EXTERNALITY, MARKET IMPERFECTION, AND OPTIMAL ENVIRONMENTAL POLICY

by

Nicole L. Hunter

January 25, 2018

A dissertation submitted to the Faculty of the Graduate School of the University at Buffalo, State University of New York in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

Department of Economics
Copyright by
Nicole L. Hunter
2018

ii
Acknowledgements

I first want to express my sincere gratitude to my main thesis advisor Professor Winston Chang who has provided me with guidance in so many forms throughout the years. He has offered support, encouragement, and advice when I needed it most and has spent countless hours of his valuable time teaching and working with me throughout this program. I have never learned so much from one individual and working with him has been the apex of this experience, something I cherish beyond measure. I would like to acknowledge Professor Peter Morgan who has been a continual source of encouragement, inspiration and humor. As someone whom I have admired since my days as an undergraduate, being able to continually learn and grow under his guidance has been one of the most rewarding experiences I could have hoped for. I would like to thank Professor Yun Pei for agreeing to be on my dissertation committee and for the valuable time he has offered. Professors William Lorenz and James Holmes also warrant recognition as Professor Lorenz was responsible for facilitating my choice to become an economics major all those years ago, and Professor Holmes offered invaluable advice and research opportunities which helped me to build a strong foundation to succeed in a rigorous graduate program.

To my family, especially my mother Anne Ferraraccio, father James Hunter, step-parents Mary Pat Hunter and Jerry Ferraraccio, and my siblings Alicia Beard and Sara Ferraraccio, your love, encouragement, patience and support has served as a grounding presence in all areas of my life, but especially throughout these past several years. It is appreciated beyond measure. To my grandmother ”Mama” Mary Lou Zsiros who endlessly amazed me as a child
with knowing the most obscure answers to trivial pursuit questions, and had inspired me from a very young age to collect knowledge and never stop learning. I am eternally grateful to her for kindling those initial sparks of curiosity and intellect as they have brought me quite a long way. And lastly, I am grateful to all of my friends and colleagues who have, in various ways, helped and supported me along the way, all while offering endless laughs and memories. Special thanks are due to several individuals, each of whom has significantly impacted my graduate experience in some way: Sara LaBelle, Twisha Chatterjee, Ievgenii Kudko, Sayan Desarkar, Kriti Singh, Xinyan Chen, Alex Itterman, Bibaswan Chatterjee, Christina Wilsey, and Chiawei Liang.

January 25, 2018

Nicole L. Hunter
Department of Economics, University at Buffalo
415 Fronczak Hall
Buffalo, New York 14260
United States of America
Contents

Acknowledgements iii

Contents v

List of Tables viii

Abstract ix

1 The Effects of Entry under a Pollution Tax 1

1.1 Introduction ......................................................... 1

1.2 Preliminary Setup .................................................... 3

1.3 Model 1: Short-run Emission Tax ................................. 4

1.3.1 Optimal Emission Tax under No Entry ....................... 6

1.4 Model 2: Long-run Emission Tax ................................. 8

1.4.1 The Effect of $t$ on $q$ and $n$ ................................. 9

1.4.2 Optimal Emission Tax under Free Entry ................. 11

1.4.3 Excess or Insufficient Entry in the Presence of an Optimal Pollution Tax 12

1.5 Model 3: Long-run Emission Tax and Business Licensing ....... 13

1.6 An Example ............................................................ 15

1.6.1 Model 1: Short-run Emission Tax ........................... 15

1.6.2 Model 2: Long-run Emission Tax ............................. 18
1.6.3 Model 3: Long-run Emission Tax and Business Licensing . . . . . . . 20
1.6.4 Comparison of Models . . . . . . . . . . . . . . . . . . . . . . . . . . 23
1.7 Conclusion . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 23
References . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 24

2 Optimal Environmental Tax Scheme 25
2.1 Introduction . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 25
2.2 G(q,a) F(q,a) T(q,z) D(Z) (Kim and Chang’s Model) . . . . . . . . . . . . 29
2.3 Non-Uniform Pollution Damage . . . . . . . . . . . . . . . . . . . . . . . . . 35
  2.3.1 The General Case . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 37
2.4 An Externality Function . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 38
2.5 Other Policy Targets . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 40
  2.5.1 G(q,a) F(q,a) T(a,z) D(Z) . . . . . . . . . . . . . . . . . . . . . . . . . 41
  2.5.2 G(q,a) F(q,a) T(q,a) D(Z) . . . . . . . . . . . . . . . . . . . . . . . . . 42
2.6 One Policy Target, Two Externalities . . . . . . . . . . . . . . . . . . . . . . 44
  2.6.1 G(q,a) F(q,a) T(z) D(Z) . . . . . . . . . . . . . . . . . . . . . . . . . . 44
  2.6.2 G(q,a) F(q,a) T(a) D(Z) . . . . . . . . . . . . . . . . . . . . . . . . . . 45
  2.6.3 G(q,a) F(q,a) T(q) D(Z) . . . . . . . . . . . . . . . . . . . . . . . . . . 46
2.7 Conclusion . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 46
References . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 47

3 Optimal Policy on Externalities with Weighted Group Interests 49
3.1 Introduction . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 49
3.2 The Model . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 51
  3.2.1 The Welfare Function . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 53
3.3 Optimal Policy Analysis . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 54
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.3.1</td>
<td>The Firms</td>
<td>54</td>
</tr>
<tr>
<td>3.3.2</td>
<td>The Government</td>
<td>56</td>
</tr>
<tr>
<td>3.3.3</td>
<td>An Illustrative Example</td>
<td>59</td>
</tr>
<tr>
<td>3.4</td>
<td>The Case of Choosing $Q$ and $A$ by the Government</td>
<td>62</td>
</tr>
<tr>
<td>3.5</td>
<td>Technology and Cost Functions in Environmental Policy Analysis</td>
<td>64</td>
</tr>
<tr>
<td>3.5.1</td>
<td>Specification of Technology and the Implied Cost Function</td>
<td>64</td>
</tr>
<tr>
<td>3.5.2</td>
<td>An Alternative Formulation</td>
<td>67</td>
</tr>
<tr>
<td>3.5.3</td>
<td>An Integrated View</td>
<td>70</td>
</tr>
<tr>
<td>3.6</td>
<td>Conclusion</td>
<td>71</td>
</tr>
<tr>
<td>References</td>
<td>72</td>
<td></td>
</tr>
</tbody>
</table>

**Conclusion** 74
List of Tables

1.1 Example Results ................................................................. 23
Abstract

This dissertation consists of three chapters. The first chapter studies the effects of emission taxation on an oligopolistic industry. It shows that, in the short run with a fixed number of firms, an increase in the tax rate always lowers all firms’ outputs, resulting in a contraction of the industry output. In the long run, the tax affects firms’ entry and exit decisions and may yield ambiguous effects on the firms’ and industry outputs. It further shows that the tax reduces the equilibrium number of firms, and also reduces the equilibrium industry output if the inverse demand function is affine, concave, or not too convex. Finally, the chapter examines the question of the optimal number of firms from the social point of view. It shows excess or insufficient entry is determined by the relationship between the social marginal benefit of production and the responses of firms’ outputs to a change in the tax rate.

The second chapter focuses on the model of Kim and Chang (Journal of Regulatory Economics 1993). Their model provides a method of deriving an optimal nonlinear tax function for polluting firms in an imperfectly competitive industry under asymmetric information between firms and the regulator. They show that it is possible to devise an optimal uniform non-linear tax formula with only two pieces of information required for the regulator, i.e., the marginal damage function and the slope of the inverse demand function. This chapter examines the scope of applicability of their approach and finds it is capable of being extended to several cases. We discover two conditions which must be assumed in order to apply their model. If these conditions are not satisfied and optimal tax function cannot be guaranteed to be generated, however, the government can always resort to a unit tax rate to achieve it’s
objectives.

The welfare function which is standard in the literature includes consumer and producer surpluses, government revenue, and externality impact. The third chapter constructs a welfare function which includes different weights assigned to each of the welfare components to reflect either the regulator’s own judgment or special interest groups’ influence. In addition to a pollution externality, a consumption externality is also introduced in the model. The optimal government policies on consumption and pollution under imperfect competition are analyzed. The chapter shows that optimal tax rates are influenced significantly by the relative magnitudes of the welfare weights. It also shows that these rates can be higher (or lower) than their Pigouvian rates, even after adjusting for the market imperfection factor. Furthermore, some counter-intuitive results on the optimal tax rates may appear when the welfare weights are altered. Finally, this chapter examines various specifications of technology on output and pollution and derives the implied cost functions. This approach sheds light on the different properties the implied cost functions must possess. It reveals that the existing literature is quite lax or overly simplistic in making assumptions on the cost functions.
Chapter 1

The Effects of Entry under a Pollution Tax

1.1 Introduction

In much of the literature a common finding is that when considering an $n$-firm oligopoly engaging in Cournot competition, the introduction of a tax decreases industry output but has an ambiguous effect on individual firm output as well as the number of firms in the industry. When entry in this model setup is endogenous it has also been shown that excessive entry, relative to the socially optimal level, can result in the presence of certain market characteristics. In this chapter we explore these findings in a more general framework. We start with an overview of the relevant literature.

Seade (1980) examines the effects of entry on the optimal tax rate and its effects on welfare. He challenged the conventional notion that entry reduced profits per firm while expanding industry output as the economy moved closer to competitive equilibrium. In his model he treats entry as an exogenous change to the number of firms in the market and finds that a tax is always welfare-reducing.

Besley (1989) examines entry under a commodity tax in the case of an $n$-firm, homogeneous output Cournot oligopoly. He finds that without entry, a small tax is always welfare
reducing, but with entry it may be welfare improving. His model does not consider any damages or negative externalities to production.

Katsoulacos and Xepapadeas (1995) examine an endogenous, homogeneous $n$-firm oligopoly model in the presence of a pollution externality when firms have a pollution abatement option. Their main finding is that in an endogenous market structure, the emission tax can exceed marginal external damages. Due to the inclusion of abatement, their model appears more general and indeed their results reduce to the perfect competition and monopoly cases under proper assumptions. They assume a linear inverse demand function, a linear cost structure and a linear damage function. Such assumptions facilitate ease of exposition and lead to quantifiable results.

Mankiw and Whinston (1986) discuss inefficient levels of entry in markets where firms incur fixed setup costs. Entry is modeled by use of the zero profit condition. They find that the business stealing effect, which occurs when an increase in the number of firms causes the output of the incumbent firms to fall, is a critical determinant of the direction of entry bias. If the post-entry price exceeds marginal cost, and if a business-stealing effect exists, then free entry leads to excessive entry from a social standpoint. The free entry number of firms can be less than the welfare-maximizing number, but not by more than one firm (when considering the integer constraint). When firms incur fixed setup costs, the regulation of entry is often desirable. However, the regulation of entry becomes unnecessary as an industry comes close to the ideal of no fixed costs. If the post entry price is equal to the marginal cost, then there is no excessive entry and the number of free entry firms is socially optimal.

Lee (1999) examines the second-best output taxes for polluting oligopolists under endogenous market structure. With general functional forms, in the presence of external costs that vary with aggregate output, he shows that the equilibrium number of firms in an industry may differ from the socially optimal number of firms and that the optimal taxes under
CHAPTER 1: THE EFFECTS OF ENTRY UNDER A POLLUTION TAX

oligopolistic competition could be less than, equal to, or greater than marginal external damages depending upon the curvature of the market demand function.

In this chapter, we consider a more general framework regarding optimal environmental taxation with entry and exit. Our more general functional forms offer valuable insights into the composition of optimal choice variables for both firms and the government, and our results are also applicable to many special cases.

Section (1.2) outlines the basic setup of our three models. Section (1.3) considers a short-run model with a fixed number of firms and derives the optimal pollution tax rate. Section (1.4) considers a long-run model with free entry and examines the optimal pollution tax rate. It compares the free entry number of firms to the socially optimal level of entry. Section (1.5) also considers a long-run model in which the government, in addition to setting a pollution tax, grants operating licences to firms. In Section (1.6), we construct a numerical example to compare the specific results of the three models. Finally, Section (1.7) provides concluding remarks.

1.2 Preliminary Setup

First, we consider a short-run model in which the number of firms is fixed at $n$. We start by denoting aggregate output and the inverse demand function as

$$Q = \sum_{i=1}^{n} q_i ,$$

$$p = p(Q) ,$$

where each $q_i$ represents firm $i$’s output. The total cost function of a firm, excluding payment of a pollution tax, is $c_i(q_i)$. For simplicity, we assume all firms have identical cost functions
so that \( c_i(q_i) = c(q) \) for all \( i \).

A firm’s production process causes a quantity \( z \) of pollution which is non-decreasing in output, \( z = z(q) \). To control pollution, the government imposes a tax rate \( t \) on each unit of \( z \). Thus, a representative firm’s profit function is

\[
\pi = p(Q)q - c(q) - tz(q) \\
= r(q,n) - c(q) - tz(q) \equiv \pi(q,n,t), \tag{1.1}
\]

where \( r = p(Q)q \) is the total revenue of a firm and \( Q = nq \). Let the total quantity of pollution produced by the industry be denoted by \( Z = nz(q) \). Aggregate pollution causes damage \( D \) to society. Social welfare is defined as the social surplus contributed by the product adjusted for the pollution damage to society:

\[
W = \int_0^Q p(\mu) d\mu - nc(q) - D(Z(q,n)) = W(q,n).
\]

Unless otherwise specified, we assume \( D' > 0 \) and \( D'' > 0 \).

### 1.3 Model 1: Short-run Emission Tax

In this short-run model, the number of firms is fixed. We assume that all firms compete in the Cournot fashion. They choose outputs to maximize profits, treating other firms’ outputs as fixed:

\[
\frac{\partial \pi(q,n,t)}{\partial q} = \pi_q(q,n,t) = r_q(q,n) - c'(q) - tz'(q) = p(Q) + qp'(Q) - c'(q) - tz'(q) = 0, \tag{1.2}
\]
CHAPTER 1: THE EFFECTS OF ENTRY UNDER A POLLUTION TAX

where subscripts denote partial derivatives; e.g., 

\[ r_q = \frac{\partial r (q, n)}{\partial q} \text{ etc..} \]

We refer to \( z'(q) \) as the marginal pollution of production, or simply as marginal pollution. The second-order necessary condition requires

\[
\frac{\partial^2 \pi (q, n, t)}{\partial q^2} = \pi_{qq} (q, n, t) = r_{qq} (q, n) - c'' (q) - tz'' (q) = 2p' (Q) + qp'' (Q) - c'' (q) - tz'' (q) \leq 0.
\]

From (1.2), the Cournot-Nash equilibrium output of a firm is a function of \( n \) and \( t \);

\[ q = q (n, t). \]  

The equilibrium industry output is

\[ Q = nq (n, t) = Q (n, t). \]

To examine the effects of a change in \( n \) and \( t \) on the equilibrium value of \( q \), we apply the Implicit Function Theorem to obtain

\[
\frac{\partial q (n, t)}{\partial n} \bigg|_t = -\frac{\pi_{qn}}{\pi_{qq}} = -r_{qn} \frac{\pi_{qq}}{\pi_{qq}} = -q (p' + qp'') \frac{\pi_{qq}}{\pi_{qq}}, \quad (1.4a)
\]

\[
\frac{\partial q (n, t)}{\partial t} \bigg|_n = -\frac{\pi_{qt}}{\pi_{qq}} = \frac{z'}{\pi_{qq}} < 0. \quad (1.4b)
\]

We conclude that the sign of \( q_n \) is the same as that of \( r_{qn} \), or \( p' + qp'' \). If \( p'' \leq 0 \), then \( q_n < 0 \); but if \( p'' > 0 \), then the sign of \( q_n \) is in general indeterminate. The sign of \( q_t \) is negative since \( z' > 0 \) and \( \pi_{qq} < 0 \).
At the aggregate level, we have
\[
\frac{\partial Q(n, t)}{\partial n} = q + nq_n = q \left(1 - n \frac{p' + qp''}{\pi_{qq}}\right),
\]
\[
\frac{\partial Q(n, t)}{\partial t} = nq_t = n \frac{z'}{\pi_{qq}} < 0.
\]

Even if \(q_n < 0\), it seems that \(Q_n\) may still be positive. But if \(q_t < 0\) as is usual, then \(Q_t < 0\).

**Proposition 1** Consider a short-run, symmetric equilibrium. If \(p' + qp'' < 0\), demand is not too convex and an exogenous increase in \(n\) will decrease firm-level output \(q\) but aggregate output \(Q\) may increase or decrease. Furthermore, an increase in the pollution tax rate \(t\) will decrease both \(q\) and \(Q\).

### 1.3.1 Optimal Emission Tax under No Entry

Let the social welfare function in the present case be denoted by
\[
W_1(t; n) = \int_0^{nq(t)} p(\mu) \, d\mu - nc(q(t)) - D(nz(q(t))).
\]  

Assume that the government knows the symmetric industry equilibrium. Under a fixed \(n\), it chooses \(t\) to maximize \(W_1\). The first-order condition is
\[
W'_1(t) = [p - c'(q) - D'(Z)z'(q)] nq'(t) = 0,
\]
which requires
\[
p - c' = D'z',
\]  
since \(q'(t) < 0\) as shown in (1.4b). By interpreting \(p\) as the marginal social benefit of consuming an additional unit of the product, we see that this condition indicates that, at
the social optimum, the marginal social surplus of production, \( p - c' \), must equal the marginal pollution damage. From (1.2), it follows that the socially optimal tax rate, denoted by \( t_1 \), is

\[
t_1 = D' + \frac{qp'}{z'} = D' - \frac{p}{nz'\varepsilon} ,
\]

(1.7)

where \( \varepsilon (= -pdQ/Qdp) \) is the elasticity of market demand, and (1.7) implements the social optimum.\(^1\)

There are two main effects influencing the tax rate. One effect captures the distortion of output and price resulting from the presence of oligopolistic market power. This effect is negative given the inverse demand function is downward sloping, and pulls the tax rate towards subsidy. The other effect is positive due to marginal pollution damage. An additional term, \( z' \), is present. This marginal pollution term acts upon the market power term by reducing its impact on the tax rate. The regulator considers the positive impact to welfare that can be gained by bringing output closer to its competitive equilibrium level. However, as marginal pollution rises, the negative impact of pollution on welfare continually increases, and the tax rate becomes more positive. As \( \varepsilon \rightarrow \infty \), the tax rate approaches the marginal damage \( D' \).

In viewing (1.2), we can write the firms optimal level of output as

\[
q = \frac{c' + tz' - p}{p'} .
\]

A constraint on \( t \) exists in order to keep firm’s output level positive. For \( q > 0 \):

\[
t < \frac{1}{z'} (p - c') .
\]

\(^1\)Ebert (1992), Requate (1994) and Lee (1999) obtain a similar formula. Ebert (1992) also shows in the symmetric case that the social optimum can additionally be implemented by a tax on output.
Proposition 2 Given there is no entry or exit of firms, the optimal emission policy must aim at controlling pollution and at the same time alleviating market imperfection due to oligopoly. The resulting optimal rate, as shown in (1.7), is less than the marginal pollution damage $D'$. The optimal rate varies positively with the elasticity of demand, marginal pollution, and the number of firms.

1.4 Model 2: Long-run Emission Tax

In this section, we consider free entry and exit of firms. This pertains to a long-run situation in which firms enter a market if there is positive profit but leave if there is a loss. For simplicity, we assume that the number of firms is continuous. With (1.3) and (1.1) we obtain $\pi (q, n, t) = \pi (q (n, t), n, t) \equiv \tilde{\pi} (n, t)$. We postulate a simple entry-exit dynamic process:

$$\dot{n} = \tilde{\pi} (n, t).$$

For uniqueness and stability of an equilibrium $n$, it is required that

$$\frac{\partial \tilde{\pi} (n, t)}{\partial n} := \tilde{\pi}_n (n, t) < 0.$$

Using the first-order condition $\pi_q (q, n, t) = 0$ in (1.2), we have

$$\tilde{\pi}_n (n, t) = \pi_q (q, n, t) q_n (n, t) + \pi_n (q, n, t) = \pi_n (q, n, t) = q^2 p' < 0.$$

Thus the equilibrium $n$ is unique and stable.

The long-run equilibrium can be characterized by the following two-equation system.

---

2 Although $\dot{n}$ is conventionally expressed as a time derivative of $n$ with respect to $t$ ($= dn/dt$), please bear in mind that the $t$ in $\tilde{\pi} (n, t)$ is the tax rate, not time. Thus, the dynamic process here is autonomous.
(profit maximization and no new entry or exit):\(^3\)

\[
\pi_q (q, n; t) = 0, \quad (1.8a)
\]

\[
\pi (q, n; t) = 0, \quad (1.8b)
\]

from which we obtain equilibrium \(q\) and \(n\) as functions of \(t\).

### 1.4.1 The Effect of \(t\) on \(q\) and \(n\)

To examine the effects of changing \(t\) on the equilibrium \(q\) and \(n\) we totally differentiate (1.8) to obtain

\[
\begin{bmatrix}
\pi_{qq} & \pi_{qn} \\
\pi_q & \pi_n
\end{bmatrix}
\begin{bmatrix}
q' (t) \\
n' (t)
\end{bmatrix}
= -
\begin{bmatrix}
\pi_{qt} \\
\pi_t
\end{bmatrix},
\]

which is solved for

\[
q' (t) = \frac{\pi_t \pi_{qn} - \pi_n \pi_{qt}}{\pi_n \pi_{qq}}, \quad n' (t) = - \frac{\pi_t}{\pi_n}. \quad (1.9)
\]

From the first-order condition (1.8a), we have

\[
\pi_{qq} = 2p' + q p'' - c'' - t z'' < 0,
\]

\[
\pi_{qn} = q (p' + q p'') ,
\]

\[
\pi_{qt} = -z' < 0.
\]

From the zero-profit condition (1.8b), we have

\[
\pi_q = 0, \quad \pi_n = p' q^2 < 0, \quad \pi_t = -z < 0.
\]

\(^3\)Note that in a Cournot model, maximization of \(\pi\) with respect to \(q\) is under the assumption that all other firms’ outputs are constant. The resulting Cournot-Nash equilibrium allows us to treat \(Q\) as \(nq\) and hence express \(\pi_q\) in (1.2) as \(\pi_q (q, n, t) = 0\).
Utilizing these expressions to for further expansion of (1.9) yields

\[
q'(t) = -\frac{z(p' + qp'') - qp'z'}{qp' (2p' - c'' + qp'' - tz'')}, \quad (1.10a)
\]
\[
n'(t) = \frac{z}{q^2p'} < 0. \quad (1.10b)
\]

If the inverse demand function is linear and \( z = \delta q \), where \( \delta \geq 0 \), then \( q'(t) = 0 \), indicating that the output level is not dependent upon \( t \). If, however, the inverse demand function is non-linear and we still have \( z = \delta q \), then \( q'(t) \) reduces to

\[
-\frac{zp''}{qp'(2p' - c'' + qp'' - t z'')},
\]

the result obtained in Lee (1999) for \( z = q \).

For the aggregate, using \( Q'(t) = nq'(t) + qn'(t) \), we thus obtain

\[
Q'(t) = \frac{n \pi_t \pi_{qn} - qn \pi_{qq} - n \pi_n \pi_{qt}}{\pi \pi_{qq}} \quad (1.11a)
\]
\[
= \frac{zq (1 - n) p'' - z (p' (n - 2) + c'' + tz'') + nqp'z'}{qp' (2p' - c'' + qp'' - tz'')} \quad (1.11b)
\]

By inspection of signs in (1.11a), we see that if \( \pi_{qn} = q(p' + qp'') \geq 0 \), then \( Q'(t) < 0 \). We define \( \mu = qp''/p' \) to be the elasticity of the slope of the inverse demand, so that we can write \( p' + qp'' = 1 + \mu \).

**Lemma 1** The aggregate level of output is decreasing in the tax rate \( t \) if \( p' + qp'' \geq 0 \), or equivalently, \( \mu \geq -1 \). If \( p' + qp'' < 0 \), or \( \mu < -1 \), then the sign of \( Q' \) is ambiguous.

Recall that \( \varepsilon = -p/qp' \) is the elasticity of demand. Here we further define \( \eta = qz'/z \) to be the pollution elasticity of output. It follows that the sign of the numerator of (1.10a) is the same as \( \eta - \mu - 1 \). Given \( \eta \) is non-negative and \( p' < 0 \), we infer that \( q' > 0 \) if \( p'' < 0 \). However, \( p'' > 0 \) is not sufficient to guarantee \( q' > 0 \).

\[\footnote{Lee (1999) in a simpler model shows that \( p'' < 0 \) is sufficient to guarantee \( q'(t) > 0 \). This is due to his assumptions which lead to \( \varepsilon = 1 \).}\]
Proposition 3  The number of firms is always decreasing in the tax rate, \( n' < 0 \), but output per firm may increase, decrease or remain unchanged according to whether \( \eta \) is greater than, less than, or equal to \( 1 + \mu \).

In the case where \( q' > 0 \), the pollution elasticity of output has to be sufficiently greater than the elasticity of the slope of demand. If aggregate output falls, it opens the possibility that a surviving firm’s output may go up, as it would face a higher output price. If, however, aggregate output increases, the fall in \( n \) due to the higher tax must be relatively small.

1.4.2 Optimal Emission Tax under Free Entry

The tax rate which is consistent with both the first-order condition for firms and the zero-profit condition satisfies

\[
t = \frac{pq - c}{z} = \frac{p + qp' - c'}{z'}.\]

We call \( pq - c \) a firm’s operating profit (without taking into account of the tax), and \( p + qp' - c' \), its marginal operating profit.

Lemma 2  In the long-run equilibrium under a pollution tax, the tax rate will be equal to the ratio of a firm’s operating profit to its pollution amount, and is also equal to the ratio of its marginal operating profit to its marginal pollution amount.

The welfare function in the present case, denoted by \( W_2 \), is

\[
W_2 (t) = \int_{0}^{n(t)q(t)} p (\mu) d\mu - n (t) c (q (t)) - D (n (t) z (q (t))).
\]
The first-order condition is

\[ W'_2(t) = n \left( p - c' - D'z' \right) q' + (pq - c - D'z) n' \]

\[ = n \left( t z' - p'q - D'z' \right) q' + z \left( t - D' \right) n' = 0, \]  

(1.12a)

(1.12b)

by use of (1.2) and the zero-profit condition. The optimal tax rate under entry, denoted by \( t_2 \), is

\[ t_2 = D' + \frac{nq'p'}{Z'(t)} = D' + \frac{nqp'q'}{zn' + nq'z'} = D' + \frac{qp'}{hz'}, \]  

(1.13)

where\(^5\)

\[ h = 1 + \frac{1}{q' \cdot nz'}. \]

Since we only use a pollution tax/subsidy instrument, the optimal \( t \) seeks a balance between pollution damage and the market imperfection distortion in output due to oligopoly. Notice that the pollution effect is not only revealed in the \( D' \) term, the element \( z'(q) \) also appears in the market imperfection term. We can view the second term as the "modified" market imperfection effect. With only one instrument, the formula shows there is no complete decomposition of the two market externalities.\(^6\)

1.4.3 Excess or Insufficient Entry in the Presence of an Optimal Pollution Tax

Here we compare the free entry number of firms under the tax regime to that which would prevail with no taxation. In the latter case, there is still damage caused by pollution

\(^5\)Requate (1994) obtained a result similar to ours, which is seen when we set his \( e(t) \) to be our \( z(q(t)) \). Also, Lee (1999) obtained a form akin to ours in a simpler model in which \( z(q) = q \) so that \( z' = 1 \). If \( n \) is invariant to \( t \), then the optimal tax rate is the same as the short-run rate as shown in (1.7).

\(^6\)Requate (1994, p.10) considered a model using a pollution tax and a production subsidy and showed that each instrument aimed at its own intended target.
which affects welfare, but a firm does not internalize such a cost.

When considering the tax regime we can write, in view of (1.12a),

\[ W_2'(t) = W_q q'(t) + W_n n'(t) = n(p - c' - D'z')q'(t) + (pq - c - D'z)n'(t) = 0. \]

A closer look at the components of \( W_q \) shows that this is the social marginal benefit of production. When \( q \) changes in response to a change in \( t \), its impact on welfare takes into account the net benefit that a unit of output contributes to society, both the cost to the producer, \( c' \), and the cost to the environment, \( D'z' \). The component \( W_n \) is the social marginal benefit of entry. When \( n \) changes in response to a change in \( t \), the impact on welfare takes into account the net benefit of its production, \( pq - c \), and also its contribution to environmental damage, \( D'z \).

**Proposition 4** Excess or insufficient entry, relative to the socially optimal number of firms, depends upon the sign of \( q'(t) \):

**Case 1:** \( q'(t) < 0 \). If the social marginal benefit of production is positive (negative), then there is excess (insufficient) entry.

**Case 2:** \( q'(t) > 0 \). If the social marginal benefit of production is positive (negative), then there is insufficient (excess) entry.

**Case 3:** \( q'(t) = 0 \). In this case there is neither excess nor insufficient entry.

### 1.5 Model 3: Long-run Emission Tax and Business Licensing

In this section, we consider the case in which the government, in addition to setting an emission tax, also regulates the number of firms in the industry; *i.e.*, controls the number of
business operating licenses. Denote the social welfare function in this case as $W_3$:

$$W_3(n, t) = \int_0^{\eta(n, t)} p(\mu) d\mu - nc(q(n, t)) - D(nz(q(n, t))). \quad (1.14)$$

The first-order conditions are

$$\frac{\partial W_3}{\partial n} = n(p - c' - z'D')q_n(n, t) + (pq - c - zD') = 0, \quad (1.15a)$$

$$\frac{\partial W_3}{\partial t} = (p - c' - z'D')nq_t(n, t) = 0. \quad (1.15b)$$

By (1.4b), we infer in the present case that $q_t(n, t) < 0$. Thus, (1.15b) implies $p - c' - z'D' = 0$ and, therefore, (1.15a) implies that $pq - c - zD' = 0$. Since $p - c' - z'D' = tz' - qp' - z'D'$ by the firm’s profit-maximization condition (1.2), the system in (1.15) is equivalent to

$$pq - c - zD' = 0, \quad (1.16a)$$

$$tz' - qp' - z'D' = 0. \quad (1.16b)$$

Note that $q = q(n, t)$ here, so that (1.16) can be solved for the two optimal policies, denoted by $n_3$ and $t_3$.\footnote{We assume that the second-order conditions are satisfied.}

Some interesting optimal properties can be derived. As expected, (1.16b) can be solved for

$$t_3 = D' + \frac{qp'}{z'}, \quad (1.17)$$

which has the form of $t_1$ in (1.7). (1.16a) can be solved for $pq - c = zD'$, which implies

$$n(pq - c) = ZD'.$$
Thus, at the optimum, the total operating profit (without considering the tax payment or subsidy receipt) is equal to the total pollution damage. This is in contrast to the result obtained in (1.6) in the no-entry case where the marginal social surplus, $p - c'$, contributed by a firm’s output is equal to its marginal pollution damage, $D'z'$.

**Proposition 5** In the case where the government can choose both the tax rate and the number of operating licences, the optimal number of firms balances the marginal firm’s contribution to social surplus, given by its operating profit, with its social marginal cost to society, measured by its contribution to overall environmental damage.

In the next section, we provide a numerical example to compare the explicit solutions of the three models.

### 1.6 An Example

In our example, we consider a model with a linear inverse demand and quadratic cost function. Let $p(Q) = a - bQ$ and $c(q) = \alpha q + \beta q^2 / 2 + k$, where $k$ denotes the fixed cost. All parameters are strictly positive.

#### 1.6.1 Model 1: Short-run Emission Tax

We examine the special case in which $z(q) = \delta q$, where $\delta \geq 0$. The profit function of a firm is

$$
\pi = \left(a - b(q + \bar{Q})\right) q - \left(\alpha q + \frac{1}{2} \beta q^2 + k\right) - t\delta q,
$$

where $\bar{Q}$ is the total output of all of the other firms. The first-order condition is

$$
\frac{\partial \pi}{\partial q} = a - 2bq - \alpha - \beta q - t\delta - b\bar{Q} = 0,
$$

15
which yields the reaction function

\[ q = \frac{1}{2b + \beta} \left( a - \alpha - t\delta - b\bar{Q} \right). \]  (1.19)

The second-order condition requires

\[ \frac{\partial^2 \pi}{\partial q^2} = -2b - \beta < 0. \]

In a symmetric equilibrium, \( \bar{Q} = (n - 1)q \). Thus, (1.19) can be solved for the short-run equilibrium output:

\[ q_1 = \frac{a - \alpha - \delta t}{b + \beta + bn} \equiv q_1(n,t). \]  (1.20)

For \( q > 0 \), we require \( a > \alpha + t\delta \). It can be shown that \( \partial q(n,t)/\partial n \) and \( \partial q(n,t)/\partial t \) are both negative, and \( \partial Q(n,t)/\partial n > 0 \) but \( \partial Q(n,t)/\partial t < 0 \). The equilibrium profit in this model 1, denoted by \( \pi_1 \), is

\[ \pi_1 = \frac{(2b + \beta)(a - \alpha - t\delta)^2}{2(b + \beta + bn)^2} - k \equiv \pi_1(n,t). \]  (1.21)

Clearly, if \( k = 0 \), then \( \pi > 0 \) for any \( n \) and any \( t < (a - \alpha)/\delta \). Direct observation confirms the following lemma:

**Lemma 3** In the short run, an exogenous increase in \( t \) lowers the output and profit of each firm, and hence the industry output as well. A larger \( n \) yields a lower output and profit level for each firm, but a higher industry output.

The government chooses \( t \) to maximize welfare. For simplicity, we assume that the damage function is \( D(Z) = Z \). Since \( z = \delta q \), we have \( D(Z) = nz = n\delta q = \delta Q \). The social
welfare function in this model, denoted by $W_1$ becomes

$$W_1 = aQ - \frac{1}{2}bQ^2 - n\left(\alpha q + \frac{1}{2}\beta q^2 + k\right) - \delta Q. \quad (1.22)$$

Direct application of our formula in (1.7) yields the short-run optimal emission tax rate:

$$t_1 = 1 - \frac{b(a - \alpha - \delta)}{\delta(\beta + bn)} . \quad (1.23)$$

The firm’s output in (1.20) under (1.23) is

$$q_1 = \frac{a - \alpha - \delta}{\beta + bn} .$$

It is seen that $q_1$ does not depend on $k$. For $q_1 > 0$, we require

$$a - \alpha - \delta > 0. \quad (1.24)$$

The following proposition can be easily verified:

**Proposition 6** If demand is linear, cost is quadratic, and the number of firms is fixed, then the optimal emission tax rate:

1. Decreases with the market size (higher $a$) or the size of the slope of inverse demand curve (higher $b$).

2. Increases with the emission rate (higher $\delta$), the cost parameters (higher $\alpha$ and $\beta$) and the number of firms in the market (higher $n$).

3. Is independent of the fixed cost ($k$).
CHAPTER 1: THE EFFECTS OF ENTRY UNDER A POLLUTION TAX

The equilibrium profit of a representative firm is

\[ \pi_1 = \frac{1}{2} (2b + \beta) \left( \frac{a - \alpha - \delta}{\beta + bn} \right)^2 - k = \frac{1}{2} (2b + \beta) q^2 - k. \]

Notice that at the optimum \( t \), if \( k = 0 \), then \( \pi > 0 \).

Finally, the optimal value of \( W \) is

\[ W_1 = \frac{n (a - \alpha - \delta)^2}{2 (\beta + bn)} - nk. \]

**Corollary 1** If demand is linear, cost quadratic and the number of firms is fixed, then, under optimal emission tax rate:

1. Profits and welfare both increase if the market size is larger (higher \( a \)), and both decrease if \( b \), \( \alpha \), and/or \( \delta \) increase.

2. An increase in \( \beta \) lowers welfare, but will raise, leave unchanged, or lower a firm’s profit according to whether \( n \) is greater than, equal to, or lower than \((4b + \beta)/b\).

3. An increase in \( n \) lowers a firm’s profit, but has an ambiguous effect on optimized social welfare.

### 1.6.2 Model 2: Long-run Emission Tax

In the long-run, the industry is governed by firms’ profit maximization and free entry or exit. Since entry is assumed to occur whenever profit if positive, we see there is an issue in the present special model. Recall that in our analysis of the short-run equilibrium in the preceding section, if there is no fixed cost, then the Cournot-Nash equilibrium with positive production will have a positive profit for all firms for any \( n \) and any \( t < (a - \alpha)/\delta \), as
revealed in (1.21). This implies that, in the special example here, we must have \( k > 0 \), otherwise there will be continual entry. In what follows, we assume \( k > 0 \).

The two conditions shown in (1.8) can now be expressed as

\[
\pi_q = (-2b - \beta - b(n - 1))q + (a - \alpha - t\delta) = 0,
\]

\[
\pi = (a - bnq)q - \left(\alpha q + \frac{1}{2}\beta q^2 + k\right) - t\delta q = 0,
\]

which can be solved for \( q \) and \( n \):

\[
q_2 = \sqrt{\frac{2k}{2b + \beta}},
\]

\[
n_2 = \sqrt{\frac{2k}{2b + \beta}} \frac{(2b + \beta)(a - \alpha - t\delta) - 2k(b + \beta)}{2bk}.
\]

A higher fixed cost increases the output of each firm, however, the output of a firm falls when either the demand curve becomes more elastic (\( b \) increases) or the marginal cost, \( \beta \), rises. Output is invariant to the tax rate due to the specification of a linear inverse demand, a well known result. Since \( n \) falls with a rise in \( t \) while \( q \) is invariant to any change, we can infer that aggregate output \( Q \) must fall (\( Q'(t) < 0 \)).

Using our optimal tax formula in (1.13), we obtain

\[
t_2 = 1.
\]

(1.25)

This value occurs as a result of our specifications on the inverse demand, emission and damage functions. The first two specifications cause \( q' = 0 \) leaving us with \( t = D' \). Since our damage function treats all marginal contributions of pollution the same, we get the above result.
Proposition 7 In the long-run equilibrium for the present example in which free entry and exit are allowed,

1. $k$ must be positive.

2. The equilibrium firm output $q$ is independent of $t$ and is solely determined by $k$, $b$, and $\beta$. $q$ increases with $k$, but decreases with $b$ and $\beta$.

3. The optimal $t$ is independent of model parameters.

4. The free entry number of firms is increasing in the size of the market $a$, and is decreasing $k$, $\delta$, $b$, and $\alpha$ and may increase or decrease with an increase in $\beta$.

Given (1.25) it follows that

$$q_2 = \sqrt{\frac{2k}{2b + \beta}},$$

$$\pi_2 = 0,$$

$$W_2 = \frac{(a - \alpha - \delta) (2b + \beta) \left( \frac{1}{2} (a - \alpha - \delta) - \sqrt{\frac{2k}{2b + \beta}} (b + \beta) \right)}{b (2b + \beta)} + (b + \beta)^2 k.$$

1.6.3 Model 3: Long-run Emission Tax and Business Licensing

In this section, we consider the case in which the government both imposes an emission tax and regulates the number of firms in the industry; i.e., controls the number of operating licences. Using (1.14) and setting $\partial W/\partial n = 0$, we obtain the optimal level of $n$, denoted by $n_3$, as

$$n_3 = \frac{1}{2bk} \left( \sqrt{2\beta k (a - \alpha - \delta)} - 2\beta k \right).$$

For $n_3 > 0$, we require

$$a - \alpha - \delta > \sqrt{2\beta k}. \quad (1.26)$$
CHAPTER 1: THE EFFECTS OF ENTRY UNDER A POLLUTION TAX

This condition is more stringent than the one in (1.24) whenever $k > 0$.

Note that the effects of changing parameter values on the optimal $n_3$ can be easily determined except for \( \partial n_3 / \partial b = (a - \alpha - \delta - 2\sqrt{\beta}) / (2b\beta) \geq 0 \) according to \( a - \alpha - \delta \geq 2\sqrt{\beta} \). Since (1.26) must hold for $n_3 > 0$, we require that for $\partial n_3 / \partial b < 0$, $k$ must be smaller than 2.

The corresponding optimal tax rate in (1.17), denoted by $t_3$, is

\[
t_3 = 1 - \frac{b\sqrt{2k}}{\delta \sqrt{\beta}}.
\]

The impacts of changing the parameter values on the optimal policy $(n_3, t_3)$ are as follows:

**Proposition 8** When the policy maker uses both $n_3$ and $t_3$ to maximize social welfare:

1. An increase in $b, \alpha, k$ or $\delta$ lowers $n$ but an increase in $a$ raises $n$.

2. An increase in $\beta$ increases, leaves unchanged, or decreases $n_3$ depending upon if $a - \alpha - \delta$ is greater than, equal to, or less than $2\sqrt{\beta}$.

3. $t_3$ is independent of $a$ and $\alpha$. It is an increasing function of $\beta$ and $\delta$, and is a decreasing function of $b$ and $k$.

4. If there is no fixed cost, business licensing becomes unnecessary since $n$ will correspond to the socially optimal level.

Finally, in the case of a positive $k$, we use (1.19) and (1.17) to obtain

\[
q_3 = \sqrt{\frac{2k}{\beta}}.
\]

The resulting firm’s profit, using (1.18), is

\[
\pi_3 = \frac{bk}{\beta},
\]
CHAPTER 1: THE EFFECTS OF ENTRY UNDER A POLLUTION TAX

and the social welfare, using (1.22), is

\[ W_3 = \frac{1}{2b} (a - \alpha - \delta - \sqrt{2\beta k})^2 = \frac{b kn_3^2}{\beta} = \pi_3^2 n_3. \]

**Corollary 2** When the policy maker uses both \( n_3 \) and \( t_3 \) to maximize social welfare:

1. A firm’s output is only determined by \( k \) and \( \beta \) but its profit is additionally determined by \( b \). The larger is the fixed cost \( k \) or the smaller is the marginal cost \( \beta \), the larger are \( q \) and \( \pi \). Furthermore, the larger is \( b \), the higher is \( \pi \).

2. A firm’s profit is positive when \( k > 0 \) and is zero when \( k = 0 \).

It is of interest to note that when \( n \) is optimized, the licensed firms will earn a positive profit whenever there is a fixed cost. Compared with the free entry case, this is a phenomenon of insufficient entry in the present example.

The optimal number of licences (firms) is a monotonic decreasing function of the fixed cost, implying that there exists some fixed cost level beyond which the government should not grant any operating licenses. Specifically, the maximum \( k \) is given by

\[ k = \frac{1}{2\beta} (a - \alpha - \delta)^2. \]

If the government did not choose \( n \), then the maximum exogenous \( n \) that could occur is

\[ n = \frac{1}{2bk} \left( \sqrt{2} \sqrt{k(2b + \beta)} (a - \alpha - \delta) - 2k\beta \right). \]

Larger values of \( n \) cause negative profits.

**Lemma 4** In the short run, when the government chooses both the tax rate and number of permits to issue, firms earn positive profits.
1.6.4 Comparison of Models

For a quick, visual comparison of results from the three models, we include the following table:

<table>
<thead>
<tr>
<th></th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q )</td>
<td>( \frac{a-\alpha-\delta}{\beta+bn} )</td>
<td>( \sqrt{\frac{2k}{2b+\beta}} )</td>
<td>( \sqrt{2k} )</td>
</tr>
<tr>
<td>( \pi )</td>
<td>( (2b + \frac{\beta}{2}) \left( \frac{a-\alpha-\delta}{\beta+bn} \right)^2 - k )</td>
<td>0</td>
<td>( \frac{b\beta}{\beta} )</td>
</tr>
<tr>
<td>( t )</td>
<td>( 1 - \frac{b(a-\alpha-\delta)}{\delta(b+bn)} )</td>
<td>1</td>
<td>( 1 - \frac{b\sqrt{2k}}{\delta\sqrt{\beta}} )</td>
</tr>
<tr>
<td>( W )</td>
<td>( \frac{n(a-\alpha-\delta)^2}{2(\beta+bn)} - nk )</td>
<td>( \frac{(a-\alpha-\delta)(2b+\beta)}{b(2b+\beta)} \left( \frac{1}{2} (a-\alpha-\delta) - \sqrt{\frac{2k}{2b+\beta} (b+\beta)} \right) + \left(b+\beta\right)^2 k )</td>
<td>( n \left( a-\alpha-\delta-\sqrt{2\beta k} \right)^2 )</td>
</tr>
<tr>
<td>( n )</td>
<td>( \sqrt{\frac{n}{2k+\beta}} (2b+\beta)(a-\alpha-\delta-2k(b+\beta)) )</td>
<td>( \sqrt{2k \left( a-\alpha-\delta - 2\beta k \right)} )</td>
<td>( \sqrt{2k \left( a-\alpha-\delta - 2\beta k \right)} )</td>
</tr>
</tbody>
</table>

1.7 Conclusion

This paper examines the effects of emission taxation on an oligopolistic industry. It shows that, in the short run with a fixed number of firms, an increase in the tax rate always lowers all firms’ outputs, resulting in a contraction of the industry output. In the long run, the tax affects firms’ entry and exit decisions and may yield ambiguous effects on the firms’ and industry outputs. It further shows that the tax reduces the equilibrium number of firms, and also reduces the equilibrium industry output if the inverse demand function is affine, concave, or not too convex. Finally, the chapter examines the question of the optimal number of firms from the social point of view. It shows excess or insufficient entry is determined by the relationship between the social marginal benefit of production and the responses of firms’ outputs to a change in the tax rate. An example is constructed to illustrate the general results in practice.
References


2.1 Introduction

Finding a first-best solution in the presence of market externalities has been a task economists have devoted much time to. The literature has covered Pigouvian taxation in many model setups. In a perfectly competitive market with a pollution externality it is well known that a unit tax equal to marginal damage will correct the market imperfection. This tax rate causes the firms to internalize their external cost imposed on society. Buchanan (1969) was the first to point out that this result fails to hold once the assumption of perfect competition is relaxed. In studying the case of monopoly, he showed that setting a tax rate equal to marginal damage causes the firm to restrict output too much. The monopolist, by nature, is already producing less than the socially desirable level of output due to its market power which itself is another externality. As a result, when the tax rate is simply set at the marginal damage of pollution, output is overly curtailed. When solving for an optimal pollution tax rate, one must also consider market imperfection. Therefore, the optimal tax rate in this case should be less than marginal damage. Requate (2005) extensively covers the case of Cournot oligopoly with a pollution externality and also finds that the optimal
unit pollution tax rate is lower than marginal damage.

Moving beyond the unit tax consideration, there is another body of literature which examines the use of nonlinear tax schemes to implement policy objectives. These schemes have much in common with, and often times are derived from, the mechanism design literature. This body of literature examines the formulation of optimal policies and/or contracts in the presence of asymmetric or imperfect information. A typical model setup involves a government or a regulator whose goal is to design a scheme which induces agents or firms to truthfully reveal variables or parameters the regulator seeks to know. These models may also aim to shift the objectives of agents by incentivizing them to act cooperatively, rather than individually\(^1\). Whichever way the problem is formulated, the methodology is the same: provide the right incentives to induce desired behavior.

In dealing with asymmetric or imperfect information there have been many contributions in the literature devising optimal schemes to extract information. Groves (1973) derives an optimal compensation structure imposed by an organization leader whose goal is to induce individual employees to behave like a team. The scheme incentivizes the employees to simultaneously maximize the organizations’ payoff function while maximizing their own payoff functions. The scheme developed to induce truth telling contains agents’ individual objectives as well as the conditional expected value of the other firm’s objectives, including an adjustment term. Essentially, the scheme transforms the agents’ objectives into that of the regulator. As the adjustment term is constant it does not affect the marginal condition, but rather serves as a lump sum transfer.

Kwerel (1977) examines pollution control in an imperfectly competitive market with asymmetric information. The regulator’s objective is to minimize the social cost of pollution without knowing the pollution cost functions of the firms and seeks to design a mechanism

\(^{1}\)For more on the theory of teams see, *e.g.*, Marschak and Radner (1972).
to induce firms to reveal their true cost functions. Kwerel finds that the regulator’s best option is to implement a hybrid quantity-price policy. The policy first issues transferable pollution permits and then offers a subsidy on each licence held in excess of the firms demand. The scheme perfectly balances the incentive to over report and under report and is a direct revelation mechanism. The outcome is a truth telling Nash equilibrium.

Dasgupta, Hammond and Maskin (1980) point out that when firms’ pollutants are not perfect substitutes in the damage function, the scheme proposed by Kwerel (1977) becomes very cumbersome, and possibly unworkable. They in turn develop a similar model with the same goal of minimizing the social cost of pollution. They find a solution in which it is always in a firm’s best interest to be truthful no matter how the other firms behave. Truthfulness in their model is a dominant strategy—a stronger result than was obtained by Kwerel.

Montero (2008) points out that Kwerel’s scheme has additional limitations. He explains that there exist many other inefficient equilibria that are more profitable for firms. He claims that, ”the unique Nash equilibrium in Kwerel’s scheme under a free allocation of licences is for firms to over report their demand curves to ensure the maximum possible number of licences and subsidy level.” In light of this observation, Montero designs an auction-based scheme that retains some characteristics of Kwerel’s and introduces a revenue-sharing function dependent upon the number of licenses. This function is used to determine the optimal share of auction revenues which is to be returned to firms. It is through this revenue-sharing mechanism that the social optimum is obtained.3

Using properties from Montero (2008), Shrestha (2017) additionally shows that an optimal menu of price and quantity contracts can be offered to firms to induce firms to truthfully reveal their private information as a dominant strategy. The price that firms end up paying

---

2 He assumes that the permit market is perfectly competitive.

3 Weitzman (1978) implements a similar scheme and shows that a mixed price-quota system serves as the optimal reward for a polluting firm under uncertainty.
is a unit price, even though the menu schedule itself may be a nonlinear function.

This chapter is devoted to analyzing a specific type of tax scheme constructed in Kim and Chang (1993). Their model’s methodology has similar features common to the works discussed above. They provide a method of deriving an optimal nonlinear tax function for polluting firms in an imperfectly competitive industry under asymmetric information between firms and the regulator. The beauty of their model is that only two pieces of information—the marginal damage function and the slope of the inverse demand function—are needed for the regulator to construct a nonlinear tax scheme. The scheme is applicable to all firms and is capable of implementing the socially optimal values of output and abatement.

In light of the elegant result obtained in Kim and Chang (1993) (hereafter K&C), it is tempting for us to examine how widely their method can be applied when some modifications are made to their model. This is the primary aim of this chapter. We extend their scheme in a number of ways, including modifying the regulator’s and firms’ choices of control variables, generalizing the damage function, and altering the arguments in the tax function which is posted to the firms. We find that their model holds up well under all of the modifications, except when the number of arguments in the tax function is altered.

In the next section, we outline the model of K&C and discuss its shortcomings. In Section (2.3), we examine a model of non-uniform pollution damage and explain how our new tax scheme differs from K&C’s original scheme. In Section (2.4) we extend the analysis to a model in which an additional consumption externality is present. Section (2.5) examines cases where the tax function targets alternative variables. Section (2.6) highlights some general cases in which K&C’s tax scheme fails to implement the social optimum. Lastly, Section (2.7) concludes.
2.2 $G(q,a) \ F(q,a) \ T(q,z) \ D(Z)$ (Kim and Chang’s Model)

First we introduce the notation used throughout this chapter. We let $G$ denote the government’s problem, $F$ the firm’s problem, $T$ the tax scheme, and $D$ the pollution damage function. $G$ and $F$ are denoted as functions of their respective choice variables. For example, when the choice variables are a representative firm’s output $q$ and it’s abatement level $a$, we write $G(q,a)$ and $F(q,a)$. Similarly, $T$ when written as $T(q,z)$, is a tax function generated by a firm’s output $q$ and its level of pollution $z$. Finally, $D$ can be a function of aggregate pollution, $Z = \sum_{i=1}^{n} z_i$ where $z_i$ is firm $i$’s pollution, or a function of the vector of firms’ pollutions, $\bar{z} = (z_1, z_2, ..., z_n)$.

In this section we outline the model developed by Kim and Chang (1993). This model is related to the tax scheme developed by Loeb and Magat (1979).

K&C consider an $n$-firm Cournot oligopoly with a pollution externality on production. The inverse demand function is

$$p = p(Q) = p(q_i + Q_{-i}),$$

and aggregate output is

$$Q = \sum_{i=1}^{n} q_i = q_i + Q_{-i},$$

where $Q_{-i} = \sum_{j \neq i} q_j$ denotes the total output of all other firms, excluding firm $i$. The cost to firm $i$ of producing $q_i$ output units and $a_i$ abatement units is

$$c_i(q_i, a_i).$$
K&C assume that the pollution function of firm $i$ is

$$z_i = z_i(q_i, a_i).$$

The government sets a uniform tax function for all firms which is

$$t_i = t(q_i, \bar{q}_{-i}, z_i, \bar{z}_{-i}),$$

where $\bar{q}_{-i}$ is the vector of outputs other than $q_i$. Similarly, $\bar{z}_{-i}$ is the vector of pollution other than $z_i$.

Firm $i$’s net profit is

$$\pi_i = p(q_i + Q_{-i}) q_i - c_i(q_i, a_i) - t(q_i, \bar{q}_{-i}, z_i, \bar{z}_{-i})$$

$$= p(q_i + Q_{-i}) q_i - c_i(q_i, a_i) - t(q_i, \bar{q}_{-i}, z_i(q_i, a_i), \bar{z}_{-i}(\bar{q}_{-i}, \bar{a}_{-i})).$$ (2.1a)

$$= p(q_i + Q_{-i}) q_i - c_i(q_i, a_i) - t(q_i, \bar{q}_{-i}, z_i(q_i, a_i), \bar{z}_{-i}(\bar{q}_{-i}, \bar{a}_{-i})).$$ (2.1b)

Firm $i$ maximizes it’s profit with respect to $q_i$ and $a_i$. The first-order conditions are

$$\frac{\partial \pi_i}{\partial q_i} = p + q_i p' - c_i q_i - t_{q_i} - t_{z_i} z_{i q_i} = 0,$$ (2.2a)

$$\frac{\partial \pi_i}{\partial a_i} = -c_i a_i - t_{z_i} z_{i a_i} = 0,$$ (2.2b)

where the subscripts denote partial derivatives.

Damage is a function of aggregate emissions, $D = D(Z)$. We write

$$Z = \sum_{i=1}^{n} z_i(q_i, a_i) = z_i(q_i, a_i) + Z_{-i}(\bar{q}_{-i}, \bar{a}_{-i}).$$

The social welfare function is given as the sum of consumer and producer surpluses, less
damage:
\[ W = \int_0^Q p(\mu) \, d\mu - \sum_{i=1}^n c_i(q_i, a_i) - D(Z). \]

The government maximizes \( W \) by choosing \( \bar{q} \) and \( \bar{a} \). The first-order conditions with respect to firm \( i \)'s output and abatement are

\[
\frac{\partial W}{\partial q_i} = p - c_{iq_i} - D'z_{iq_i} = 0, \tag{2.3a}
\]
\[
\frac{\partial W}{\partial a_i} = -c_{ia_i} - D'z_{ia_i} = 0. \tag{2.3b}
\]

It follows from (2.2) and (2.3) that \( q_ip' - t_{q_i} - t_{z_i}z_{iq_i} = -D'z_{iq_i} \) and \( t_{z_i}z_{ia_i} = D'z_{ia_i} \). Solving the system K&C obtain

\[
t_{q_i} = t_{q_i}(q_i, \bar{q}_i) = q_ip'(q_i + Q_{-i}),
\]
\[
t_{z_i} = t_{z_i}(z_i, \bar{z}_i) = D'(z_i + Z_{-i}).
\]

The resulting value of \( t \) is recovered through these marginal conditions, and follows as

\[
t = \int_0^{q_i} \mu p' (\mu + Q_{-i}) \, d\mu + \int_{z_i}^{\bar{z}_i} D'(\nu + Z_{-i}) \, d\nu
\]
\[
= \left( q_ip(Q) - \int_0^{q_i} p(\mu + Q_{-i}) \, d\mu \right) + (D(z_i, \bar{z}_i) - D(0, \bar{z}_i)).
\]

This result was termed "ingenious" by Requate (2005).

It should be noted that in K&C’s tax scheme this is a uniform tax schedule, published for all firms. It depends not only on firm \( i \)'s levels of output and pollution, but also on those of all other firms. The tax is nondiscriminatory in the sense that all firms are treated in the same way; i.e., if firms \( i \) and \( j \) have the same technology and produce the same output and pollution levels, then the tax they pay is the same.
K&C illustrate their results with an example. They assume the following functional forms:

\[
p(Q) = 2 - q_i - Q_{-i}, \tag{2.4a}
\]

\[
c_i(q_i, a_i) = q_i - \frac{1}{3} a_i, \tag{2.4b}
\]

\[
z_i(q_i, a_i) = q_i - a_i, \tag{2.4c}
\]

\[
D(Z) = \frac{1}{2} (z_i + Z_{-i})^2. \tag{2.4d}
\]

It follows that

\[
t_{q_i} = q_i p' = -q_i,
\]

\[
t_{z_i} = D' = Z = z_i + Z_{-i}.
\]

The corresponding tax rate is

\[
t_i = t(q_i, z_i, Z_{-i}) = -\frac{1}{2} q_i^2 + \frac{1}{2} z_i^2 + Z_{-i} z_i.
\]

When firms are given this tax rate, they maximize net profit. The first-order conditions are

\[
\frac{\partial \pi_i}{\partial q_i} = p + q_i p' - c_i q_i - t_{q_i} - t_{z_i} z_i q_i
\]

\[
= (2 - Q) + q_i (-1) - 1 + q_i - (z_i + Z_{-i}) \cdot 1 = 1 - Q - Z = 0,
\]

\[
\frac{\partial \pi_i}{\partial a_i} = -c_i a_i - t_{z_i} z_i a_i
\]

\[
= -\frac{1}{3} - (z_i + Z_{-i}) (-1) = -\frac{1}{3} + Z = 0.
\]
If the firms are identical, then

\[ q = \frac{2}{3n}, \quad (2.5a) \]
\[ a = \frac{1}{3n}, \quad (2.5b) \]
\[ z = \frac{1}{3n}, \quad (2.5c) \]

and

\[ t = \frac{1}{18} \frac{2n - 5}{n^2}. \]

The values of \( q, a \) and \( z \) chosen by the firms match the welfare maximizing levels chosen by the government. Of course, this methodology of recovering a uniform tax scheme which implements optimal choices is dependent upon both the number of policy target variables and the mathematical properties of the functions used to describe the governments’ and firms’ optimization problems.

We note that a unit tax can always be implemented which will achieve the optimum, even when a tax function constructed in the K&C fashion fails to do so. In the cases that we examine in the following sections, the government can still be ignorant of the firms’ cost functions, but information additional to the marginal damage function and the slope of the inverse demand function may be needed in order for them to implement the socially optimal choices of output, abatement and pollution. To illustrate the unit tax option, let us assume that we have constant unit tax rates of \( k_{zi} \) on pollution and \( k_{qi} \) on output instead of a tax scheme presented to the firms in the form of \( t (q_{i}, q_{-i}, z_{i}, z_{-i}) \). Not surprisingly, the optimum can be reached if

\[ k_{qi} = q_{i}p' (q_{i} + Q_{-i}), \quad (2.6a) \]
\[ k_{zi} = D' (z_{i} + Z_{-i}), \quad (2.6b) \]
evaluated at the optimal solution. We show this by considering the following problem.

The firms know these \( k_{zi} \) and \( k_{qi} \) rates set by the government and include them in their profit maximization problems so that (2.1b) becomes

\[
\max_{q_i, a_i} \pi_i = p(q_i + Q_{-i}) q_i - c_i(q_i, a_i) - k_{qi}q_i - k_{zi}z_i .
\]

The firms’ first-order conditions are

\[
\frac{\partial \pi_i}{\partial q_i} = p + q_i p' - c_{qi} - k_{qi} - k_{zi}z_{qi} = 0, \\
\frac{\partial \pi_i}{\partial a_i} = -c_{ia_i} - k_{zi}z_{ia_i} = 0.
\]

When we use (2.6) in these conditions we see

\[
\frac{\partial \pi_i}{\partial q_i} = p + q_i p' - c_{qi} - k_{qi} - k_{zi}z_{qi} \\
= p + q_i p' - c_{qi} - q_i p' - D' z_{qi} \\
= p - c_{qi} - D' z_{qi} \\
= \frac{\partial W}{\partial q_i}, \\
\frac{\partial \pi_i}{\partial a_i} = -c_{ia_i} - k_{zi}z_{ia_i} \\
= -c_{ia_i} - D' z_{ia_i} \\
= \frac{\partial W}{\partial a_i}.
\]

Essentially, these unit tax rates makes firms’ choices of \( q_i \) and \( a_i \) equivalent to those of the government\(^4\). The unit rate placed on \( q_i \) is firm-specific, and depends upon their choice of  

\(^4\)Note that in order for firms to specifically choose the socially optimal level of abatement, the \( a_i \) term must not enter the profit function linearly. If this occurs, marginal revenue equalling marginal cost of abatement is independent of the firms choice of \( a_i \), so that the firm itself is indifferent about it choice of \( a_i \). There is no
$q_i$. A larger firm will receive a larger subsidy rate on output. This is because a larger firm is more efficient. The regulator capitalizes on that efficiency by encouraging production. The unit rate on pollution, however, is not firm-specific; the same rate applies to all firms. This is because each $z_i$ has the same impact on damage.

The methodology used to derive (2.6) differs slightly from the standard literature. Typically, the problem is constructed with the government internalizing the behavior of firms and directly choosing the tax rate from its objective function. Although essentially the same, our government chooses $q_i$ and $a_i$ and uses the "unknown" cost functions to internalize the firms’ decisions.

### 2.3 Non-Uniform Pollution Damage

This section investigates the role that assumptions made on the damage function play in the recovery of a uniform, or even an individualized, nonlinear tax scheme. In K&C’s model firms’ pollutions are perfect substitutes in the damage function. As they assume homogeneous goods, this assumption is standard. It could be the case, however, that even identical goods could affect damage in asymmetric ways. As an example consider the case of location. If the government implementing the tax is local, and seeks to reduce ambient air pollution, it seems reasonable to assume that a firm which is located closer to, or within, the locale may impact ambient pollution by a factor greater than that of a firm downwind or located outside the locale. If this is the case, then the tax rate which is given to firms is no longer uniform, but becomes firm specific. Two firms may still have the same tax schedule, but that would require their respective pollutions to impact aggregate damage in the same manner, for all levels of pollution. The marginal condition of the tax scheme with respect to guarantee that the firm will choose the socially optimal amount. In this case there is a clear advantage to the K&C nonlinear tax scheme.
output \( q_i \) will remain the same, i.e., \( t_{q_i} = q_ip' \). For a quantity \( z_i \) of pollution, however, the marginal condition for firm \( i \) now becomes \( t_{z_i} = D'Z_{z_i} \). The presence of the \( Z_{z_i} \) term is the cause of differentiated rates.

Before looking at the general case, we consider an illustrative example when pollution does not affect damage uniformly and each firms’ pollutants harm with differing intensity. We still denote total pollution as \( Z = \sum z_i \), but replace \( D(Z) \) in the welfare function with the more general functional form \( D(\bar{z}) \). For simplicity we consider a duopoly. We denote the damage function as

\[
D(\bar{z}) = \delta_1 z_1^{\theta_1} + \delta_3 z_1^{\gamma_1} z_2^{\gamma_2} + \delta_2 z_2^{\theta_2},
\]

where \( \delta, \theta, \) and \( \gamma \) are parameters. We utilize all other specific functional forms from (2.4).

The optimal tax rates for firm 1 and firm 2 are

\[
t_1 = \delta_1 z_1^{\theta_1} + \delta_3 z_1^{\gamma_1} z_2^{\gamma_2} - \frac{1}{2} q_1^2,
\]

\[
t_2 = \delta_2 z_2^{\theta_2} + \delta_3 z_1^{\gamma_1} z_2^{\gamma_2} - \frac{1}{2} q_2^2.
\]

Alternatively, we can write these as

\[
t_1 = (D(Z) - \delta_2 z_2^{\theta_2}) - \frac{1}{2} q_1^2,
\]

\[
t_2 = (D(Z) - \delta_1 z_1^{\theta_1}) - \frac{1}{2} q_2^2.
\]

With differing parameter values in \( t_i \) it becomes evident that the tax schemes are now firm-specific. The terms in parentheses represent each firms’ respective residual damages. This shows that firm \( i \) is responsible for paying its share of pollution damage, taking the other firms’ pollution levels as given. The second term in the tax rate comes from imperfect competition in the market. Here we have the standard result that the firm is subsidized on
output up to the point where the competitive market equilibrium would be reached.

Providing these tax schemes to the firms transforms their profit maximization problems into functions which yield the same set of first-order conditions that the regulator faces. Their profit functions are not transformed into the exact problem of the regulator, since the regulator sees aggregate damage while the firms see only their residual damage. The crucial result is that the two yield the same optimal choices of output, abatement and pollution at both the individual firm and aggregate levels.

2.3.1 The General Case

Here we assume that the damage function depends on the vector of pollution levels instead of the aggregate pollution level. That is, we consider $D(\bar{z})$ instead of $D(Z)$. With this departure we must emphasize that instead of obtaining and using $(\partial D/\partial Z)(\partial Z/\partial z_i) = \partial D/\partial z_i = D'(Z)$ we now have $\partial D(\bar{z})/\partial z_i = D_i(z_i, \bar{z} - i)$, which can differ across firms. The marginal conditions are

$$t_{q_i} = t_{q_i}(q_i, \bar{q} - i) = q_i p'(q_i + Q - i),$$

$$t_{z_i} = t_{z_i}(z_i, \bar{z} - i) = \frac{\partial D(\bar{z})}{\partial z_i}.$$

Two things are immediately apparent. The first is that we are able to recover a nonlinear tax scheme. Of course we can set a linear rate, but the recovery method applies here. The second point is that the tax scheme which is recovered is no longer applicable to all firms, but now is firm specific. Each firm faces their own $t_i$ function of the following general form:

$$t_i = \int_0^{q_i} (\mu p'(\mu + Q - i)) d\mu + \int_0^{z_i} D_i(\nu, \bar{z} - i) d\nu.$$
The caveat here is that the regulator needs to know each firms’ pollution damage intensity (as opposed to only marginal damage) in addition to the slope of the aggregate damage function.

**Theorem 1** A uniform tax scheme can be recovered if the damage function takes the form \( D(Z) \) where \( Z = \sum_{i=1}^{n} z_i \). If the damage function takes the form \( D(\bar{z}) \) then the recovered tax scheme becomes firm specific except in the special case where \( D(\bar{z}) \) is additively separable and \( \partial Z/\partial z_i = \partial Z/\partial z_j, \forall \ i, j = 1, ..., n; \ i \neq j \).

### 2.4 An Externality Function

In this section we generalize the damage function, and instead view it as an "externality" function where, in addition to pollution, it also contains output. This may occur due to positive externalities such as network effects, for example. We write this function as \( E = E(Q, Z) \). All else is the same, so that we are considering \( G(q,a) F(q,a) T(q,z) E(Q, Z) \).

In this more general set up the modified marginal conditions are

\[
\begin{align*}
    t_{q_i} &= t_{q_i}(Q,Z) = q_i p'(Q) + E_{q_i}(Q,Z), \\
    t_{z_i} &= t_{z_i}(Q,Z) = E_{z_i}(Q,Z).
\end{align*}
\]

With this generalization the optimal tax scheme follows as

\[
t = \left( q_i p(Q) - \int_{0}^{q_i} p(\mu + Q_{-i}) \, d\mu \right) + E(Q,Z) - E(Q_{-i},Z_{-i}).
\]

The result is similar to the case where the externality was only on pollution and the damage function depended upon the pollution vector. The tax function here is uniform and it is based on each firms’ residual externality. Although output appears in this residual externality
function, we can still view the tax rate as the sum of two parts: the residual externality term, which now also includes \( q_i \), and the imperfect competition term. Notice that this tax function follows a Vickrey-Clark-Groves (VCG) payoff rule; each firm pays for its residual damage.

**Theorem 2** A uniform tax scheme can be recovered when the externality function takes the form \( E = E(Q, Z) \).

Generalizing this externality function to \( E = E(\bar{q}, \bar{z}) \), while retaining additive separability, gives

\[
t = \left( q_i p (Q) - \int_0^{q_i} p (\mu + Q_{-i}) d\mu \right) + E(q_i, \bar{q}_{-i}, z_i, \bar{z}_{-i}) - E(0, \bar{q}_{-i}, 0, \bar{z}_{-i})
\]

and the following corollary to Theorem (2):

**Corollary 3** A uniform tax scheme can be recovered when the externality function takes the form \( E = E(\bar{q}, \bar{z}) \).

Here we include an illustrative example. Define the externality function as

\[
E(\bar{z}, \bar{q}) = \delta_1 \bar{z}_1^{\theta_1} + \delta_3 \bar{z}_1^{\gamma_1} \bar{z}_2^{\gamma_2} + \delta_2 \bar{z}_2^{\theta_2} - \lambda_1 \bar{q}_1^{\varphi_1} - \lambda_3 \bar{q}_1^{\sigma_1} \bar{q}_2^{\sigma_2} - \lambda_2 \bar{q}_2^{\sigma_2}.
\]

Using the same method and functional forms as in (2.4a), it follows that the tax rates for firm 1 and 2 must satisfy

\[
t_{q_1} = -q_1 - \lambda_1 \varphi_1 \bar{q}_1^{\varphi_1-1} - \sigma_1 \lambda_3 \bar{q}_1^{\sigma_1-1} \bar{q}_2^{\sigma_2},
\]

\[
t_{z_1} = \theta_1 \delta_1 \bar{z}_1^{\theta_1-1} + \gamma_1 \delta_3 \bar{z}_1^{\gamma_1-1} \bar{z}_2^{\gamma_2},
\]

\[
t_{q_2} = -q_2 - \lambda_2 \varphi_2 \bar{q}_2^{\varphi_2-1} - \sigma_2 \lambda_3 \bar{q}_1^{\sigma_1} \bar{q}_2^{\sigma_2-1},
\]

\[
t_{z_2} = \theta_2 \delta_2 \bar{z}_2^{\theta_2-1} + \gamma_2 \delta_3 \bar{z}_1^{\gamma_1} \bar{z}_2^{\gamma_2-1}.
\]
CHAPTER 2: OPTIMAL ENVIRONMENTAL TAX SCHEME

These yield optimal tax schemes

\[ t_1 = \delta_1 z_1^{\theta_1} + \delta_3 z_1^{\gamma_1} z_2^{\gamma_2} - \frac{1}{2} q_1^2 - \lambda_1 q_1^{\sigma_1} - \lambda_3 q_1^{\sigma_1} q_2^{\sigma_2} \]

\[ = (E(\bar{z}, \bar{q}) - \delta_2 z_2^{\theta_2}) + \left(\lambda_2 q_2^{\sigma_2} - \frac{1}{2} q_2^2\right) \]

= (residual externality) + (imperfect competition term),

\[ t_2 = \delta_2 z_2^{\theta_2} + \delta_3 z_1^{\gamma_1} z_2^{\gamma_2} - \frac{1}{2} q_2^2 - \lambda_2 q_2^{\sigma_2} - \lambda_3 q_1^{\sigma_1} q_2^{\sigma_2} \]

\[ = (E - \delta_1 z_1^{\theta_1}) + \left(\lambda_1 q_1^{\sigma_1} - \frac{1}{2} q_1^2\right) \]

= (residual externality) + (imperfect competition term).

2.5 Other Policy Targets

In practice there could be many factors influencing a government’s choice of policy variables. In some instances, levying a tax on pollution may be ideal, e.g., due to budgetary issues. In other cases, abatement subsidies might be more easily implemented due to industry lobbying, political pressure, or perhaps due to an added (positive) externality of spurring innovation. Whatever the reasoning, governments may want to focus on different objectives. In K&C we have the functional relationship \( z = z(q, a) \) so mathematically speaking, the government should be able to achieve its policy objective of reaching optimal output and abatement by targeting any two of \( z, q \) and \( a \). In the unit tax case this is no problem. Using K&C’s tax scheme, however, it is required that any one of the variables be uniquely determined by the other two. If this is the case, we can use change of variables in order to apply their method. The optimal tax rates can be transformed depending upon which variables the government wishes to target.
2.5.1 $G(q,a)$ $F(q,a)$ $T(a,z)$ $D(Z)$

In the case where a tax on output is not feasible, the regulator may want the policy instrument to target abatement and pollution. The choice variables of the government and firm are still output and abatement levels.

The marginal conditions on the tax rate are

$$t_z = t_z(z_i, \bar{z}_i, q_i, \bar{q}_i, a_i) = \frac{1}{z_{iq_i}} (D'z_{iq_i} + p'q_i), \hspace{1cm} (2.7a)$$

$$t_a = t_a(z_i, q_i, a_i) = -p'q_i \frac{z_{ia_i}}{z_{iq_i}}. \hspace{1cm} (2.7b)$$

In this general form the tax function cannot be explicitly written.

Using the special example from K&C the tax function would become

$$t = (Z_i - a_i) z_i - \frac{1}{2} a_i^2.$$

Alternatively the government may also use a unit tax scheme on $z_i$ and $a_i$. The rates are set according to the above conditions, evaluated at the socially optimal levels of $q_i$ and $a_i$.

To show that this unit tax scheme will implement the social optimum we use the profit function $\pi_i = p(q_i + Q_{-i}) q_i - c_i(q_i, a_i) - t_z z_i - t_a a_i$. Note that $t_z$ and $t_a$ are constants here.

The first-order conditions for firm $i$ are

$$\frac{\partial \pi_i}{\partial q_i} = p + q_i p' - c_{iq_i} - t_z z_{iq_i} = 0,$$

$$\frac{\partial \pi_i}{\partial a_i} = -c_{ia_i} - t_a - t_z z_{ia_i} = 0.$$
Through substitution, we obtain

\[
\frac{\partial \pi_i}{\partial q_i} = p + q_ip' - c_{iqi} - tz_{ziqi} = p + q_ip' - c_{iqi} - \left(\frac{1}{z_{iqi}} (D'z_{iqi} + p'q_i)\right) z_{iqi} = p - c_{iqi} - D'z_{iqi} = \frac{\partial W}{\partial q_i}.
\]

For the abatement level,

\[
\frac{\partial \pi_i}{\partial a_i} = -c_{iai} - ta_i - tz_{ziai} = -c_{iai} - \left(-p'q_i \frac{ziai}{z_{iqi}} \right) - \left(\frac{1}{z_{iqi}} (D'z_{iqi} + p'q_i)\right) z_{iai} = -c_{iai} + p'q_i \frac{ziai}{z_{iqi}} - \left(\frac{ziai}{z_{iqi}} (D'z_{iqi} + p'q_i)\right) = -c_{iai} - D'ziai = \frac{\partial W}{\partial a_i}.
\]

As is expected, the unit scheme obtains the social optimum.

### 2.5.2 \mathbf{G(q,a) F(q,a) T(q,a) D(Z)}

For further insight we consider the case where the tax is implemented on output and abatement. Here, instead of penalizing pollution, the government rewards abatement efforts undertaken by firms. We note that from a budgetary point of view, this scheme may not be as attractive. It may be easier to enact due to preexisting pressures on firms or as a result of lobbying efforts.

The government’s problem and first-order conditions remain unchanged. Under the new
tax scheme $t(a_i, \bar{a}_i, q_i, \bar{q}_i)$ the marginal conditions become

$$t_{q_i} = t_{q_i} (q_i, \bar{q}_i, z_i, \bar{z}_i, a_i) = q_i p' + D' z_{iq_i},$$

$$t_{a_i} = t_{a_i} (z_i, \bar{z}_i, a_i) = D' z_{ia_i}.$$ 

Generally we can not write the function but again using the functional form from K&C we get

$$t = (Z_{-i} - a_i) q_i - Z_{-i} a_i + \frac{1}{2} a_i^2$$

As in the case of $t(a_i, z_i, \bar{a}_i, \bar{z}_i)$ we are able to implement a linear scheme, setting $k_{q_i} = q_i p' + D' z_{iq_i}$ and $k_{a_i} = D' z_{ia_i}$. The government must additionally know how abatement efforts affect each firms’ pollution level ($z_{ia_i}$), as well as how pollution intensive is each firms output ($z_{iq_i}$). When firms are presented with these rates, it follows that

$$\frac{\partial \pi_i}{\partial q_i} = p + q_i p' - c_{iq_i} - k_{q_i}$$

$$= p + q_i p' - c_{iq_i} - (q_i p' + D' z_{iq_i})$$

$$= p - c_{iq_i} - D' z_{iq_i} = \frac{\partial W}{\partial q_i},$$

$$\frac{\partial \pi_i}{\partial a_i} = -c_{ia_i} - k_{a_i} = -c_{ia_i} - D' z_{ia_i} = \frac{\partial W}{\partial a_i}.$$ 

The firms choose the socially optimal levels as their problem is transformed, on the margin, into that of the government.
CHAPTER 2: OPTIMAL ENVIRONMENTAL TAX SCHEME

2.6 One Policy Target, Two Externalities

2.6.1 $G(q,a)$ $F(q,a)$ $T(z)$ $D(Z)$

This subsection examines the case where the government only targets pollution, $z$. When using K&C's methodology to recover the nonlinear tax scheme, we obtain two simultaneous conditions which must be met for $t_z$. Specifically, this tax scheme must satisfy both

$$t_z = \frac{1}{z_i q_i} p' + D',$$

$$t_z = D'.$$

In other words, we must have

$$\frac{1}{z_i q_i} p' + D' = D',$$

which implies that $p'q_i = 0$. This is a model-specific internal constraint which arises from the set up of this system. This condition says that the markup firms receive from selling their product must be zero, or in other words, that the product market is perfectly competitive. Clearly this is a contradiction to our assumptions, and we discover that this type of tax scheme will only work in the presence of one externality.

In consideration of a unit tax rate, we would expect the standard result that, in the presence of oligopoly, the pollution tax should contain marginal damage as well a term relating to the market distortion from imperfect competition. Indeed this holds true in the unit tax case; we get the same generalized result derived in chapter 1 of this dissertation.
Here we would have

\[
\begin{align*}
  t_{zi} &= \frac{1}{ziq_i}q_ip' + D' \\
        &= -q_i + z_i + Z_i,
\end{align*}
\]

where the first equality is in general form, and the second uses (2.4a).

The first condition is what results in the unit tax case, as can be seen in Lee (1999) and Requate (2005). This comes from pollution being a function of output. The second case focuses solely on abatement. As there is only one tool, both conditions cannot be satisfied and we see K&C’s method cannot be used when the government only targets pollution.

### 2.6.2 G(q,a) F(q,a) T(a) D(Z)

This subsection examines the case where the government only targets pollution, a. Again we note that a first-best outcome should not result due to the presence of two externalities and only one policy target. In the general set-up we get that the marginal condition which must be satisfied is

\[
t_{ai} = D'z_{iai}.
\]

We are able to recover a tax function, however, it will yield a suboptimal outcome as compared to the social optimum. From the example in (2.4a) we obtain

\[
\begin{align*}
  t_{ai} &= -(z_i + Z_{-i}) \\
        &= -(q_i - a_i + Z_{-i}).
\end{align*}
\]
This would yield a tax function of the form

\[ t = \frac{1}{2} a_i^2 - (q_i + Z_{-i}) a_i, \]

from which it can be shown, the resulting choices of \( q_i, a_i \) and therefore \( z_i \) are suboptimal unless it is the case that \( q_i p' = D' z_{iq_i} \). This condition follows from the first-order condition with respect to \( a_i \) for the government and the firm. We note that mathematically this equation could be met, but economically speaking both sides would need to be 0 in order for this to make sense. If they are non zero values the left-hand side is a negative value while the right-hand side is a positive value. Furthermore, if both of these values are zero that is equivalent to saying that we have no externalities in the first place.

### 2.6.3 \( G(q,a) \ F(q,a) \ T(q) \ D(Z) \)

For consistency we include this last case where the tax function targets only \( q \). The tax scheme derived must satisfy the marginal condition

\[ t_{q_i} = D' z_{iq_i} + p' q_i, \]

again leading to a suboptimal outcome. Only in the case where \(-D' z_{iq_i} = 0\) will the optimum be reached.

### 2.7 Conclusion

Kim and Chang (1993) provide a method for deriving an optimal nonlinear tax function for polluting firms in an imperfectly competitive industry under asymmetric information between firms and the regulator. This method, elegant and clever in its construction, is
capable of being extended to cases where firms’ pollutions may not be perfect substitutes, where additional variables enter the damage (or externality) function and where the government may have different choice variables than the firms. We find two caveats to the model. First, we find that the number of policy targets must match the number of externalities in the market. If this is not the case, conflicting conditions on the properties of the function may arise. The resulting tax function would not achieve the social optimum, a result that is well known in the literature. When their method cannot be applied, we suggest that unit tax rates be imposed to achieve the regulator’s objective. These unit rates, however, require that the regulator know additional information not needed in Kim and Chang’s model. The second caveat is that firms and the government must focus on choosing the same set of choice variables unless the relationship between all variables is linear. If this is not the case, unique solutions for each variable in terms of the others is not guaranteed and suboptimal tax schemes will result.

References


Kim, Jae-Cheol and Ki-Bok Chang (1993). An Optimal Tax/Subsidy for Output and Pollution Control under Asymmetric Information in Oligopoly Markets. *Journal of


Chapter 3
Optimal Policy on Externalities with Weighted Group Interests

3.1 Introduction

Pigouvian taxation has been studied extensively in the literature. Buchanan (1969), in studying the case of a polluting monopolist, showed that the optimal pollution tax rate is less than the marginal damage of pollution by an amount equal to the markup the monopolist charges on the output. The monopolist, by its nature, produces less than the socially desirable level of output. If the pollution tax rate was simply set equal to the marginal damage, output would be further restricted. In a survey article Requate (2005) covers, in detail, environmental policy analysis in an oligopolistic market structure. In this article he shows that the optimal pollution tax rates can be lower than marginal damage.

The literature on optimal environmental policy traditionally uses the standard welfare function, which weighs surplus values and externalities equally. In this chapter, we propose a modification to this welfare function. We construct a weighted welfare function, where the government or the regulator places various weights on consumer and producer surpluses, and on externalities. This could be the result of mimicking political interests, or the regulator’s
own judgment, for example. We study the resulting optimal tax structure and compare it to that which is obtained in the standard literature with an unweighted social welfare function.

Neary (1994) examines an international Cournot duopoly that competes in a third market. His model structures the social welfare function in a way to weigh public funds differently than private funds. Such a scheme may reflect the policy maker’s preference in taking into account pure distributional considerations, the deadweight loss of raising taxes to finance the subsidy from other parts of the economy, or may result due to limited budget availability for financing trade policy. He finds that the optimal subsidy rate is likely to become negative when the subsidy payment weight is high.

In this chapter, we consider an oligopolistic Cournot model in which firms compete in output and abatement of pollution. The government employs a unit tax on the output externality which can be caused by consumption or production, and another unit pollution tax on the pollutants emitted in the production process. We derive the optimal tax formulae and examine their properties. Our formulae reveal new features that the policy maker must consider. As a result, the traditional results in the literature are special cases of our model’s. To understand better the intricate relationships of the various factors affecting the optimal policies, we construct a numerical example and obtain several interesting results.

With our two policy instruments, we also consider, instead of the government choosing the tax rates directly, choosing the socially-desired quantity of output and pollution level with the aim of finding the optimal implied tax rates. We obtain different implicit forms of the optimality conditions and show that they imply the same optimal tax rates. The idea of choosing quantities by the government mimics that of Kim and Chang (1993), although our model is different from theirs. Their aim is to derive a nonlinear tax function that depends on output and pollution levels rather than the use of two unit tax instruments which we examine in this chapter. Moreover, in their model, since the welfare function is unweighted,
they resort to using only the quantity approach.

Finally, in this chapter, we also examine the technological relationships between output, abatement and pollution, and their implied cost functions. Much of the literature makes assumptions regarding the signs of cross-derivatives between output and abatement in the cost function without further investigation into technological relationships. Our examination of the technology reveals an over-simplification in many of the assumptions made on the cost functions. Our results should be useful for those who directly rely on cost functions in studying optimal environmental policies.

The remainder of this paper is structured as follows: Section (3.2) outlines our model. In Section (3.3) we derive the optimal tax formulae and discuss their implications. Section (3.4), in light of Kim and Chang (1993), looks at conditions under which the government can choose quantity variables as opposed to policy variables to maximize welfare. Section (3.5) examines production, abatement and pollution technologies, and their implied cost functions. Our results show that the cost functions are far more complex than those usually assumed in the literature. Section (3.6) concludes.

### 3.2 The Model

Consider a symmetric $n$-firm Cournot oligopoly model. The firms compete in output. Each firm $i$’s production activity produces a quantity $z_i$ of pollution, which is determined by its output level $q_i$ and its level of abatement $a_i$, so that $z_i = z_i(q_i, a_i)$. Under symmetry, the total output is $Q = nq$ and the aggregate pollution is $Z = \sum_{i=1}^n z_i = nz(q, a) = nz\left(\frac{Q}{n}, \frac{A}{n}\right) = Z(Q, A; n)$, where $A = \sum_{i=1}^n a_i = na$. Consumption has an external cost associated with it, denoted by $\Omega (Q)$. If $\Omega$ is positive (negative) we view the externality as a cost (benefit) to society. Because it creates pollution, production also imposes a social external cost that we denote by $\Phi (Z (Q, A))$. 

51
CHAPTER 3: OPTIMAL POLICY ON EXTERNALITIES WITH WEIGHTED GROUP INTERESTS

When aiming to correct the externalities, we assume that it is difficult for the government to observe the individual consumption effects. As a result the government imposes a uniform consumption tax/subsidy rate, $\tau$, on each unit of consumption. On the other hand, we assume that due to easier monitoring, the government imposes a firm-specific unit pollution tax $t_i$ based on the damage to the environment caused by an individual firm’s production process. We assume that the damage is directly measured by $z_i$.

Let a firm’s cost of production (including the abatement cost but excluding the tax) be $c_i(q_i, a_i)$, and let the aggregate total cost in the industry be $C(Q, A) = \sum_{i=1}^{n} c_i(q_i, a_i)$.

Let the consumer price be $p$, which includes the tax paid by the consumers due to the consumption externality. Let the producer price be $p_p$ which is the net price received by each producer. Since the output is both produced and consumed, a tax on it can be interpreted either as a consumption tax or as an output tax levied on producers. Because we do not distinguish between quantity consumed and produced, in what follows, the tax rate $\tau$ can be called either a consumption tax or an output tax.\(^1\) Regardless of how $\tau$ is interpreted, it creates a wedge between the consumer price $p$ and the producer price $p_p$:

$$p_p = p - \tau.$$

Let $Z$ be the total pollution from production, and let $\Delta$ be the total external damage to society. Total external damage is the sum of the damage due to consumption, $\Omega(Q)$, and the damage due to pollution, $\Phi(Z)$:

$$\Delta = \Omega(Q) + \Phi(Z(Q,A)).$$

\(^1\)For consistency, we will refer to it as a consumption tax throughout the paper.
The government collects tax revenues in the amount of

\[ R = \tau Q + tZ. \]  

(3.1)

We count \( tZ \) as part of the production cost. Thus, the producer surplus is\(^2\)

\[ S = (p - \tau)Q - C - tZ, \]

and the consumer surplus is

\[ U = \int_0^Q p(\mu) \, d\mu - pQ. \]

### 3.2.1 The Welfare Function

Let \( \alpha \) be the weight the government places on consumer surplus, \( \beta \) be the weight placed on producer surplus, and \( \delta \) be the weight placed on externalities. We assume that \( \alpha, \beta \) and \( \delta \) are all nonnegative. We define the weighted welfare function as

\[ W = \alpha U + \beta S + R - \delta \Delta. \]  

(3.2)

In general, one can assign another weight to \( R \). Since all weights are assumed constant, we can, without loss of generality, normalize that weight to be 1. This indeed will have some advantage compared to, say, choosing \( \delta \) to be one, and having a weight different from one on \( R \). The advantage of the specification in (3.2) is that it yields a simpler form for the optimal policy formulae.

\(^2\)Without loss of generality, we choose to neglect counting fixed cost in the producer surplus so that the latter is simply profits.
Clearly, the traditional welfare function is a special case of (3.2) with $\alpha = \beta = \delta = 1$:

$$W = U + S + R - \Delta = \int_0^Q p(\mu) d\mu - C - \Delta.$$ 

### 3.3 Optimal Policy Analysis

#### 3.3.1 The Firms

An individual firm’s profit function is

$$\pi_i = (p(Q) - \tau) q_i - tz_i(q_i, a_i) - c_i(q_i, a_i).$$

We assume the firm chooses $q_i$ and $a_i$ to maximize $\pi_i$. The first-order conditions are

$$\pi_{iq_i} = p - \tau + q_i p' - tz_{iq_i} - c_{iq_i} = 0, \quad (3.3a)$$
$$\pi_{ia_i} = -tz_{ia_i} - c_{ia_i} = 0, \quad \forall \ i = 1, \ldots, n, \quad (3.3b)$$

where $z_{iq_i} = \partial z_i/\partial q_i$, $z_{ia_i} = \partial z_i/\partial a_i$, $c_{iq_i} = \partial c_i/\partial q_i$ and $c_{ia_i} = \partial c_i/\partial a_i$.

System (3.3) contains $2n$ equations and $2n$ unknowns. We denote $\bar{q}$ and $\bar{a}$, as the $n$-dimensional output and abatement vectors of the firms. The resulting Nash solutions, $q_i^*$ and $a_i^*$, are functions of $\tau$ and $t$.

For simplicity, assume that the firms are identical. Then, the first-order conditions can now be expressed as

$$\pi_q = p(nq) - \tau + qp'(nq) - tz_q(q, a) - c_q(q, a) = 0, \quad (3.4a)$$
$$\pi_a = -tz_a(q, a) - c_a(q, a) = 0, \quad (3.4b)$$
and the second-order conditions are

\[
\pi_{qq} = 2p' + qp'' - t z_{qq} - c_q \leq 0, \\
\pi_{aa} = -t z_{aa} - c_{aa} \leq 0.
\]

Equations (3.4) can be solved for the equilibrium \(q\) and \(a\):\(^3\)

\[
q = q(\tau, t), \\
a = a(\tau, t).
\]

It follows that the industry equilibrium \(Q\) and \(A\) are also functions of \(\tau\) and \(t\):

\[
Q = Q(\tau, t), \quad (3.5a) \\
A = A(\tau, t). \quad (3.5b)
\]

The effects of changing the parameters on the firm and industry equilibrium configurations are as follows:

\[
Q_r = n q_r = \frac{n \pi_{aa}}{|J|} < 0, \quad (3.6a) \\
Q_t = n q_t = -n \frac{-\pi_{aa} \bar{z}_q + \pi_{qa} \bar{z}_a}{|J|} < 0, \quad (3.6b) \\
A_r = n a_r = -n \frac{\pi_{aq}}{|J|} > 0, \quad (3.6c) \\
A_t = n a_t = -n \frac{\pi_{aq} \bar{z}_q - \pi_{qq} \bar{z}_a}{|J|} > 0, \quad (3.6d)
\]

where \(|J| \equiv \pi_{qq} \pi_{aa} - \pi_{qa} \pi_{aq} > 0.\)

\(^3\)To economize the use of notation, we do not introduce a new notation such as \(q^*\) for the equilibrium Nash solution. We just express such \(q\) as \(q(\tau, t),\) etc.
We assume that $c_q > 0$, $c_a > 0$ and $z_q > 0$, $z_a < 0$. By direct calculation, $\pi_{qa} = -tz_{qa} - c_{qa}$ and $\pi_{aq} = -tz_{aq} - c_{aq}$. A priori there seems to be no ground to assume that $c_{qa} = c_{aq}$ and $z_{qa} = z_{aq}$. But, if both of them hold, then $\pi_{qa} = \pi_{aq}$. For the sake of definiteness, in what follows, we further assume:

$$\pi_{qa} = \pi_{aq} < 0.$$

Proposition 9 If $\pi_{qa} = \pi_{aq} < 0$, then an increase in $\tau$ and $t$ will lower both $q$ and $Q$, and raise both $a$ and $A$.

3.3.2 The Government

The government internalizes the output and abatement decisions of the firms by taking (3.5) into consideration when optimizing welfare with respect to $\tau$ and $t$. Welfare remains defined as in (3.2), and can now be expressed as

$$\tilde{W}(\tau, t) = \alpha U(\tau, t) + \beta S(\tau, t) + R(\tau, t) - \delta \Delta(\tau, t).$$

(3.7)

The first-order conditions when the industry is in Nash equilibrium become

$$\tilde{W}_\tau = \alpha U_\tau + \beta S_\tau + R_\tau - \delta \Delta_\tau = 0,$$

(3.8a)

$$\tilde{W}_t = \alpha U_t + \beta S_t + R_t - \delta \Delta_t = 0,$$

(3.8b)

where, after simplification, we have

$$U_\tau = -Qp'Q_\tau, \quad U_t = -Qp'Q_t,$$

$$S_\tau = Qp'^{n-1}_nQ_\tau - Q, \quad S_t = Qp'^{n-1}_nQ_t - Z,$$

$$R_\tau = (\tau + tZ_Q)Q_\tau + tZ_AA_\tau + Q, \quad R_t = (\tau + tZ_Q)Q_t + tZ_AA_t + Z,$$

$$\Delta_\tau = (\Omega' + \Phi'Z_Q)Q_\tau + \Phi'Z_AA_\tau, \quad \Delta_t = (\Omega' + \Phi'Z_Q)Q_t + \Phi'Z_AA_t.$$

(3.9)
CHAPTER 3: OPTIMAL POLICY ON EXTERNALITIES WITH WEIGHTED GROUP INTERESTS

From (3.6), all of $U_\tau$, $U_t$, $\Delta_\tau$, and $\Delta_t$ are negative. The signs of $S_\tau$, $S_t$, $R_\tau$ and $R_t$ are indeterminate. However, if $p'Q_\tau < n/(n-1)$, then $S_\tau < 0$.

We can alternatively write (3.8) as

$$
\tilde{W}_\tau = A_1\tau + A_2t + A_3 = 0, \quad (3.10a)
$$

$$
\tilde{W}_t = B_1\tau + B_2t + B_3 = 0, \quad (3.10b)
$$

where

$$
A_1 = Q_\tau,
$$

$$
A_2 = Z_A A_t + Z_Q Q_\tau,
$$

$$
A_3 = -Q(\beta - 1) - Q_\tau \left( \delta (\Omega' + \Phi'Z_Q) + Qp' \left( \alpha - \beta \frac{n-1}{n} \right) \right) - \delta \Phi'Z_A A_\tau,
$$

$$
B_1 = Q_t,
$$

$$
B_2 = Z_A A_t + Z_Q Q_t,
$$

$$
B_3 = -Z(\beta - 1) - Q_t \left( \delta (\Omega' + \Phi'Z_Q) + Qp' \left( \alpha - \beta \frac{n-1}{n} \right) \right) - \delta \Phi' Z_A A_t.
$$

We then solve (3.10) to obtain the optimal consumption and pollution tax rates:

$$
\tau^* = \delta \Omega' + \left( \alpha - \beta \frac{n-1}{n} \right) Qp' + (\beta - 1) \frac{Z_A (QA_t - ZA_\tau) + Z_Q (QQ_t - ZQ_\tau)}{Z_A (A_t Q_\tau - Q_t A_\tau)}, \quad (3.11a)
$$

$$
t^* = \delta \Phi' - (\beta - 1) \frac{QQ_t - ZQ_\tau}{Z_A (A_t Q_\tau - Q_t A_\tau)}. \quad (3.11b)
$$

The signs of $QQ_t - ZQ_\tau$ and $QA_t - ZA_\tau$ in the numerators, and $A_t Q_\tau - Q_t A_\tau$ in the denominator, are indeterminate unless specific functional forms for cost, pollution and damages are specified.

Note that the formulae in (3.11) are implicit since the firms’ decision variables are func-
tions of $\tau$ and $t$.

Also note that when the government favors consumers over firms, (i.e., $\alpha > \beta$) it must take into account that firms’ profits are not to be squeezed to become negative. It is conceivable that such a consideration may become binding. In order to carry out the true social optimum in the eyes of the policy makers, should such a case arise, the government could impose lump-sum taxes on the consumers and subsidize the firms to keep them in the market.

Several conclusions about the externality taxes in the weighted welfare case are apparent from (3.11).

Let

\begin{align*}
I_1 &= \frac{QQ_t - ZQ_{\tau}}{Z_A (Q_{\tau}A_t - Q_t A_{\tau})}, \\
I_2 &= I_1 Z_Q + \frac{QA_t - ZA_{\tau}}{Q_{\tau}A_t - Q_t A_{\tau}},
\end{align*}

so that we can express (3.11) as

\begin{align*}
\tau^* &= \delta \Omega' + \left(\alpha - \beta \frac{n - 1}{n}\right) Qp' + (\beta - 1) I_2, \\
t^* &= \delta \Phi' - (\beta - 1) I_1.
\end{align*}

First, if $\alpha = \beta = \delta = 1$, then $\tau^* = \Omega' + Qp'/n = \Omega' - p/n \varepsilon$, where $\varepsilon = -pdQ/Qdp$ is the elasticity of demand, and $t^* = \Phi'$. The consumption (output) tax is less than the direct marginal damage of consumption (production) since the policy maker makes the adjustment of offsetting the oligopolistic market imperfection which causes less output than the competitive case. The adjusted amount is $qp'$, which is the gap between the output price, $p$, and a firm’s marginal revenue, $\partial (q_ip (q_i + Q_{-i})/\partial q_i)$, where $Q_{-i}$ represents other firms’ output. The pollution tax is the standard Pigouvian marginal pollution damage.
CHAPTER 3: OPTIMAL POLICY ON EXTERNALITIES WITH WEIGHTED GROUP INTERESTS

Proposition 10  1. If \( \alpha = \beta = \delta = 1 \), then \( \tau^* = \Omega' + Qp' / n \), the conventional rate.

2. If \( \beta = \delta = 1 \), then \( t^* = \Phi' \).

Second, it is conceivable that the optimal tax rates may be higher or lower than their respective Pigouvian rates.

Finally, with our formulae in (3.11) established, it would be useful to examine some special functional forms to see how the optimal externality policies are affected by the parameter values in the model. We present an example in the next section.

3.3.3 An Illustrative Example

We consider an example with the special functional forms:

\[
p = a_1 - a_2 Q, \\
c = \frac{1}{2} q^2 + baq + \frac{1}{2} a^2, \\
z = q - a, \\
\Omega = \omega Q, \\
\Phi = \phi Z,
\]

where \( a_1, a_2, \omega \) and \( \phi \) are constant parameters. We assume that \( \phi > 0 \) since pollution causes a negative externality. However, we allow \( \omega \) to be positive or negative, to indicate a negative or positive consumption (production) externality, respectively.\(^4\)

We consider the symmetric case. A firm’s profit function is

\[
\pi = (p - \tau) q - c - tz.
\]

\(^4\)It is required \( b^2 < 1 \) for the cost function in the present example to be positive definite.
From the first-order profit maximization conditions the firms’ optimal levels of output and abatement are:

\[ q = -\frac{(b + 1) t + \tau - a_1}{a_2 (n + 1) - b^2 + 1}, \]
\[ a = \frac{(a_2 (n + 1) + b + 1) t + b\tau - ba_1}{a_2 (n + 1) - b^2 + 1}. \]

The aggregate Nash equilibrium \( Q \) and \( A \) are:

\[ Q = -n\frac{(b + 1) t + \tau - a_1}{a_2 (n + 1) - b^2 + 1}, \quad (3.12a) \]
\[ A = n\frac{(a_2 (n + 1) + b + 1) t + b\tau - ba_1}{a_2 (n + 1) - b^2 + 1}. \quad (3.12b) \]

The government’s welfare function is (3.2). Its components’ are:

\[ U = \frac{1}{2} a_2 Q^2, \]
\[ S = (p - \tau) Q - C - tZ, \]
\[ R = \tau Q + tZ, \]
\[ \Delta = \Omega + \Phi. \]

Using (3.12) in (3.2) we obtain this example’s equivalent of (3.7). Maximizing this function with respect to \( \tau \) and \( t \), gives

\[ \tau^* = \frac{E_1 \omega + E_2 \phi + E_3}{2(\beta - 1)(2\beta - 2a_2 - 2na_2 + 2\beta a_2 - 2b^2\beta + 2b^2 + n\alpha a_2 - 2)}, \quad (3.13a) \]
\[ t^* = -\delta \frac{\phi}{2(\beta - 1)}, \quad (3.13b) \]
where

\[ E_1 = 2\delta (\beta - 1) (-a_2 - na_2 + b^2 - 1), \]
\[ E_2 = n\delta a_2 (\alpha - 2\beta) (b + 1), \]
\[ E_3 = 2a_1 (\beta - 1) (2\beta - a_2 - na_2 + 2\beta a_2 - 2b^2\beta + b^2 + naa_2 - 1). \]

Several interesting implications result from this example. First, notice that \( \beta \neq 1 \). This stems from our treatment of tax revenues in the welfare function. Since \( S = (p - \tau)Q - C - tZ \) and \( R = \tau Q + tZ \), we can write \( S = pQ - R - C \). Then, when \( \beta = 1 \), \( W \) becomes

\[ W = \alpha U + (pQ - C) - \delta \Delta, \]

revealing that \( \tau \) and \( t \) no longer appear explicitly. Therefore, our solution in (3.13) requires that \( \beta \neq 1 \).

Second, the effects of changing the weights \( \alpha \) and \( \beta \) on the optimal policy values are typically indeterminate. In this example, all are indeterminate except for \( \partial t/\partial \alpha \) and \( \partial t/\partial \beta \). The value of \( \partial t/\partial \alpha \) is zero. This can be explained by the presence of \( \tau \). Since \( t \) is applied to pollution and \( \tau \) to consumption directly, \( \tau \) corrects to the full extent the consumption externality without having to involve \( t \). The sign of \( \partial t/\partial \beta \) is positive. This result may be counter-intuitive, but it is not abnormal. Recalling (3.8b) and (3.9), we know that \( U_t < 0 \) and \( \Delta_t < 0 \), while \( S_t \) and \( R_t \) can have either sign. An increase in \( \beta \) affects each of the preceding four values. A higher \( \beta \) means the government places a higher valuation on \( S \). Therefore, in the case where \( S_t > 0 \), it is conceivable that the government would choose to raise \( t \).


3.4 The Case of Choosing $Q$ and $A$ by the Government

In light of Kim and Chang (1993)'s method of choosing quantity variables, in this section we now consider the case where the government chooses $Q$ and $A$ to maximize social welfare. A typical firm’s optimization problem is the same as in Subsection (3.3.1). Recall that $\tau$ and $t$ are constant parameters for the firms.

To this end, we revisit firms’ first-order-conditions in (3.4) in our model:

$$\pi_q (q, a; \tau, t, n) = 0,$$
$$\pi_a (q, a; t) = 0.$$

$$|K| \equiv \left| \begin{array}{ll} \pi_{q\tau} & \pi_{qt} \\ \pi_{a\tau} & \pi_{at} \end{array} \right| = \left| \begin{array}{cc} -1 & -z_q \\ 0 & -z_a \end{array} \right| = z_a < 0, \text{ so } \tau \text{ and } t \text{ are locally functions of } Q \text{ and } A$$

making it possible to locally express the tax functions as

$$\tau = \tau (q, a),$$
$$t = t (q, a).$$

For the local inversion to work, the $K$ matrix must have the property that the number of policy instruments is at least the number of firms’ decision variables.

In the symmetry case the government views $\tau = \tau (Q, A)$ and $t = t (Q, A)$ after utilizing the firms first-order conditions. Given $\alpha, \beta$ and $\delta$, the government chooses $Q$ and $A$ to maximize the social welfare function:

$$\hat{W} (Q, A) = \alpha U (Q, A) + \beta S (Q, A) + R (Q, A) - \delta \Delta (Q, A). \quad (3.14)$$
The first-order conditions are

\[ \hat{W}_Q = \alpha U_Q + \beta S_Q + R_Q - \delta \Delta_Q = 0, \quad (3.15a) \]
\[ \hat{W}_A = \alpha U_A + \beta S_A + R_A - \delta \Delta_A = 0, \quad (3.15b) \]

where after simplification the components become

\[ U_Q = -Q p', \quad U_A = 0, \]
\[ S_Q = Q \left( p' \frac{n-1}{n} - \tau Q \right) - Z t_Q, \quad S_A = - (Z t_A + Q \tau_A), \]
\[ R_Q = Q \tau_Q + Z t_Q + \tau + t Z_Q, \quad R_A = Q \tau_A + Z t_A + t Z_A, \]
\[ \Delta_Q = \Omega' + \Phi' Z_Q, \quad \Delta_A = \Phi' Z_A. \]

These expressions were obtained by use of (3.4) and the homogeneity property assumed for the cost and pollution functions.

The system (3.15) can be expressed as

\[ \hat{W}_Q = 0 = Q \left( 1 - \beta \right) \tau_Q + Z \left( 1 - \beta \right) t_Q - \alpha Q p' + \beta \left( (p - \tau) + Q p' - (p - \tau + q p') \right) + \tau + t Z_Q - \delta \left( \Omega' - \Phi' Z_Q \right), \]
\[ \hat{W}_A = 0 = Q \left( 1 - \beta \right) \tau_A + Z \left( 1 - \beta \right) t_A + t Z_A - \delta \Phi' Z_A. \]

From this, we derive the optimal tax rates:

\[ \hat{\tau} = \delta \Omega' + Q p' \left( \alpha - \beta \frac{n-1}{n} \right) + (\beta - 1) \frac{Z t_Q + Q \tau Q}{Z_A} Z_A - \left( Q \tau_A + Z t_A \right) Z_Q, \]
\[ \hat{t} = \delta \Phi' + (\beta - 1) \frac{Q \tau_A + Z t_A}{Z_A}. \]
Similarly, we let
\[ J_1 = \frac{Qr_A + Zt_A}{Z_A}, \]
\[ J_2 = Zt_Q + Qr_Q - J_1 Z_Q, \]
so that we can express the optimal tax rates as
\[ \hat{\tau} = \delta \Omega' + \left( \alpha - \beta \frac{n-1}{n} \right) Qp' + (\beta - 1) J_2, \]
\[ \hat{t} = \delta \Phi' + (\beta - 1) J_1. \]

Although the formulae do not look exactly the same as in (3.11), as long as the \( t \) and \( \tau \) can be uniquely obtained from the firms’ first-order conditions as functions of \( Q \) and \( A \), then the two models will yield the same optimal solution.\(^5\)

The interpretations of these tax rates remain the same as in (10). The term attached to \( \beta - 1 \) differs due to the change in choice variables by the government.

### 3.5 Technology and Cost Functions in Environmental Policy Analysis

#### 3.5.1 Specification of Technology and the Implied Cost Function

Assume pollution is a by-product of the production process. To produce output \( q \), a representative firm needs to use labor, \( l \), and pollution-control input \( m \) according to the production function:
\[ q = f (l, m), \]

\(^5\)We have used the example in Subsection (3.3.3) to illustrate.
where \( f_l > 0, f_{ll} < 0 \). It is not clear that \( f_m \) is positive or negative, as the sign depends on how the production process functions. In one scenario, adding pollution-control machines can be in the way of labor’s operation in the factory, so that \( f_m < 0 \) and \( f_{lm} < 0 \). In another scenario, adding \( m \) makes worker’s working environment better (less pollutant flowing around in the factory), so that \( f_m > 0 \) and \( f_{lm} > 0 \). Lacking better terms, we shall call the case \( f_m < (>) 0 \) negative (positive) marginal productivity of \( m \).

Let the quantity emitted of pollutant be

\[
z = h (l, m),
\]

(3.16)

where \( h_l > 0, h_m < 0 \). There is no \( a \ priori \) ground to know the signs of \( h_{ll} \) and \( h_{mm} \).

If there is no pollution control, then \( m = 0 \) and pollution is

\[
z_0 = h (l, 0).
\]

One simple case is that \( z \) and \( q \) are always in a constant proportion, \( i.e., z = kq \). Then \( z = kf (l, m) \).

Assume that \( m \) is purchased by the firm in the open market at a unit price of \( v \). The input cost function is

\[
c = wl + vm = \hat{c}(l, m),
\]

where \( w \) is the wage rate. \( w \) and \( v \) are constants.

The firm chooses \( l \) and \( m \) to minimize its direct production cost \( \hat{c}(l, m) \) subject to \( f (l, m) \geq \bar{q} \) and \( h (l, m) \leq \bar{z} \). The cost function here does not include a possible tax/subsidy from the government’s environmental policy. Let the Lagrangian function be

\[
\mathcal{L} (l, m; \lambda, \mu; \bar{q}, \bar{z}, w, v) = (wl + vm) - \lambda (f (l, m) - \bar{q}) - \mu (\bar{z} - h (l, m)).
\]
Assume an interior solution. Then the necessary first-order conditions are

\[ \mathcal{L}_l = w - \lambda f_1 + \mu h_1 = 0, \]
\[ \mathcal{L}_m = v - \lambda f_2 + \mu h_2 = 0, \]
\[ \mathcal{L}_\lambda = \bar{q} - f(l, m) = 0, \]
\[ \mathcal{L}_\mu = h(l, m) - \bar{z} = 0. \]

From the last two equations above, we can solve

\[ l = l(\bar{q}, \bar{z}), \quad m = m(\bar{q}, \bar{z}). \quad (3.17) \]

The minimized cost function is

\[ c^*(\bar{q}, \bar{z}; w, v) = \min \hat{c} \equiv wl(\bar{q}, \bar{z}) + vm(\bar{q}, \bar{z}). \]

By the Envelope Theorem,

\[ c_q^*(\bar{q}, \bar{z}; w, v) = \lambda, \quad (3.18a) \]
\[ -c_2^*(\bar{q}, \bar{z}; w, v) = \mu. \quad (3.18b) \]

\( \lambda \) is the marginal cost of production. \( \mu \) is the marginal cost of abatement. With \( w \) and \( v \) being constant, we shall write the minimized cost function as

\[ c^* = c^*(q, z). \]

Assume that there is no consumption tax or subsidy, but that pollution is taxed at the
rate $t$. Then, the profit function is

$$
\pi (q, z; Q_{-i}, t) = p(Q)q - c(q, z) - tz = r(q; Q_j) - c(q, z) - tz = \pi (q, z; Q_{-i}, t),
$$

where $r = p(Q)q = r(q; Q_{-i})$ is the firm’s total revenue, and $Q_{-i}$ is all other firms’ output. Maximizing $\pi$ by choosing $q$ and $z$, under the Nash conjecture, yields

$$
\pi_q = r' (q; Q_j) - c^*_q (q, z) = 0 \quad (3.19a)
$$

$$
\pi_z = -c^*_z (q, z) - t = 0, \quad (3.19b)
$$

where $r' = p + qp'$ is the marginal revenue. Thus, (3.19a) equates marginal revenue and marginal cost, and (3.19b) indicates the firm will pollute up to the amount at which its marginal abatement cost ($-c^*_z$) equals the tax rate.

### 3.5.2 An Alternative Formulation

Let $a$ be the amount of pollutant abated. We then have

$$
z = z_0 - a = h(l, 0) - a \equiv \tilde{h}(l, a),
$$

where $\tilde{h}_a(l, a) = -1$. From (3.16) we have $z = h(l, m) = \tilde{h}(l, a)$. Solving for $m$, gives

$$
\tilde{m} = \tilde{m}(l, a).
$$

Since $h(l, m) = \tilde{h}(l, a)$,

$$
h_t dl + h_m d\tilde{m} = \tilde{h}_t dl + \tilde{h}_a da = h_t (l, 0) dl - da,
$$
from which we obtain

\[ h_m d\tilde{m} = (h_l (l, 0) - h_l (l, m)) \, dl - da. \]

We then have

\[ \tilde{m}_l = \frac{h_l (l, 0) - h_l (l, m)}{h_m}, \]
\[ \tilde{m}_a = -\frac{1}{h_m}. \]

It is reasonable to assume \( h_l (l, 0) > h_l (l, m) \). Since \( h_l > 0 \) and \( h_m < 0 \) as assumed, we can infer that \( \tilde{m}_l < 0 \) and \( \tilde{m}_a > 0 \).

The cost function now becomes

\[ \hat{c} = w l + v \tilde{m} (l, a) \equiv \hat{c} (l, a). \]

We have \( \hat{c}_l = w + v \tilde{m}_l \gtrless 0 \) and \( \hat{c}_a = v \tilde{m}_a > 0 \). Let \( q = f (l, \tilde{m} (l, a)) \equiv f (l, a) \). We have

\[ dq = \tilde{f}_l dl + \tilde{f}_a da = (f_l + f_m \tilde{m}_l) \, dl + f_m \tilde{m}_a da, \]
\[ dz = \tilde{h}_l dl + \tilde{h}_a da = (h_l + h_m \tilde{m}_l) \, dl + h_m \tilde{m}_a da. \]

Although we know that \( f_l > 0 \) and \( \tilde{m}_l < 0 \), we have argued that the sign of \( f_m \) is indeterminate. Thus, the sign of \( \tilde{f}_l \) remains unknown. Similarly, although we know \( \tilde{m}_a > 0 \), the sign of \( \tilde{f}_a \) is also unknown.

Consider the problem of choosing \( l \) and \( a \) to minimize \( \hat{c} (l, a) \) subject to \( \tilde{f} (l, a) \geq \bar{q} \) and \( \tilde{h} (l, a) \leq \bar{z} \). Let the Lagrangian function be

\[ \tilde{\mathcal{L}} (l, a, \gamma, \theta; \bar{q}, \bar{z}, w, v) = (w l + v \tilde{m} (l, a)) - \gamma (\tilde{f} (l, a) - \bar{q}) - \theta (\bar{z} - \tilde{h} (l, a)). \]
CHAPTER 3: OPTIMAL POLICY ON EXTERNALITIES WITH WEIGHTED GROUP INTERESTS

We consider only the case where the constraints are binding. Then the first-order conditions are:

\[ \tilde{L}_l = w + v \tilde{m}_l - \gamma \tilde{f}_l + \theta \tilde{h}_l = 0, \]  
\[ \tilde{L}_a = v \tilde{m}_a - \gamma \tilde{f}_a + \theta \tilde{h}_a = 0, \]  
\[ \tilde{L}_\gamma = \bar{q} - \tilde{f}(l,a) = 0, \]  
\[ \tilde{L}_\theta = \tilde{h}(l,a) - \bar{z} = 0. \]

From (3.20c) and (3.20d), we can solve for

\[ l = l(\bar{q}, \bar{z}), \quad a = a(\bar{q}, \bar{z}). \]  

The minimized cost function is

\[ \bar{c} = \min \hat{c} = w l(\bar{q}, \bar{z}) + v a(\bar{q}, \bar{z}) \equiv \bar{c}(\bar{q}, \bar{z}; w, v). \]

By the Envelope Theorem,

\[ \bar{c}_q(\bar{q}, \bar{z}; w, v) = \gamma, \]  
\[ -\bar{c}_a(\bar{q}, \bar{z}; w, v) = \theta. \]

From (3.18) and (3.22), we see that \( \lambda = \gamma \) and \( \mu = \theta. \)
3.5.3 An Integrated View

Most of the literature specifies the cost function \( \bar{c}(\bar{q}, \bar{z}; t, w, v) \), or simply
\[
c = c(q, z).
\]
(See, e.g., Requate (2005 Assumption 4, p.8)). From the above analysis, we arrive at the same cost function by either using the system \( q = f(l, m) \) and \( z = h(l, m) \), or the system \( q = \tilde{f}(l, a) \) and \( z = \tilde{h}(l, a) \). It is often assumed that the cost function is twice continuously differentiable and that \( c_{qz} < 0 \).

Kim and Chang (1993) specify the cost function
\[
c^{KC} = c^{KC}(q, a),
\]
and the pollution generating function
\[
z = z^{KC}(q, a).
\]

First, their pollution generating function can be derived from our framework as follows. We have
\[
z = h(l, m) = h(l, \tilde{m}(l, a)) = \tilde{h}(l, a) = \tilde{h}(l(q, z), a),
\]
by (3.17). Solving \( z = \tilde{h}(l(q, z), a) \), we obtain \( z = z(q, a) \). Next, by our specification, \( a = z_0(l, 0) - z \) or \( z = z_0(l, 0) - a = \tilde{h}(l, a) \). Then, under the specification \( c = c(q, z) \), we have
\[
c = c(q, z) = c(q, \tilde{h}(l, a)) = c(q, l, a).
\]
By (3.21), we have \( l = l(q, z) \) and by Kim and Chang’s pollution generating function \( z =
\( z^{KC}(q,a) \), we obtain
\[
l = l(q,z) = l(q,z^{KC}(q,a)) = \hat{l}(q,a).
\]

It follows that
\[
c = c(q,l,a) = c(q,\hat{l}(q,a),a) = \hat{c}(q,a).
\]

We interpret this as Kim and Chang’s cost function, \( c^{KC}(q,a) \) so that \( \hat{c}(q,a) = c^{KC}(q,a) \).

We then notice that assuming the sign of \( c^{KC}_a \) (as \( \hat{c}_a \)) is not so straight-forward, as it would involve signing \( c_l(q,\hat{l}(q,a),a) \hat{l}_a + c_a \). An additional assumption would have to be made upon the sign of \( \hat{l}_a \) as there are no economic grounds for assuming that this value is positive or negative.

### 3.6 Conclusion

This paper constructs a welfare function which includes different weights assigned to each of the components to reflect either the regulator’s own judgment or special interest groups’ influence. In addition to a pollution externality, a consumption externality was also introduced in the model. The optimal government policies on consumption and pollution under imperfect competition were analyzed. We show that optimal tax rates are influenced significantly by the relative magnitudes of the welfare weights. Additionally we show that these rates can be higher (or lower) than their Pigouvian rates, even after adjusting for the market imperfection factor. Furthermore, we show that some counter-intuitive results on the optimal tax rates may appear when the welfare weights are altered. An example was constructed to further illustrate our results.

In the last portion of the paper we examine various specifications of the technology for output and pollution and derive the implied cost functions. This approach has allowed us to shed light on the different properties the implied cost functions must possess. It reveals that
the existing literature is quite lax or overly simplistic in making assumptions on the cost functions. These new insights should be useful for those who rely on cost functions when studying optimal environmental policies.

References


Conclusion

This dissertation examined optimal environmental policies in an imperfect market set up. Chapter 1 examined the effects of emission taxation on an oligopolistic industry. It showed that, in the short run with a fixed number of firms, an increase in the tax rate always lowers all firms’ outputs, resulting in a contraction of the industry output. In the long run, the tax affects firms’ entry and exit decisions and may yield ambiguous effects on the firms’ and industry outputs. It further showed that the tax reduces the equilibrium number of firms, and also reduces the equilibrium industry output if the inverse demand function is affine, concave, or not too convex. Finally, the chapter examined the question of the optimal number of firms from the social point of view. It showed excess or insufficient entry is determined by the relationship between the social marginal benefit of production and the responses of firms’ outputs to a change in the tax rate. An example was constructed to illustrate the general results in practice.

Chapter 2 analyzed a model proposed by Kim and Chang (1993) in which they provide a method for deriving an optimal nonlinear tax function for polluting firms in an imperfectly competitive industry under asymmetric information between firms and the regulator. It was discovered that their method, elegant and clever in its construction, is capable of being extended to cases where firms’ pollutions may not be perfect substitutes, where additional variables enter the damage (or externality) function and where the government may have different choice variables than the firms. Two caveats to the model were found. First, the number of policy targets must match the number of externalities in the market, otherwise
conflicting conditions on the properties of the function could arise. The resulting tax function would not achieve the social optimum, a result that is well known in the literature. When their method cannot be applied, we suggested that unit tax rates be imposed to achieve the regulator’s objective. The unit rates, however, require additional information not needed in Kim and Chang’s model. The second caveat found was that firms and the government must focus on choosing the same set of choice variables unless the relationship between all variables is linear. If it is not the case, unique solutions for each variable in terms of the others is not guaranteed and suboptimal tax schemes will result.

Chapter 3 constructed a welfare function which included different weights assigned to each of the components of the welfare function, to reflect either the regulator’s own judgment or special interest groups’ influence. In addition to a pollution externality, a consumption externality was also introduced in the model. The optimal government policies on consumption and pollution under imperfect competition were analyzed. It was shown that optimal tax rates are influenced significantly by the relative magnitudes of the welfare weights. Additionally, the chapter demonstrated that these rates can be higher (or lower) than their Pigouvian rates, even after adjusting for the market imperfection factor. Some counter-intuitive results on the optimal tax rates were shown to appear when the welfare weights are altered, and an example was constructed to further illustrate our results.

The last portion of the chapter examined various specifications of the technology for output and pollution. It derived the implied cost function which sheds light on the different properties the implied cost functions must possess. The analysis revealed that the existing literature is quite lax or overly simplistic in making assumptions on the cost functions. These new insights should be useful for those who rely on cost functions when studying optimal environmental policies.