A Program to Generate Site Dependent Time Histories: EQGEN

by

G.W. Ellis, M. Srinivasan and A.S. Cakmak

Technical Report NCEER-90-0009
January 30, 1990

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A PROGRAM TO GENERATE SITE
DEPENDENT TIME HISTORIES: EQGEN

by

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PREFACE

The National Center for Earthquake Engineering Research (NCEER) is devoted to the expansion and dissemination of knowledge about earthquakes, the improvement of earthquake-resistant design, and the implementation of seismic hazard mitigation procedures to minimize loss of lives and property. The emphasis is on structures and lifelines that are found in zones of moderate to high seismicity throughout the United States.

NCEER’s research is being carried out in an integrated and coordinated manner following a structured program. The current research program comprises four main areas:

- Existing and New Structures
- Secondary and Protective Systems
- Lifeline Systems
- Disaster Research and Planning

This technical report pertains to Program 3, Lifeline Systems, and more specifically to the study of dams, bridges and infrastructures.

The safe and serviceable operation of lifeline systems such as gas, electricity, oil, water, communication and transportation networks, immediately after a severe earthquake, is of crucial importance to the welfare of the general public, and to the mitigation of seismic hazards upon society at large. The long-term goals of the lifeline study are to evaluate the seismic performance of lifeline systems in general, and to recommend measures for mitigating the societal risk arising from their failures.

In addition to the study of specific lifeline systems, such as water delivery and crude oil transmission systems, effort is directed toward the study of the behavior of dams, bridges and infrastructures under seismic conditions. Seismological and geotechnical issues, such as variation in seismic intensity from attenuation effects, faulting, liquefaction and spatial variability of soil properties are topics under investigation. These topics are shown in the figure below.

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In developing engineering models for earthquake ground motion in seismically active zones, the study focuses on the use of multivariate autoregressive moving average methods to model earthquakes. The modeling parameters used in the analysis of the accelerograms are related to physical variables such as earthquake magnitude, epicentral distance and site geology and a set of regression relations derived for the region. This enables the simulation of site-faithful time histories for regions without ground motion records. This study also deals with the extension of this procedure to model multiple event earthquakes. Studies are conducted on multiple event earthquakes in Mexico and Japan and suitable regression relations between the modeling parameters and site conditions are developed. This report further includes an alternative procedure to model the time series based on the digital simulation of the power spectral density function.
ABSTRACT

This paper presents the final report of a study on the modelling of earthquake ground motion in seismically active zones. The first part of the study focusses on the use of parametric time series methods to model earthquakes. The modelling parameters used in the analysis of the accelerograms are related to physical variables such as earthquake magnitude, epicentral distance and site geology and a set of regression relations derived for the region. This enables the simulation of site-faithful time histories for regions without ground motion records. The modelling of the earthquakes is done using a multivariate Autoregressive Moving Average process. The second part of the study deals with the extension of this procedure to model multiple event earthquakes. Studies are conducted on multiple event earthquakes in Mexico and Japan and suitable regression relations between the modelling parameters and the site conditions are developed. This report also includes an alternative procedure to model the time series based on the digital simulation of the power spectral density function. The fundamental advantage of these procedures is that the time invariant parameters for both the power spectral density and the ARMA models can be easily related to physical variables at the site. Further, confidence intervals for the modelling parameters can be used to generate an ensemble of earthquakes corresponding to the mean, mean plus one standard deviation etc. for design purposes.
ACKNOWLEDGEMENTS

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SECTION 1

INTRODUCTION

The fundamental goal of the structural engineer in designing a structure is to ensure that it performs adequately in the face of the various loadings it might experience during its lifetime. The prediction of some types of loadings such as earthquake effects is not very easy given the paucity of data. When recordings of ground motion at a site are known, the designer can simulate future loadings with these to obtain an estimate of future seismic loadings. However, there are very few sites with sufficient recordings of strong ground motion. Simulated or synthetic accelerograms are therefore a very useful tool in design. Further, earthquakes are random events both in their occurrence as well as their time history. Many of the approaches taken to model earthquakes have been using stochastic models (Housner, 1964; Jennings, 1968; Kozin, 1977; Chang et. al, 1982, Gersch and Kitagawa, 1985; Ellis and Cakmak, 1987).

A very important part of the modelling procedure is to relate the model parameters to the physical parameters which affect ground motion. This will enable the designer to generate an artificial accelerogram corresponding to an earthquake at that site with a given magnitude, distance from epicenter and other geological site conditions. This paper presents a simplified procedure for the simultaneous modelling of the three components of an earthquake, starting with a stationary series and introducing nonstationarity in amplitude and frequency and is a summary of earlier work done by the authors in this area. The modelling of the stationary series is done through a choice of time series procedure (ARMA) or through digital simulation using the power spectral density function in the form of the Kanai-Tajimi (KT) spectrum. The modelling parameters are related to physical variables through empirical relations derived from the study of the extensive databases of free-field strong motion
accelerograms recorded in California, Mexico, Taiwan and Japan (Ellis and Cakmak, 1987,88). Sample accelerograms are also shown and compared.
SECTION 2
GROUND MOTION PROPERTIES

Recorded accelerograms show nonstationarity in two areas - amplitude and frequency. Amplitude nonstationarity is usually seen as a buildup in the magnitude of the acceleration record over time as well as a decay. One way to achieve this is to multiply a stationary series by a deterministic function of time which changes amplitude like the original accelerogram.

Frequency nonstationarity is also noticeable in accelerograms. The zero crossing rate of the accelerogram is a good indicator of the changes in the frequency content. Typically, the predominant frequency decreases with time.

Multiple shock earthquakes can occur when the rupture reaches a barrier that stops the faulting. The subevents in the earthquake can manifest themselves as individual periods of strong shaking in the accelerogram.

A general procedure to simulate accelerograms should include the effects of nonstationarity in the simulation process so that the simulated records look similar to past accelerograms as well as have similar effects upon structures subjected to them. Multiple peak earthquakes have been modelled for the first time by Ellis and Cakmak (1988).
SECTION 3
MODELLING PROCEDURE

The modelling procedure as described below analyses the earthquake time history through its three components (2 horizontal and 1 vertical). The accelerogram is first truncated to its period of strong motion. The amplitude nonstationarity is then removed using a combination of a standard deviation and vertical angle functions. The resulting series is then rescaled in time using a frequency function dependent on the crossing rate and is then a stationary series both in amplitude as well as in frequency. This stationary or stabilized series can then be analyzed to yield time invariant parameters for regression analysis.

3.1 Determining the Duration

The multivariate earthquake model is given by the two horizontal components $H_1$ and $H_2$ and the vertical component $V$. These can be transformed into spherical coordinates ($\rho$, $a$, and $\gamma$) as:

$$H_1(t) = \rho(t)\cos\gamma(t)\cos a(t)$$
$$H_2(t) = \rho(t)\cos\gamma(t)\sin a(t)$$
$$V(t) = \rho(t)\sin\gamma(t)$$

This is shown in Fig. (3-1). The vector magnitude $\rho(t)$ and the vertical angle $\gamma(t)$ are used to stabilize the acceleration series.

The first step in the modelling procedure is the determination of the duration of strong motion $T$. The Arias intensity curve (Arias, 1970) which is a plot of the cumulative energy of the earthquake with time is computed and the first 1% and the
FIGURE 3-1 Components Of The Earthquake
last 2% of the cumulative energy is eliminated. The cumulative energy is given by:

$$I_0 = \int_{t=0}^{t=T} \rho^2(t) dt$$  \hspace{1cm} (3.4)

For an accelerogram digitized at a discrete time interval $\Delta t$, the cumulative energy is given by:

$$I_0 = \sum \rho_i^2 \Delta t$$  \hspace{1cm} (3.5)

The time between the 1st and 98% of the energy values is the duration of the earthquake. This process is shown in Fig. (3-2).

### 3.2 Estimating the Standard Deviation Envelope

The variance envelope is estimated by squaring the acceleration and calculating its running average by using an equally weighted two-second time window. For accelerograms digitized at increments of 0.02 seconds, this can be given by:

$$\sigma_i^2(t) = \frac{1}{101} \sum_{i=r-50}^{i=t+50} Z_i^2$$  \hspace{1cm} (3.6)

The two second window was arrived at by checking various window sizes to obtain a smooth estimate of the variance function. A standard deviation function $\hat{\sigma}(t)$, of the form suggested by Ellis, DeVeaux and Cakmak (1988)

$$\hat{\sigma}(t) = c_1(\alpha - k_1)(\frac{t}{\tau})^{p}e^{-(\frac{\tau}{\tau^2})} + k_1$$  \hspace{1cm} (3.7)

where

$$c_1 = \frac{(2\sqrt{3})^{p}e^{p}}{p^p}$$  \hspace{1cm} (3.8)

$$c_2 = 2\sqrt{3}$$  \hspace{1cm} (3.9)

is fitted to the standard deviation envelope which is given by the square root of the variance envelope. This is shown in Fig. (3-3). The parameter $\alpha$ is a measure of the strong shaking. $k_1$ is the standard deviation of the weak shaking. Parameter $\tau$
calculate cumulative energy of vector magnitude

\[ I_0 = \sum \rho_i^2 \Delta t \]
FIGURE 3-3 Vertical Angle, Standard Deviation And Zero Crossing Envelopes
measures the duration of strong shaking and the product $p\tau$ is a measure of the
time to the maximum of the function, $t_{max}$. These parameters are fitted by minimizing
the error between the standard deviation envelope and the function to be fit using a
nonlinear least squares routine.

### 3.3 Estimating the Vertical Angle Envelope

The vertical angle envelope is calculated by first estimating the vertical angle by using
a running two-second window over the absolute value of the vertical angle $\gamma_i$ by:

$$
\gamma(t) = \frac{1}{101} \sum_{i=t+50}^{i=t+50} |\gamma_i|
$$

A smooth function, $\hat{\gamma}(t)$ of the form is then fitted to this average function value as:

$$
\hat{\gamma}(t) = (c_3 - \gamma_f)(1 + \frac{t}{b_3})e^{-\frac{t}{b_3}} + \gamma_f
$$

Here, $c_3$ is the initial value of the function, $b_3$ is the rate of decay and $\gamma_f$ is the lower
limit of the function. The value of $c_3$ is estimated as the mean value of $\gamma_t$ during
the first 10% of the record and $\gamma_f$ as the mean value of $\gamma_t$ during the final 1/3 of the
record. Finally, $b_3$ is estimated so that the areas under $\gamma(t)$ and $\hat{\gamma}(t)$ are equal. Figure
(3-3) also shows the calculation of the vertical angle envelope.

### 3.4 Stabilizing the Accelerograms - Amplitude

The three nonstationary components of the accelerogram $H_1(t), H_2(t)$ and $V(t)$ are
transformed into three stationary time series $\hat{H}_1(t), \hat{H}_2(t)$ and $\hat{V}(t)$ by dividing them
by a function of the standard deviation and vertical envelope functions. This proce-
dure removes the nonstationarity in amplitude. The variance of these stabilized series
is then equal to 1.

$$
\hat{H}_1(t) = \frac{H_1(t)}{\sqrt{2}\sigma_p(t)\cos\hat{\gamma}(t)}
$$

3-6
\[
\hat{H}_2(t) = \frac{H_2(t)}{\sqrt{2} \sigma_p(t) \cos \gamma(t)} \\
\hat{V}(t) = \frac{V(t)}{\sigma_p(t) \sin \gamma(t)}
\] (3.13) (3.14)

3.5 Stabilizing the Accelerograms - Frequency

The frequency content of the accelerogram is stabilized by using a frequency envelope function to change the time scale of the record as shown in Fig. (3-3). The frequency envelope shows the time varying behavior of the zero crossing rate. The zero crossing rate \( F_c(t) \) is found by using a two second window as

\[
F_c(t) = \frac{\text{total crossings of all components between } t \pm 1 \text{ second}}{2 \text{ seconds}}
\] (3.15)

A smooth function of the form

\[
\hat{F}_c(t) = (c_2 - k_2)(1 + \frac{t}{b_2})e^{-\frac{t}{b_2}} + k_2
\] (3.16)

is fitted to the zero crossing rate. The value of \( c_2 \) is estimated as the mean of \( F_c(t) \) during the first 10% of the record and the lower limit \( k_2 \) is estimated as the mean value of \( F_c(t) \) during the final 1/3 of the record. The parameter \( b_2 \) is estimated so that the areas under \( F_c(t) \) and \( \hat{F}_c(t) \) are equal.

The frequency function \( \hat{F}_c(t) \) is then used to change the time increments of the variance stabilized acceleration components by

\[
\Delta t' = (\Delta t) \hat{F}_c(t)
\] (3.17)

where \( \Delta t' \) is the new time increment and \( \Delta t \) is the original time increment of 0.02 seconds. The transformed records are then reduced to the same length of time as the original records by

\[
\Delta t'' = \frac{\Delta t'}{\text{duration of original record}} \times \frac{\text{duration of transformed record}}{\text{duration of original record}}
\] (3.18)

and redigitized at 0.02 second increments using linear interpolation.
3.6 Stabilization - Multiple Periods of Shaking

In order to model accelerograms with more than one segment of strong shaking, only the standard deviation function needs to be modified. The standard deviation average function and a standard deviation function with several peaks is fit to the data. For example, if the earthquake had two peaks, the standard deviation envelope would be:

\[
\hat{\sigma}_p(t) = c_1(\alpha_k - k_1)(\frac{t}{\tau_1})^p e^{-\left(\frac{\alpha_1}{\tau_1}\right)t} + c_1\alpha_2\left(\frac{t - \Delta'}{\tau_2}\right)^p e^{-\left(\frac{\alpha_2}{\tau_2}\right)(t - \Delta')} + k_1
\]

(3.19)

This function is a summation of the function in Eq. (3-7) with a second function of maximum intensity \(\alpha_2\) with a time lag of \(\Delta'\) between the peaks of the two periods of strong shaking. The maximum of each period of strong shaking is estimated by \(\alpha_1\) and \(\alpha_2\) and the durations by \(\tau_1\) and \(\tau_2\). Finally, \(p\) can be estimated.

3.7 Modelling the Stabilized Series

The three components of the original accelerogram have been converted into three stationary series \(\hat{H}_1(t), \hat{H}_2(t)\) and \(\hat{V}(t)\). These series can be modelled in two ways.

(i) Kanai-Tajimi Model

The three components can be modelled as stationary processes with a power spectral density function of the form of the Kanai-Tajimi spectrum (Tajimi, 1960; Kanai, 1961). The Kanai-Tajimi spectrum is of the form:

\[
S(\omega) = G_0 \frac{1 + 4\xi_g^2 \left( \frac{\omega}{\omega_g} \right)^2}{\left[ 1 - \left( \frac{\omega}{\omega_g} \right)^2 \right]^2 + 4\xi_g^2 \left( \frac{\omega}{\omega_g} \right)^2}
\]

(3.20)

where

\[
\omega_g = \text{characteristic ground frequency (rad/s)}
\]

(3.21)

\[
\xi_g = \text{characteristic ground damping}
\]

(3.22)

\[
G_0 = \text{intensity of gaussian white noise}
\]

(3.23)
The three parameters of the Kanai-Tajimi spectrum can be estimated from the stabilized records for each of the three stationary series as follows:

The Kanai-Tajimi spectrum has a maximum value of

\[ S_{\text{max}} = G_0 \frac{16 \xi_g^4}{16 \xi_g^4 - 8 \xi_g^2 + 2(1 + 8 \xi_g^2)^{1/2} - 2} \]  \hspace{1cm} (3.24)

at

\[ \omega_m = \left( (1 + 8 \xi_g^2)^{1/2} - 1 \right)^{1/2} \frac{\omega_g}{2 \xi_g} \]  \hspace{1cm} (3.25)

Further, the variance of the process is given by:

\[ \sigma^2 = \pi G_0 \omega_g \frac{(1 + 4 \xi_g^2)}{4 \xi_g} \]  \hspace{1cm} (3.26)

From the power spectrum of the stabilized records, the values of \( S_{\text{max}} \) and \( \omega_m \) can be estimated easily. Further, the variance of the stabilized series is 1. For the three unknowns \( G_0 \), \( \omega_g \) and \( \xi_g \), there are three nonlinear equations relating them to \( S_{\text{max}} \), \( \omega_m \) and the variance \( \sigma^2 = 1 \). The three parameters of the Kanai Tajimi spectrum can then be estimated as shown in Appendix A.

(ii) ARMA Model

In Ellis et al. (1988), the use of an ARMA (3,1) model to fit the stabilized series has been discussed. The ARMA model describes a linear relationship between the present and past values of a time series \( Z_t \) and a white noise shock \( a_t \) as

\[ Z_t - \phi_1 Z_{t-1} - \phi_2 Z_{t-2} - \phi_3 Z_{t-3} = a_t - \theta_1 a_{t-1} \]  \hspace{1cm} (3.27)

where \( \theta_1 \) was constrained to be 0.99. From the stabilized series, the ARMA constants as well as the white noise variance can be calculated using standard techniques (Box and Jenkins, 1976).
SECTION 4

RELATING MODEL PARAMETERS TO PHYSICAL VARIABLES

Once an accelerogram from a given region has been analyzed and the modelling parameters obtained, it is relatively easy to simulate many realizations of the stochastic process underlying that record. However, it is much more important to relate the parameters estimated from the recorded accelerograms to the physical parameters affecting ground motion. Ellis and Cakmak, (1987) applied the modelling procedures to large databases of free-field strong motion accelerograms recorded in California, Mexico and Taiwan. Ellis and Cakmak (1988) further applied these modelling procedures to a set of multiple event earthquakes in Japan. By using the relations they developed, it is possible to generate simulations for earthquakes in these regions given physical variables such as earthquake magnitude, epicentral distance and geological site variables. Tables 4-I and 4-II show examples of the type of regression relations developed for Taiwan and Mexico. In these tables, $f_{\text{max}}$ is the frequency at which the Fourier spectrum is maximum and $F(f_{\text{max}})$ is the peak value of the Fourier spectrum.

Once the various physical variables have been selected for a given site, the modelling parameters can be calculated for the region using functional relations of the type shown in Tables 4-I and 4-II. Other available tables can be found in Ellis and Cakmak (1987,88). These parameters can then be used to simulate a set of accelerograms as outlined in the section on the simulation procedure.
TABLE 4-I
Parametric Relations for Taiwan

\[ H = \text{horizontal components} \]
\[ V = \text{vertical distance} \]
\[ d = \text{epicentral distance (km)} \]
\[ d_h = \text{hypocentral distance (km)} \]
\[ M = \text{magnitude} \]
\[ h = \text{depth of epicenter (km)} \]

\[ \alpha = (-42 \pm 1.26) + (6.94 \pm 1.26) \ln \left( \frac{10^M}{d_h^{1.9} k_3^3} \right) \]
\[ \ln(\tau) = (0.574 \pm 0.095) + (0.011 \pm 0.0013) d \]
\[ k_1 = (-10.6 \pm 1.19) + (5.39 \pm 0.36) \alpha \]
\[ \ln(t_{\max}) = (1.527 \pm 0.24) + (0.130 \pm 0.007) d \]
\[ r_2 = 1.42 \pm 0.45 \]
\[ b_2 = 1.67 \pm 0.82 \]
\[ r_3 = (2.45 \pm 0.34) - (0.92 \pm 1.10) \tan^{-1} \frac{d}{h} \]
\[ b_3 = 1.44 \pm 0.6 \]
\[ k_3 = 0.338 \pm 0.078 \]
\[ f_{\max} = (17.2 \pm 1.21) - (2.08 \pm 0.19) M \]
\[ F(f_{\max}) = (0.029 \pm 0.038) - (0.026 \pm 0.0059) M \]
\[ \phi_1 + \phi_2 - \phi_3 = 0.977 \pm 0.160 \]
Table 4-II
Parametric Relations for Mexico

d = epicentral distance (km)
M = magnitude

\[ \alpha = (-1.3 \pm 13.1) + (13.24 \pm 2.22) \ln \left[ \frac{10^M}{d^3 k_3^3} \right] \]

\[ \ln(\tau) = (0.512 \pm 0.75) + (0.392 \pm 0.11)M - (3.63 \pm 1.17)k_3 \]

\[ k_1 = (2.04 \pm 2.0) + (0.247 \pm 0.023)\alpha \]

\[ \ln(t_{max}) = (1.249 \pm 0.22) + (0.00377 \pm 0.00086)d \]

\[ r_2 = 1.16 \pm 0.37 \]

\[ b_2 = 3.29 \pm 6.76 \]

\[ r_3 = 1.60 \pm 1.48 \]

\[ b_3 = 5.43 \pm 12.6 \]

\[ k_3 = \text{function(soiltype)} \]

\[ \ln(f_{max}) = (2.31 \pm 0.51) + (3.14 \pm 0.58)k_3 - (0.321 \pm 0.09)M - (0.00262 \pm 0.00 \]

\[ \ln[F(f_{max})] = (-1.08 \pm 0.11) + (0.0031 \pm 0.00023)d - (2.25 \pm 0.29)k_3 \]

\[ \phi_1 + \phi_2 + \phi_3 = (1.01 \pm 0.0081) - (0.0159 \pm 0.0019)\hat{f}_{max} \]
SECTION 5
SIMULATION PROCEDURE

The process for the generation of artificial accelerograms is a reversal of the modelling process. The procedure is outlined below and Figure (5-1) shows a flow chart of the process.

(a) Generate a Stationary Series

Once the site variables and the magnitude and epicentral distance of the earthquake have been chosen, the peak value of the Fourier spectrum as well as the frequency at which this value occurs can be obtained from the relationships between the physical and modelling variables (an example is shown in Table 4-I). The peak value of the spectral density can be obtained from the peak value of the Fourier spectrum. The stationary series can be generated in two ways as outlined before:

Kanai-Tajimi Model:

From the equations (3.24-26), the three parameters of the Kanai-Tajimi spectrum can be obtained as outlined in Appendix A. Once this is known, the stationary series can be simulated using the technique outlined by Shinozuka and Jan (1972). A stationary series $f(t)$ with a power spectral density $S(\omega)$ can be simulated by computing:

$$f(t) = \sqrt{2} \sum_{k=1}^{N} [S(\omega_k)\Delta\omega]^\frac{1}{2} \cos \left( \omega_k' + \phi_k \right)$$

where $N$ is the number of points to be summed over, $\Delta\omega$ is the frequency interval, $k$ is the summation index, $\omega_k$ is the frequency at index $k$, $\omega_k' = \omega_k + \Delta\omega$. The quantity $\phi_k$ is a random phase angle uniformly distributed between 0 and $2\pi$. As the function $S(\omega)$ is the Kanai-Tajimi spectrum with the calculated parameters, the stationary process can be calculated for each of the three components.
SIMPLIFIED SIMULATION PROCEDURE

Physical Relationships \[ \omega_{\text{max}} \] \[ S_{\text{max}} \] \[ \text{Generate Stationary Ground Motion} \]

Kanai Tajimi Model \[ \text{ARMA Model} \]

Transform Stationary motion

\[ F(t) \] \[ \gamma_{\text{max}} \] \[ \text{Add nonstationary frequency - redigitize} \]

\[ \text{Add nonstationary variance} \]

SIMULATED ACCELEROGRAM

FIGURE 5-1 Earthquake Simulation Procedure
The ARMA model:

Given the peak value of the spectral density and the frequency at this peak as well as the sum of the three parameters \( \phi_1, \phi_2 \) and \( \phi_3 \), the three multiplicative parameters \( \phi'_{1-3} \) can be back calculated as shown in Appendix B. From these, the three ARMA parameters \( \phi_{1-3} \) can be calculated as well as the variance of the white noise \( \sigma_w^2 \). From this, three series with stable variance and frequency content can be generated by:

\[
Z_t = \phi_1 Z_{t-1} + \phi_2 Z_{t-2} + \phi_3 Z_{t-3} - \theta_1 a_{t-1} + a_t
\]

(5.2)

(b) Add Nonstationary Frequency Content

A zero crossing frequency function \( \hat{F}_c(t) \) is computed from the model parameters and Eq. (3.16). The time axis of each component is rescaled by:

\[
\Delta t'_s = \frac{0.02}{\hat{F}_c(t)}
\]

(5.3)

After changing the time scale of each stationary component, the records are reduced to their original duration by:

\[
\Delta t''_s = \frac{\Delta t'_s \text{ duration of original record}}{\text{duration of transformed record}}
\]

(5.4)

The three components are then digitized into equal increments of 0.02 seconds.

(c) Add Nonstationary Amplitude

The standard deviation envelope and the vertical angle envelope are calculated from Equations (3.7) and (3.11). The standard deviation envelopes for the horizontal and vertical components are calculated as:

\[
\sigma_h(t) = \frac{1}{\sqrt{2}} \sigma_p(t) \cos \gamma(t)
\]

(5.5)

\[
\sigma_v(t) = \sigma_p(t) \sin \gamma(t)
\]

(5.6)

For the case with multiple periods of shaking, two other parameters \( \Delta' \) and \( \alpha_2 \) are needed. They can be calculated from Equation (3.19) by setting the derivative of
Equation (3.19) equal to zero yielding the time of the second peak,

\[ \frac{d\hat{\gamma}_p}{dt} = 0 \]  \hspace{1cm} (5.7)

and by setting the value of the function at the second peak equal to \( \alpha_2 \),

\[ \hat{\gamma}_p \left[ \frac{\rho_1}{2\sqrt{3}} + \Delta \right] = \alpha_2 \]  \hspace{1cm} (5.8)

u The two horizontal components are then multiplied by \( \sigma_h(t) \) and the vertical component by \( \sigma_v(t) \) to yield a set of artificial accelerogram components nonstationary in amplitude and frequency content as well as being consistent with the given site and physical conditions.

### 5.1 Examples

Figure (5-2b) shows the three components of the accelerogram of the September 19, 1985 Michoacan earthquake which occurred along the Pacific coast of Mexico and recorded a magnitude of 8.1 on the Richter scale. The damage in Mexico city at a distance of 350 kilometers from the epicenter was severe. At least 8000 people were killed and over several hundred buildings destroyed or badly damaged with an estimate of 4 billion dollars for the damage (Anderson et al, 1986). The three components shown in Figure (5-2b) were recorded at station SCTI which is located on soft sediment in the lakebed zone of the city. In order to simulate this earthquake, the model parameters were calculated for the same magnitude and distance. From Ellis and Cakmak (1987), the value of the constant \( \gamma_f \) was taken to be 0.14.

Figure (5-2a) shows the simulation of the three components of the SCTI accelerogram using the parameters listed above. Figure (5-3) shows a comparison of the response spectrum for velocity at 5% damping and the spectra are in good agreement over a range of periods.
FIGURE 5-2 Simulated And Recorded Motions - Mexico
Station SCTI (lakebed)

FIGURE 5-3 Comparison Of Response Spectrum
SECTION 6
CONCLUSIONS

This work presents a simplified and realistic procedure for the computation of artificial accelerograms using relationships between modelling variables and physical variables at the site. The advantage of using a stationary time series as the basis lies in the ease of relating physical variables to time invariant constants in the time series model. Also, these series can be simulated using either frequency domain (Kanai-Tajimi) or time domain (ARMA) simulation techniques.

Further, this technique can be extended to any region by following the matching process used in Ellis and Cakmak (1987) to relate modelling parameters (obtained from the analysis of existing accelerograms) to physical parameters (obtained from a site study). Relationships similar to those used in the above study can then be utilized in the simulation procedure to generate a set of accelerograms for the site for use in design. Also, regression errors and hence confidence intervals can be generated for the modelling parameters such as $f_{\text{max}}$ and an ensemble of earthquakes corresponding to the mean, mean plus one standard deviation etc. can be generated for design purposes (Ellis and Cakmak, 1987).
SECTION 7
SOFTWARE

The generation of the earthquake accelerograms as shown in this and earlier reports was done using a software package called **EQGEN**. This package was developed at Princeton University and is written in FORTRAN. It can be run on a microvax or other mainframes supporting standard fortran and does not require specialized libraries such as IMSL. The software is available through NCEER to interested researchers. **EQGEN** consists of two preprocessors which query the user for data and create an input file which is read by the main program for time history generation. Alternatively, the main program can be run interactively. The package can also be run on a personal computer having a FCRTRAN compiler although the run time may be large. The user’s guide for the program is included at the end of this report.
SECTION 8
REFERENCES


15. Polhemius, N.W., and Cakmak A.S., “Simulation of Earthquake ground Motions using Autoregressive Moving Average (ARMA) Models”, Earthquake Engineer-

APPENDIX A

ESTIMATION OF KT SPECTRUM PARAMETERS

The Kanai-Tajimi spectrum is given by:

\[ S_\omega = G_0 \frac{1 + 4\xi_g^2(\frac{\omega_g}{\omega_p})^2}{[1 - (\frac{\omega_g}{\omega_p})^2]^2 + 4\xi_g^2(\frac{\omega_g}{\omega_p})^2} \] (A-1)

where the quantities \( \omega_p \), \( \xi_g \) and \( G_0 \) represent the characteristic ground frequency, the ground damping and the white noise intensity. The three unknowns are these three parameters. The quantities known are the peak value of the spectral density function \( S_{max} \) and the frequency at this peak \( \omega_m \). Further the variance of the process is set to be 1.0. The three nonlinear equations then become:

\[ \frac{16\xi_g^4G_0}{16\xi_g^4 - 8\xi_g^2 + 2(1 + 8\xi_g^2)^{1/2} - 2} = S_{max} \] (A-2)

\[ \frac{(1 + 8\xi_g^2)^{1/2} - 1}{2\xi_g} \omega_g = \omega_m \] (A-3)

\[ \pi G_0 \omega_g \frac{(1 + 4\xi_g^2)}{4\xi_g} = 1 \] (A-4)

Substituting the value of \( G_0 \) from the first equation and the value of \( \omega_g \) from the second equation into the third equation, the following nonlinear equation in \( \xi_g \) is obtained:

\[ \frac{16\xi_g^4[(1 + 8\xi_g^2)^{1/2} - 1]^{1/2}}{(8\xi_g^4 - 4\xi_g^2 + (1 + 8\xi_g^2)^{1/2} - 1)(1 + 4\xi_g^2)} = \frac{1}{\pi \omega_m S_{max}} \] (A-5)

For a given value of \( \omega_m \) and \( S_{max} \) and knowing that the damping \( \xi_g \) falls between 0 and 1.0, the above equation can be solved to yield \( \xi_g \). Next, the ground frequency \( \omega_g \) can be calculated from the second of the three nonlinear equations. From these two
parameters, the white noise intensity $G_0$ can be found and the Kanai-Tajimi spectrum is now known completely.
APPENDIX B

ESTIMATION OF THE AR PARAMETERS

The AR parameters $\phi_{-3}$ are related to the multiplicative parameters $\phi'_{1-3}$ by:

\[
\begin{align*}
\phi_1 &= \phi'_1 + \phi'_3 \\
\phi_2 &= \phi'_2 - \phi'_1 \phi'_3 \\
\phi_3 &= -\phi'_2 \phi'_3
\end{align*}
\] (B-1, B-2, B-3)

The three quantities that are known from the relationship between the modelling and physical parameters are the frequency at which the power spectrum is the maximum, the peak value of the power spectrum and the sum of the AR parameters $\phi_{1-3}$. The first equation is given by:

\[
\begin{align*}
\omega_m &= \frac{50}{2\pi} \cos^{-1} \left[ \frac{\phi'_1}{2\sqrt{-\phi'_2}} \right] \\
\eta &= \cos \frac{2\pi \omega_m}{50}
\end{align*}
\] (B-4, B-5)

Using these two definitions, the parameter $\phi'_2$ can be expressed as:

\[
\phi'_2 = -\frac{\phi'_1^2}{4\eta^2}
\] (B-6)

Note that $\phi'_2$ must always be less than or equal to zero.

The equation for the sum of the three AR parameters is given by

\[
\phi_1 + \phi_2 + \phi_3 = C
\] (B-7)

where the quantity $C$ is related to the physical variables and is obtained by regression analysis of the site records. In terms of the multiplicative AR parameters, the same
relation can be expressed as:

\[ \phi'_1 + \phi'_2 + \phi'_3 - \phi'_1 \phi'_3 - \phi'_2 \phi'_3 = C \]  \hspace{1cm} (B-8)

From this equation, the parameter \( \phi'_3 \) can be expressed as:

\[ \phi'_3 = \frac{C - \phi'_1 - \phi'_2}{1 - \phi'_1 - \phi'_2} \]  \hspace{1cm} (B-9)

Finally, the peak value of the power spectrum is given by:

\[ S_{max} = \frac{2\sigma^2(1 + \theta_1^2 - 2\theta_1\eta)}{50(2\pi)\left[1 + \phi'_1^2 + \phi'_2^2 - 2\phi'_1(1 - \phi'_2)\eta - 2\phi'_2(2\eta^2 - 1)\right]\left[1 + \phi'_3^2 - 2\phi'_3\eta\right]} \]  \hspace{1cm} (B-10)

The parameter \( \sigma^2 \) is the variance of the white noise and is a function of the parameters \( \phi_{1-3} \) as given in Appendix C. The constant \( \theta_1 \) has been set equal to 0.99. Substituting the expressions for \( \phi'_2 \) and \( \phi'_3 \) in terms of \( \phi'_1 \), \( f_m \) and \( C \) into the expression for \( S_{max} \), a highly nonlinear expression in terms of \( \phi'_1 \) is obtained of the form:

\[ \Psi(f_m, C, \phi'_1) = S_{max} \]  \hspace{1cm} (B-11)

where \( \phi'_1 \) is the only unknown. This can be solved to yield \( \phi'_1 \) and then used to solve the other two multiplicative parameters. Then the AR coefficients \( \phi_{1-3} \) can be solved.
APPENDIX C

ESTIMATION OF THE WHITE NOISE VARIANCE

This section describes the relationship between the variance of the white noise and the ARMA parameters for the ARMA (3,1) model used in Ellis and Cakmak, (1987).

From Box and Jenkins (1976), the relationship between the variance of the time series and the variance of the white noise can be given by:

\[
\gamma_0 = \frac{\sigma^2 \frac{c_6 - c_7}{c_1 - c_2 - c_3 - c_4 - c_5}}{(C-1)}
\]

where

\[
c_1 = 1 - \phi_2 - \phi_3(\phi_1 + \phi_3) - \phi_1(\phi_1 + \phi_2\phi_3) \quad (C-2)
\]

\[
c_2 = \phi_2 [\phi_1(\phi_1 + \phi_3) + \phi_2(1 - \phi_2)] \quad (C-3)
\]

\[
c_3 = \phi_1 \phi_3 [\phi_1(\phi_1 + \phi_3) + \phi_2] \quad (C-4)
\]

\[
c_4 = \phi_3(1 - \phi_2)(\phi_1 \phi_2 + \phi_3) \quad (C-5)
\]

\[
c_5 = \phi_3^2 [\phi_2^2 - \phi_3(\phi_1 + \phi_3)] \quad (C-6)
\]

\[
c_6 = [1 - \theta_1(\phi_1 - \theta - 1)][1 - \phi_2 - \phi_1 \phi_3 - \phi_3^2] \quad (C-7)
\]

\[
c_7 = \theta_1 \left[ \phi_1 + \phi_2(\phi_1 + \phi_3) + \phi_3(\phi_1^2 + \phi_1 \phi_3 + \phi_2) \right] \quad (C-8)
\]

Since the time series has a variance of 1.0, the white noise variance for any component can be calculated as:

\[
\sigma^2 = \frac{c_1 - c_2 - c_3 - c_4 - c_5}{c_6 - c_7} \quad (C-9)
\]
APPENDIX D
EQGEN USER’S GUIDE

D.1 Introduction

This section describes the program EQGEN and is intended as a user’s guide for the program. A complete description of the algorithm used in the program can be found in Ellis, Cakmak and Srinivasan (1990).

The program assumes that the designer, prior to generating the earthquake, is privy to information about the region and the earthquake. The required information in the form of parameters is:

1. The earthquake magnitude
2. The distance from the epicenter
3. The soil parameter $k_3$
4. The duration
5. The region

The simulation process is based upon the computation of a set of model parameters from regression relations for the region of interest. These relations have been obtained for Mexico and Japan and are described in Ellis and Cakmak (1987). The program can also compute time histories for multiple event earthquakes (Ellis and Cakmak, 1988). For such situations, the additional data required is:

1. The earthquake magnitude for the second event
2. The lag time between the two events
3. The distance for the second event.
D.2 The Program Algorithm

Once the region is specified, the regression relations are used to obtain 2 frequency parameters which are the peak value of the fourier spectrum and the frequency at which this peak value occurs. A stationary series for each of the three components of the earthquake can then be generated in two ways:

- The Kanai Tajimi spectrum: Using the 2 frequency parameters and keeping the series variance to be 1.0, the three parameters of the KT spectrum - the white noise intensity, the ground frequency and the damping can be calculated. Then, a stationary series with the KT spectrum can be digitally simulated using the technique elucidated by Jan and Shinozuka (1972).

- The ARMA technique: From the two frequency parameters and keeping the series variance as 1.0, the white noise variance as well as the ARMA coefficients can be calculated and the time series can be generated using the definition of the ARMA process.

Once the stabilized series are calculated, the program then introduces nonstationarity in frequency through the use of a scaling function which distorts the time intervals and then rescales them to the desired earthquake duration. The scaling function is calculated from the regression analysis.

This is followed by the introduction of a time dependent amplitude function which introduces amplitude nonstationarity into the series. The amplitude function is also computed from the regression analysis. The three series (for the two horizontal and one vertical components) are then the desired time histories for the area and reflect the site conditions as well as the earthquake magnitude and other factors desired by the designer.

The available regression relations are valid in Mexico and Japan. It might be necessary to simulate time histories in other areas also. In such cases, the procedure
is modified to include the calculation of the regression relations and these must be done by the user first. The procedure essentially involves the statistical analysis of the available data base of acceleration histories and the technique is described in Ellis, Cakmak and Srinivasan (1990).

D.3 Program Description

The program EQGEN consists of three parts - two preprocessors and one computational routine. When run, the preprocessors will prompt the user for the required data such as magnitude, duration etc. and these must be kept ready for input. Once the preprocessor runs, it creates a file called sinpar which is the input file for the main program. If the user does not desire to use the preprocessors, he or she may run the main program interactively. The two preprocessors are for single and multiple event earthquakes respectively and are called single.f and double.f. The main program is called eqgen.f.

D.4 Compiling and Running the Preprocessors

The preprocessor is compiled on a unix based system as follows:

f77 single.f -o single.out

The preprocessor is run as follows:

single.out

A sample run session is shown below. The bold characters are input by the user on to the terminal.

single.out
ENTER THE EARTHQUAKE MAGNITUDE

7.0
ENTER THE DISTANCE IN KILOMETERS

200.0

ENTER THE VALUE OF K3

0.4

ENTER THE DURATION OF THE EARTHQUAKE - seconds

20.0

ENTER THE REGION  1=mexico  2=japan

1

ENTER GENERATION METHOD 1=KT spectrum  2=ARMA

2

CALCULATING SIMULATION PARAMETERS FOR A MAGNITUDE 7 EARTHQUAKE AT A DISTANCE OF 200 KILOMETERS. THE OUTPUT IS STORED IN FILE SINPAR

The input file sinpar looks as follows for an example case: The words after the ! sign are for explanation only and do not exist in the actual file.

20.0000 ! duration in seconds
1 ! Generation technique (1 = KT spectrum, 2 = ARMA)
2.00005e-02 ! Peak value of the fourier spectrum - horizontal comp.
9.47090 ! Frequency at which this peak occurs
1.00000 ! Series variance = 1.0
2.00005e-02 ! Peak value of the fourier spectrum - vertical comp.
9.47090 ! Frequency at which this peak occurs
1.00000 ! Series variance = 1.0
2.00005e-02 ! Peak value of the fourier spectrum - horizontal comp.
9.47090 ! Frequency at which this peak occurs
1.00000 ! Series variance = 1.0
1.48000 ! parameter R2 - frequency envelope
6.21000 ! parameter B2 - frequency envelope
A ! single peak (multiple peak would be B)
64.7455 ! parameter alpha - amplitude envelope
17.3359 ! parameter k1 - amplitude envelope
9.28129 ! parameter tau - amplitude envelope
B ! choice of input parameters p or t_max (A=input p)
4.57223 ! t_max
1.12000 ! parameter r3 - vertical angle envelope
1.17000 ! parameter b3 - vertical angle envelope
0.40000 ! parameter k3 (soil condition)

This file is created by the preprocessor program single.f which contains the required regression routines for the available regions (Mexico, Japan, California). As more sites are analyzed, future regions can be incorporated into the preprocessors very easily through the addition of their regression relations.

The main program ecgen.f contains about 900 lines of code and takes about 4 minutes on a microvax to generate a 1000 point (20 second) earthquake. The program reads in the input file sinpar (or can be run interactively) and creates the following output files:

- H1.DAT, H2.DAT, V.DAT - simulated components
- H1H.DAT, H2H.DAT, VH.DAT - variance stabilized series
- H1S.DAT, H2S.DAT, VS.DAT - stabilized (frequency+variance) series
- H1HS.DAT, H2HS.DAT, VHS.DAT - variance stabilized series (new time scale)
- STD.DAT - standard deviation envelope
- GAMAV.DAT - vertical angle envelope
• TFC.DAT - frequency envelope

The three files H1.DAT, H2.DAT, V.DAT are the two horizontal and one vertical component of the desired earthquake and they may be used in structural analysis as time histories faithful to the site.

D.5 Compiling and Running the Program

The program is compiled on a unix based system as follows:

f77 eqgen.f -o eqgen.out

The program is run on a unix based system as follows:

eqgen.out

For VMS or CMS based systems, the procedure is slightly different in that the compiling and running commands may vary. Also, the file name may have to be changed to eqgen.for or eqgen.fortran.

A sample run session for the main program is given below. Words in boldface are user inputs.

eqgen.out
SELECT ONE OF THE FOLLOWING
A- Run the program interactively
B- Run the program in batch mode
E- Exit

B

ENTER THE FILE NAME TO BE READ

sinpar