating bed capacity can be mobilized within a few hours and the availability of OR can increase of 20 – 30 % (Hick et al, 2004). The external shelters can provide additional room for the triage and first aid of the injured, reducing the pressure on the hospital, allowing the staff to concentrate on the non ambulatory staff.

Triage and initial treatment at the site of injury, the so called ‘off – site patient care’ (Hick et al, 2004) can relieve pressure on the emergency transportation and care system or when the local health care is damaged. It is assumed that doctors’ skills may increase the efficiency of the hospital up to the 20%. On the other side the existence of the emergency plan, which can be applied with a certain delay, can increase the number of operating room and the number of bed respectively of the 10% and 20%.

The effect of the application of the emergency plan on the values of resilience has been investigated. It is assumed that the emergency plan increases the number of beds of 10 % and the surgery capacity (number of OR) of 30%. The values of the penalty factors before and after the application of the emergency plan for the three size classes considered are reported in Table 3-28.

<table>
<thead>
<tr>
<th>Without EP</th>
<th>With EP</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>PF_B</td>
</tr>
<tr>
<td>100</td>
<td>0.540</td>
</tr>
<tr>
<td>300</td>
<td>0.561</td>
</tr>
<tr>
<td>500</td>
<td>0.612</td>
</tr>
</tbody>
</table>
In the following figures, the variation of the organizational resilience and the community resilience is illustrated, as a function of the time of application of the emergency plan. The emergency plan generates a sudden increase in the values of the organizational parameters (in this case number of beds and number of operating room) with a certain delay from the stroke of the earthquake (0, 4, 8, 16 and 24 hours). The main consequences are that the hospital has a sudden increase of capacity in terms of critical arrival and a lower waiting time in saturated condition, as show in Figure 3-35.

![Arrival rate and Waiting time](image)

**Figure 3-35** Effect of the Emergency plan on the configuration with 10 OR and $E = 900$

The emergency plan has a benefic effect only in the case of medium and large size hospitals, with a medium high surge capacity (10 – 15 operating rooms) as shown in Figure 3-36a. From Figure 3-36 to Figure 3-38 it shown that the performance in terms of hospital resilience increases up to 20% for medium-size and high-efficiency hospitals and the number of patients is up to 20 units higher.
Figure 3-36 Effect of the Emergency plan on the configuration with 10 OR and E = 900 operation per year, for different size of hospitals

Figure 3-37 Effect of the Emergency plan on a medium size hospital with 10 OR and for different classes of efficiency
Figure 3-38 Effect of the Emergency plan on a medium size hospital with medium class of efficiency for different surgery capacities

3.10.3 Example 3: Hospital network

In order to provide informational support for coordinated disaster relief efforts, the resilience framework should be able to represent multiple hospitals in the disaster region. A discrete-event simulation model of hospitals can be developed to represent various hospitals in a disaster region. However, each hospital should be modeled individually and this can be computationally expensive. Therefore, outputs from off-line simulation runs were used to obtain a simulation metamodel, which can represent hospitals of various sizes and capabilities and could be used to estimate the hospital capacity during a disaster.

The example describes a network of two hospitals modeled using the metamodel. It is assumed that they are 40.23km far each other and they are located in urban area. The example models the damages of the two hospitals and of the network connecting the two facilities.

The importance of the Operative Center (OC), keeping the contacts between the two hospitals and delivering the patients according to the real time capacity of the emergency departments has
been investigated. The OC has the function to decide the best delivery of the patients, taking into account the real-time waiting time and capacity of the facility, the distance from the place of injury, the damage of the network and the travel time to reach the facility. Different options are considered with and without OC. The scheme of the two hospitals model is illustrated in Figure 3-39.

![Figure 3-39 Scheme of the model of two hospitals with OC](image)

where

\[ t_{0i} \] is the travel time necessary to reach an hospital i from the position 0 of the injury;

\[ t_{ij} \] is the travel time on the link \( i - j \) (\( t_{12} \) in the figure);

\( t_{inf1}, t_{inf2} \) are the time delay to collect and update the data of the hospital connected to the system to the OC;

\[ t_{el} \] is the time delay necessary to elaborate the data provided by the hospitals and make decision on the emergency policy to adopt.

Therefore

\[ t_{st} = t_{inf1} + t_{inf2} + t_{el} \]  \hspace{1cm} (3-59)

is the total time necessary to update the information related to the hospital network at the OC. \( t_{oi} \) and \( t_{ij} \) are the travel times on a given link that are evaluated by dividing its length by the travel
speed on that link. Therefore, travel time for a given link changes as the travel speed fluctuates for example because of the damage condition of the network. For estimating the travel time between the position of the injured people and the nearest hospital, the speed – volume relationship developed by the U.S. Department of Transportation is used. The travel time on a given link is given by

\[ t_i = t_a^0 \left[ 1 + \alpha \left( \frac{x_a}{C_a} \right)^\beta \right] \]  

(3-60)

where
- \( \alpha = 0.15; \)
- \( \beta = 4.0. \)
- \( t_a^0 = \) travel time at zero flow on the link \( a, \) given by the length of the link, divided by the free flow speed (FFS);
- \( x_a = \) the flow (or volume) on the link \( a \) (expressed in passengers Passenger Care Unit per day);
- \( C_a = \) the “practical capacity” of the link \( a \) (expressed in passengers Passenger Care Unit per day);

The free flow speed (FFS) of a link can be defined as the average speed of a vehicle on that link, measured under low-volume conditions when drivers tend to drive at their desired speed and are not constrained by control delay. The free flow speed is the mean speed of passengers cars measured when the equivalent hourly flow rate is no greater than 1300 pc/h/ln (passengers car / hour / line). If speed studies are not available, the FFS can be determined on the basis of specific characteristics of the freeway section including:

1. The lane width;
2. The number of lanes;
3. The right shoulder lateral clearance;
4. The interchange density.

The mathematical formulation is given by

\[ FFS = BFFS - f_{lw} - f_{lc} - f_N - f_{id} \]  

[mi/h] 

(3-61)
where
\[
\text{BFFS} = \text{base free flow speed, mi/h 70 (urban), mi/h 75 (rural)}
\]
\[
f_{LW} = \text{adjustment for lane width (Table 3-29)}
\]
\[
f_{LC} = \text{adjustment for right shoulder lateral clearance (Table 3-30)}
\]
\[
f_{N} = \text{adjustment for number of lanes (Table 3-31)}
\]
\[
f_{ID} = \text{adjustment for interchange density (Table 3-32)}.
\]

Values of each single parameter are given in the following tables.

### Table 3-29 Adjustment for line width

<table>
<thead>
<tr>
<th>Lane width (ft)</th>
<th>Reduction in Free-Flow Speed f_{LW} (mi/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>0.0</td>
</tr>
<tr>
<td>11</td>
<td>1.9</td>
</tr>
<tr>
<td>10</td>
<td>6.6</td>
</tr>
</tbody>
</table>

### Table 3-30 Adjustment for right shoulder lateral clearance

<table>
<thead>
<tr>
<th>Right – Shoulder Lateral Clearance (ft)</th>
<th>Reduction in Free – Flow Speed, f_{LC} (mi/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>≥5</td>
</tr>
<tr>
<td>≥6</td>
<td>0.0</td>
</tr>
<tr>
<td>5</td>
<td>0.4</td>
</tr>
<tr>
<td>4</td>
<td>0.8</td>
</tr>
<tr>
<td>3</td>
<td>1.2</td>
</tr>
<tr>
<td>2</td>
<td>1.6</td>
</tr>
<tr>
<td>1</td>
<td>2.0</td>
</tr>
<tr>
<td>0</td>
<td>2.4</td>
</tr>
</tbody>
</table>

### Table 3-31 Adjustment for number of lanes

<table>
<thead>
<tr>
<th>Number of lanes (one direction)</th>
<th>Reduction in Free-Flow Speed f_{N} (mi/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>≥5</td>
<td>0.0</td>
</tr>
<tr>
<td>4</td>
<td>1.5</td>
</tr>
<tr>
<td>3</td>
<td>3.0</td>
</tr>
<tr>
<td>2</td>
<td>4.5</td>
</tr>
</tbody>
</table>

Note: for all rural freeway segments, f_{N} is 0.0
Table 3-32 Adjustment for interchange density

<table>
<thead>
<tr>
<th>Interchanges per mile</th>
<th>Reduction in Free-Flow Speed f_{ID} (mi/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.50</td>
<td>0.0</td>
</tr>
<tr>
<td>0.75</td>
<td>1.3</td>
</tr>
<tr>
<td>1.00</td>
<td>2.5</td>
</tr>
<tr>
<td>1.25</td>
<td>3.7</td>
</tr>
<tr>
<td>1.50</td>
<td>5.0</td>
</tr>
<tr>
<td>1.75</td>
<td>6.3</td>
</tr>
<tr>
<td>2.00</td>
<td>7.5</td>
</tr>
</tbody>
</table>

As regards the ratio $x/C$ between the actual flow and the capacity of the link, three values are considered:
- $x/C < 0.8$
- $x/C = 0.8$
- $x/C > 0.8$ collapse

The link damage is determined by the worst performing bridge on the link. The assumed link damage impact is given by Table 3-33 (Chang, 2000).

Table 3-33 State of link damage

<table>
<thead>
<tr>
<th>State of link damage</th>
<th>$\varepsilon$ = Capacity rate change</th>
<th>$\delta$ = Free flow speed change rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>No damage</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Minor damage</td>
<td>100%</td>
<td>75%</td>
</tr>
<tr>
<td>Moderate damage</td>
<td>75%</td>
<td>50%</td>
</tr>
<tr>
<td>Major damage</td>
<td>50%</td>
<td>50%</td>
</tr>
<tr>
<td>Collapse</td>
<td>50%</td>
<td>50%</td>
</tr>
</tbody>
</table>

This detailed procedure can be used when the roadway system is very simple. Alternatively, for more realistic and complex roadway systems the program REDARS (session 3.9.3) can be used to evaluate the travel times.

The OC plays a key role in a hospital network, because it has the function to decide the best delivery of the patients, and it takes into account the real-time waiting time and capacity of the facility, the distance from the place of injury, the damage of the network and the travel time to reach the facility. For these reasons, two hospitals networks described in the following paragraphs have been considered with and without the Operative Center (OC).
3.10.3.1 Hospital network without operative center (OC)

In this case, it is assumed that the two hospitals do not know the real time condition of the components of the system. The overflow is completely absorbed by the facility, which has the highest attractiveness, i.e. the shortest distance from the epicenter of the earthquake, while the second hospital works in normal operative condition. In this case, without losing generality, we assume that hospital 1 is closer to the injured patient \( t_{01} < t_{02} \) (Figure 3-39) and that there is no damage to the road network. Therefore, the arrival rates at the two hospitals defined respectively with subscript 1 and 2 before Hospital A reaches its critical condition are defined as follows

\[
\begin{align*}
\dot{\lambda}_1(t) &= \dot{\lambda}(t) \\
\dot{\lambda}_2(t) &= \dot{\lambda}_{20}(t) \quad \text{if } \dot{\lambda}(t) < \dot{\lambda}_U(t)
\end{align*}
\] (3-62)

Hospital 1 is able to provide the requested care until it reaches the saturated condition \( \dot{\lambda}_1 = \dot{\lambda}_{1U} \), then it starts to deliver the overflow to the next health care system. Hospital A can sustain the critical condition without any external help until all the resources (drugs and medical equipment) are sufficient to satisfy the demand \( t \leq t_{sub} \), while Hospital B has an increase in the normal flow.

In the case of saturated condition of Hospital A, the arrival rates at the two hospitals are defined respectively as follows

\[
\begin{align*}
\dot{\lambda}_1(t) &= \dot{\lambda}_{1U} \\
\dot{\lambda}_2(t) &= \dot{\lambda}_{20}(t) + \dot{\lambda}(t) - \dot{\lambda}_{1U} \quad \text{if } \dot{\lambda}(t) \geq \dot{\lambda}_{1U}
\end{align*}
\] (3-63)

Therefore, the waiting time at both hospitals is given by the following expression

\[
\begin{align*}
WT_1(t) &= WT_{crit} \\
WT_2(t) &= WT_2(\dot{\lambda}_1(t)) \quad \text{if } \dot{\lambda}(t) \geq \dot{\lambda}_{1U} \quad \text{and } t \leq t_{sub}
\end{align*}
\] (3-64)

where \( \dot{\lambda}_2 \) is given in equation (3-63). If \( t \geq t_{sub} \) then Hospital A collapses because of lack of resources and it cannot handle anymore the critical arrival rate. In this case, the entire patient flow will be sent to Hospital B.
3.10.3.2 Hospital network with operative center (OC)

In the presence of an Operative Center (OC) the patient will call the OC to have information about the status of the two hospitals that is described by the real time waiting time (WT). In this case, the OC can decide the optimum distribution of the number of patients per unit of time (arrival rate $\lambda$) between the two hospitals according for example, to the following rule

$$\hat{\lambda}_1(t) = \hat{\lambda}(t) \quad \text{if} \quad WT_1(t) + t_{01} < WT_2(t) + t_{02} + t_{st} \quad \text{with} \quad t_{01} < t_{02}$$

(3-65)

In this case, it is more convenient for the injured patient going to Hospital A that is also closer to him and it will be served in a shorter time. On the other hand, the arrival rates can be redistributed according to the following rule

$$\hat{\lambda}_1(t) = \left( \frac{WT_2(t) + t_{02} + t_{st}}{WT_1(t) + t_{01}} \right) \hat{\lambda}(t)$$
$$\hat{\lambda}_2(t) = \left( 1 - \frac{WT_2(t) + t_{02} + t_{st}}{WT_1(t) + t_{01}} \right) \hat{\lambda}(t) + \lambda_{20}(t) \quad \text{if} \quad WT_1(t) + t_{01} \geq WT_2(t) + t_{02} + t_{st} \quad \text{with} \quad t_{01} < t_{02}$$

(3-66)

In this case, if the total waiting time at Hospital A exceeds the waiting time at Hospital B increased by the transportation time, the OC starts to redistribute the patients between the two facilities increasing the total resilience of the system.

3.10.3.3 Results of the analysis

The results of a hospital network model (with and without Operative Center) are presented for different configurations of the system. It is assumed that:

- only the first hospital (hospital A) is damaged;
- the distance of the two facilities is 40.23 km and the damage of the road network is moderate, according to REDARS classification (Werner et al. 2006);
- initially only the first hospital absorbs the sudden increase of patient flow, while the second one works in normal operative conditions;
• The weighting factor considered for the qualitative functionality is equal to 0.8.

Four types of hospital networks are considered:

• Small size hospital (100 beds) with small surgical capacity (OR = 5) and low efficiency (E=600 operation per operating room per year) cooperating with a small size facility with medium surgical capacity (10 OR) and medium efficiency (E= 900 operation per operating room per year);

• Small size hospital of configuration 1 cooperating with a medium size facility (300 beds), medium surgical capacity (10 OR) and medium efficiency (E= 900 operation per operating room per year);

• Small size hospital of configuration 1 cooperating with a large size facility (500 beds) with high surgical capacity (15 OR) and highest efficiency (E= 1200 operation per operating room per year);

• Two medium size hospitals (100 beds) with medium surgical capacity (OR = 5) and medium efficiency (E = 900 operation per operating room per year) cooperating each other.

In the following tables are shown the results:

The configuration of the system, which can operate without (Equation (3-62)-(3-64)) or with operating center (Equation (3-65) - (3-66)); the characteristics of the hospital of the system, in terms of size (number of beds B), surgical capacity (OR), class of efficiency (E); The value of resilience of each hospital; the community resilience; the total number of patients in the community who don’t receive any care $N_{pinj}$; The total resilience of the hospital network.

<table>
<thead>
<tr>
<th>Operative Center</th>
<th>Hosp. N.</th>
<th>B</th>
<th>OR</th>
<th>E</th>
<th>$R_{Hosp}$</th>
<th>Community resilience</th>
<th>$N_{pinj}$</th>
<th>$R_{network}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without OC</td>
<td>1</td>
<td>100</td>
<td>5</td>
<td>600</td>
<td>51.18</td>
<td>68.00</td>
<td>603</td>
<td>40.83</td>
</tr>
<tr>
<td>With OC</td>
<td>2</td>
<td>100</td>
<td>10</td>
<td>900</td>
<td>100.00</td>
<td>0.00</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>100</td>
<td>5</td>
<td>600</td>
<td>98.69</td>
<td>1.51</td>
<td>4</td>
<td>87.01</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>100</td>
<td>10</td>
<td>900</td>
<td>93.47</td>
<td>18.50</td>
<td>111</td>
<td></td>
</tr>
</tbody>
</table>
Table 3-35 Hospital network: *configuration 2*

<table>
<thead>
<tr>
<th>Operative Center</th>
<th>Hosp. N.</th>
<th>B</th>
<th>OR</th>
<th>E</th>
<th>Community resilience</th>
<th>N pinj</th>
<th>R network</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without OC</td>
<td>1</td>
<td>100</td>
<td>5</td>
<td>600</td>
<td>51.18</td>
<td>603</td>
<td>49.35</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>300</td>
<td>10</td>
<td>900</td>
<td>100.00</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>With OC</td>
<td>1</td>
<td>100</td>
<td>5</td>
<td>600</td>
<td>98.69</td>
<td>4</td>
<td>91.19</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>300</td>
<td>10</td>
<td>900</td>
<td>96.03</td>
<td>74</td>
<td></td>
</tr>
</tbody>
</table>

Table 3-36 Hospital network: *configuration 3*

<table>
<thead>
<tr>
<th>Operative Center</th>
<th>Hosp. N.</th>
<th>B</th>
<th>OR</th>
<th>E</th>
<th>Community resilience</th>
<th>N pinj</th>
<th>R network</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without OC</td>
<td>1</td>
<td>100</td>
<td>5</td>
<td>600</td>
<td>51.18</td>
<td>603</td>
<td>55.73</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>500</td>
<td>15</td>
<td>900</td>
<td>100.00</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>With OC</td>
<td>1</td>
<td>100</td>
<td>5</td>
<td>600</td>
<td>98.69</td>
<td>4</td>
<td>98.23</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>500</td>
<td>15</td>
<td>900</td>
<td>99.54</td>
<td>12</td>
<td></td>
</tr>
</tbody>
</table>

Table 3-37 Hospital network: *configuration 4*

<table>
<thead>
<tr>
<th>Operative Center</th>
<th>Hosp. N.</th>
<th>B</th>
<th>OR</th>
<th>E</th>
<th>Community resilience</th>
<th>N pinj</th>
<th>R network</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without OC</td>
<td>1</td>
<td>300</td>
<td>10</td>
<td>900</td>
<td>94.80</td>
<td>115</td>
<td>90.34</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>300</td>
<td>10</td>
<td>900</td>
<td>100.00</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>With OC</td>
<td>1</td>
<td>300</td>
<td>10</td>
<td>900</td>
<td>97.66</td>
<td>53</td>
<td>93.65</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>300</td>
<td>10</td>
<td>900</td>
<td>99.13</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

Results show that the presence of the OC improve resilience of about 50% for the first three configurations (Table 3-34, Table 3-35 and Table 3-36) when the two hospitals have different capacities. On the other hand, the presence of the OC is not that effective for two medium size hospitals of the same capacity (Table 3-37).

**3.10.4 How to track waiting time in future seismic events**

While in literature, many data can be found about the waiting time in normal operating condition only data related to Northridge earthquake are available in disaster condition.

The reason is justified by the fact that during crisis all resources including doctors and nurses are necessary to save the major number of patients. Therefore, statistical data about waiting time during a crisis are not available yet, because they become of secondary importance respect to the life of a patient.
In the future, in the case of emergency or disaster, in order to track these data, will be helpful training not only professionals (e.g. doctors, nurses etc), but also volunteers of governmental and non-governmental institutions (schools, airline companies, foundations, etc.). The use and the training, especially of volunteers, will help managing human resources, because they could track statistical data like the waiting time and alleviate the effects of stress in doctors and nurses during an emergency.

3.11 Summary and concluding remarks

An organizational metamodel for health care facilities (e.g. hospitals) has been defined and implemented. The “waiting time” before service can be received it is identified as the main parameter characterizing the technical and organizational response and it is used to evaluate the disaster resilience of health care facilities. The metamodel has been designated to cover a large range of hospital configurations and takes into account hospital resources, in terms of staff and infrastructures, operational efficiency and possible existence of an emergency plan, maximum capacity and behavior both in saturated and over capacitated conditions. The sensitivity of the model to different patients’ “arrival rates”, “patient mix”, “hospital configurations and capacities”, and the technical and organizational policies applied during, and before the strike of the disaster, has been investigated.

Uncertainties associated to the nature of the disaster (e.g. Earthquakes, hurricane etc.), to the influence of the structural damage on the organizational model and to the functionality limits are also taken considered in the formulation. Numerical examples are presented for a typical Californian hospital. The single hospital metamodel has been extended to a double hospital model and a regional system of hospitals.

The impact of an Operative Center in the global resilience of a hospital network has been investigated. Results shows that the Operative Center improves the disaster resilience of the hospital network, although this improvement may not be so evident when medium size hospitals of the same capacity are included in the network.

A regional planner could make use of the proposed model in various ways. It could be used to perform a clean-slate design for an earthquake prone region. In such a design it is assumed that no hospitals have been built and the design tells us where to build hospitals and what capacity each hospital should be. However, a more likely situation is one in which hospitals already have
been built in an earthquake prone region and a decision has to be made on capacity reallocation between these sites to best prepare for an earthquake.
SECTION 4

CONCLUSIONS

4.1 Summary and conclusions

The definition of disaster resilience combines information from technical and organizational fields, from seismology and earthquake engineering to social science and economics. The final goal is to integrate the information from these different fields into a unique function leading to results that are unbiased by uninformed intuition or preconceived notions of risk.

The fundamental concepts of disaster resilience discussed herein provide a common frame of reference and a unified terminology. This report presents a comprehensive conceptual framework to quantify resilience including both technical and organizational aspects using a hybrid/analytical model. A double exponential model with parameters estimated from regression analysis is used as a substitute of a complex discrete model to describe the transient operations in a hospital that are globally represented by the patient “waiting time”. The effect of facility damage, as well as the resources influencing functionality, is also included in the organizational model to allow the evaluation of the hospital resilience. The framework has been applied to a typical hospital facility located in California and to a network of hospitals in order to show the implementations issues.

Many assumptions and interpretations have to be made in the study of disaster resilience. However, it is important to note that the assumptions made herein are only representative for the cases presented. For other problems, users can focus on those assumptions that are mostly affecting the problem at hand, while using the case study as guidance.

4.2 Further research

Further research can be addressed on the validation of the model with real cases and on the Organizational fragility of the single health care facility and of the network as well. The optimum design of the organizational configuration during a disaster in order to maximize the resilience of the entire system can also be investigated.
Many assumptions have been made in the computation of resilience of a hospital. Further research should be carried out in order to verify or revise these assumptions. The following are specific recommendations for research on the field:

The current formulation of resilience is based on the development of a well determined metamodel, however other types of metamodel should be studied in future research and compared;

The effect of damage in the organizational model is included using penalty factors. A better effect of modeling of structural damage should be addressed in future research;

The model presented in this chapter is for a single hospital building. This is unrealistic because the road network and other hospitals are not considered. Ultimately, a regional derived metamodel should be developed that is able to describe the entire behavior of the community;

The effect of triaging should be model more properly;

Other possible measures of functionality of hospitals should be considered as topics for future research.

Comparison of different regional models of hospital networks within the resilience framework.
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Appendix A: Queue theory

The queue occurs in a system when the number of ‘customers’ desiring care service exceeds the capacity of the service facility and its presence may worsen the outcome of the patient. When an earthquake strikes, a hospital has to handle the disruption of the normal steady state operative conditions. The change in the normal flow of arrivals creates a queue, which causes a delay (waiting time) in the treatment, and a consequent crowding of the ED.

Although some mathematical formulation is provided, for the purpose of the description of the simulated data, herein it is assumed that the transient is a sequence of steady state conditions. The hospital can be treated as a single service channel, with Unlimited Queue Length.

The arrival rate is considered independent of the queue length and the distributions are stationary in time. Under these conditions, it can be proved that a steady state or an equilibrium state will be reached by the system if the “coefficient of utilization” of the system, defined as

$$\rho = \frac{\lambda}{\lambda_c}$$  \hspace{1cm} (A-1)

is less than 1. If either $\lambda$ or $\lambda_c$ decrease, that is, the intensity or rate of arrivals increases, or the mean service time increases ($\lambda_c$ decreases), the utilization of the system will increase. At the same time, it is more likely that a customer will have a long wait. If $\rho \geq 1$, the queue will grow without a bound, and the steady state condition would not be reached.

The main characteristics of the queuing problems are:

1. The average rate of customer arrivals $\lambda$ is related to the mean arrival time $T_a$ between successive arrivals by the relation

$$\lambda = \frac{1}{T_a}$$  \hspace{1cm} (A-2)

In particular the probability that a unit will arrive at the system in a very short interval $dt$ is $\lambda dt$;
While service times may show a tendency to regularity, the arrival times are in many instances completely unpredictable.

2. The average rate at which customers can be served is usually linked to the mean service time \( T_s \) by the relation

\[
\lambda_c = \frac{1}{T_s}
\]  

(A-3)

The probability of service completion in \( dt \) is \( \lambda_c dt \).

3. The probability of more than one arrival or more than one service in \( dt \) is infinitesimal and it will be disregarded.

4. Unless otherwise stated, the order in which the customers receive the service is the FIFO (first in, first out) rule.

5. In usual practice population the arrival rate is not affected by the deflection in the population itself, caused by units waiting for service and being served.

6. The state probability can be defined as the total number of customers being served at the service which will lead directly to the evaluation of the mean number of units that are waiting and the mean waiting time. Assuming that the probability distribution of customer arrival and service times is stationary (unchanging in time), it could be expected that after a reasonable period of time the system would reach ‘equilibrium’, i.e. after a transient, the system state probabilities settle down to constant values in time.

If the service time and the interarrival time distributions are exponential and the rates \( \lambda_c \) and \( \lambda \) are known, the total number of patients in the system, the average waiting time and the number of patients waiting, can be easily evaluated with the following equations, assuming the queue theory applicable locally

*Expected mean number of units in the system*

\[
N_{tot} = \frac{\rho}{1-\rho} = \frac{\lambda}{\lambda_c - \lambda}
\]  

(A-4)

denotes number in the waiting line, not yet treated \( N_{NT} \)
\[ N_{NT} = \frac{\rho^2}{1 - \rho} = \frac{\lambda^2}{\lambda_C \cdot (\lambda_C - \lambda)} \]  

**(A-5)**

mean waiting time \( W_q \) for those who are in line

\[ W_{T_q} = \frac{\lambda}{\lambda_C \cdot (\lambda_C - \lambda)} \]  

**(A-6)**

average or mean time \( W_{T_{tot}} \) in the system

\[ W_{T_{tot}} = \frac{N_{NT}}{\lambda} = \frac{1}{\lambda_C - \lambda} = W_{T_q} + \frac{1}{\lambda_C} \]  

**(A-7)**

the average waiting time for those who wait

\[ W_{T_w} = \frac{1}{\lambda_C + \lambda} \]  

**(A-8)**

**Appendix B: Statistical Analysis of a California Hospital**

In the statistical analysis, only General Medical / Surgical facilities with emergency department are considered out of the nine possible Principal Type of Service\(^5\). Data regarding Long-term Care (SN/IC), Psychiatric, Chemical Dependency (alcohol/Drug), pediatric, Physical Rehabilitation, Orthopedic or Pediatrics Orthopedics development disabled are not considered able to provide appropriate contribution during the disaster.

- **NofBed** number of beds taken from the total licensed number of beds of each hospital\(^6\);
- **NofOR** number of operating rooms of each hospital, assumed to be equal to the number of emergency stations\(^7\);
- **Effst** efficiency taken as the number of medical surgery discharges on the total number of EMS stations\(^8\)

\(^5\) Column M of the worksheet, named TYPE_SVC_PRINCIPAL download from http://www.oshpd.ca.gov/
\(^6\) Column CK named HOSP_TOTAL_BED_LIC
\(^7\) Column FF named EMS_STATION
\(^8\) Ratio between values of Column AC called MED_SURG_DIS and values of column FF named EMS_STATION
\[ E = \frac{MED\_SURG\_DIS}{EMS\_STATION} \]  

\( \alpha \)  
patient mix index, assumed equal to the ratio between the average of the daily admission to the emergency department\(^9\) and the average value of emergency department visits\(^{10} \)

\[ \alpha = \frac{EMS\_ADM\_VIS\_TOTL}{EMS\_VIS\_TOTL} \]  

EDvisits  
average number of emergency department visits\(^{11} \).

The analysis of sensitivity considers the value of the coefficient of determination \( R^2 \), defined as the proportion of variability in a data set that is accounted for by a statistical model. In this definition, the term "variability" stands for variance or, equivalently, sum of squares. There are equivalent expressions for \( R^2 \). The version most common in statistics texts is based on analysis of variance decomposition as follows

\[ R^2 = \frac{SS_r}{SS_T} \]  

where \( SS_r = \sum_n (\hat{y} - \bar{y})^2 \) is the regression sum of squares;

\[ SS_T = \sum_n (y - \bar{y})^2 \]  
is the total sum of squares.

The coefficients used in the model are reported in Table B-1 that provide the number of patients per minute arriving at the emergency department.

---

\(^9\) Column FE, titled EMS\_ADM\_VIS\_TOTL / 365  
\(^{10}\) Column FD named EMS\_VIS\_TOTL / 365  
\(^{11}\) Column FD named EMS\_VIS\_TOTL / 365
Table B-1 Coefficient of regression from the statistical analysis of National Statistics CA

<table>
<thead>
<tr>
<th>Arrival rate $\lambda_0$ (National CA Statistics 2005)</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_0 = 26.1787 + 0.2509 B$</td>
<td>0.5441</td>
</tr>
<tr>
<td>$\lambda_0 = 13.7331 + 3.8370 OR$</td>
<td>0.6617</td>
</tr>
<tr>
<td>$\lambda_0 = 59.6147 + 0.0616 E$</td>
<td>0.0459</td>
</tr>
<tr>
<td>$\lambda_0 = 56.6376 + 66640.5711 \alpha$</td>
<td>0.0555</td>
</tr>
<tr>
<td>$\lambda_0 = 8.3278 + 0.1111 B + 2.7513 OR$</td>
<td>0.7155</td>
</tr>
<tr>
<td>$\lambda_0 = 25.3980 + 0.0196 B + 1.9534 OR + 0.0033 B OR$</td>
<td>0.7436</td>
</tr>
<tr>
<td>$\lambda_0 = 11.7309 - 0.0889 B + 4.9525 OR + 0.0002 B^2 - 0.0359 OR^2 + 0.0003 B OR$</td>
<td>0.7739</td>
</tr>
<tr>
<td>$\lambda_0 = 1.8554 + 0.0936 B + 2.9045 OR + 0.0211 E$</td>
<td>0.7195</td>
</tr>
<tr>
<td>$\lambda_0 = 17.1051 + 0.0164 B + 1.1474 OR + 0.0412 E + 0.0041 B OR - 0.0002 B E + 0.0030 OR E$</td>
<td>0.7624</td>
</tr>
<tr>
<td>$\lambda_0 = -10.7908 - 0.0798 B + 4.3534 OR + 0.1080 E + 0.0004 B^2 - 0.0206 OR^2 + 0.00003 E^2 - 0.0020 B OR - 0.0002 B E + 0.0024 OR E$</td>
<td>0.8047</td>
</tr>
<tr>
<td>$\lambda_0 = 7.5416 + 0.0981 B + 3.1199 OR + 0.0485 E + 53907.4361 \alpha$</td>
<td>0.7421</td>
</tr>
<tr>
<td>$\lambda_0 = 3.1221 - 0.0253 B + 1.8378 OR + 0.0222 E + 72256.6998 \alpha - 0.0001 B OR + 0.0046 E + 114.2760 B \alpha + 0.0062 OR E + 4101.3047 OR \alpha + 46.6135 E$</td>
<td>0.8143</td>
</tr>
<tr>
<td>$\alpha$ $\lambda_0 = -13.254 - 0.0539 B + 4.5457 OR + 0.0815 E + 39486.8581 \alpha + 0.0002 B^2 - 0.0353 OR^2 + -0.0006 E^2 - 127433266.4197 \alpha^2 + 0.0002 B OR + 0.0014 B E - 74.7959 B \alpha + 0.0037 OR E + 1365.7695 OR \alpha + 125.7832 E \alpha$</td>
<td>0.8539</td>
</tr>
</tbody>
</table>

Appendix C: Fragility Curves for Different Rehabilitation Strategies

In this appendix are shown the fragility curves for structural and nonstructural (drift and acceleration sensitive) components, using HAZUS assessment data. The fragility curves are shown for two types of structures called C2M and C2L according to HAZUS definition and they are related to the four different retrofit options and four different damage states.
Figure C-1 Structural fragility curves for C2M structure: (a) No Action; (b) Rehabilitation Life Safety; (c) Rehabilitation Immediate Occupancy; (d) Rebuild

- No Action - (C2M Buildings)
  - Return Period (yrs): 0 500 1000 1500 2000 2500
  - Probability of exceeding:
    - Slight Damage: $y=413$ yrs, $\beta=0.60$
    - Moderate Damage: $y=629$ yrs, $\beta=0.48$
    - Extensive Damage: $y=1101$ yrs, $\beta=0.49$
    - Complete Damage: $y=2047$ yrs, $\beta=0.62$

- Rehabilitation Life Safety - (C2M Buildings)
  - Return Period (yrs): 0 500 1000 1500 2000 2500
  - Probability of exceeding:
    - Slight Damage: $y=475$ yrs, $\beta=0.54$
    - Moderate Damage: $y=800$ yrs, $\beta=0.50$
    - Extensive Damage: $y=1815$ yrs, $\beta=0.59$
    - Complete Damage: $y=2551$ yrs, $\beta=0.03$

- Rehabilitation Immediate Occupancy - (C2M Buildings)
  - Return Period (yrs): 0 500 1000 1500 2000 2500
  - Probability of exceeding:
    - Slight Damage: $y=547$ yrs, $\beta=0.52$
    - Moderate Damage: $y=1101$ yrs, $\beta=0.59$
    - Extensive Damage: $y=2551$ yrs, $\beta=0.03$
    - Complete Damage: $y=2551$ yrs, $\beta=0.01$

- Rebuild - (C2M Buildings)
  - Return Period (yrs): 0 500 1000 1500 2000 2500
  - Probability of exceeding:
    - Slight Damage: $y=695$ yrs, $\beta=0.53$
    - Moderate Damage: $y=1486$ yrs, $\beta=0.52$
    - Extensive Damage: $y=2551$ yrs, $\beta=0.02$
    - Complete Damage: $y=2551$ yrs, $\beta=0.01$
Figure C-2 Fragility curves of drift sensitive nonstructural components for C2M structure: (a) No Action; (b) Rehabilitation Life Safety; (c) Rehabilitation Immediate Occupancy; (d) Rebuild
Figure C-3 Fragility curves of acceleration sensitive nonstructural components for C2M structure: (a) No Action; (b) Rehabilitation Life Safety (c) Rehabilitation Immediate Occupancy; (d) Rebuild
Figure C-4 Structural fragility curves for C2L structure: (a) No Action; (b) Rehabilitation Life Safety; (c) Rehabilitation Immediate Occupancy; (d) Rebuild
Figure C-5 Fragility curves of drift sensitive nonstructural components for C2L structure: (a) No Action; (b) Rehabilitation Life Safety (c) Rehabilitation Immediate Occupancy; (d) Rebuild
Figure C-6 Fragility curves of acceleration sensitive nonstructural components for C2L structure: (a) No Action; (b) Rehabilitation Life Safety (c) Rehabilitation Immediate Occupancy; (d) Rebuild
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