Contact Dynamics Modeling and Simulation of Tether-Nets for Space Debris Capture

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A proposed method for containing the growth of space debris which jeopardizes operation of spacecraft is the active debris removal of massive derelict spacecraft and launcher upper stages by means of tether-nets. The behavior of nets in space is not well-known; therefore, numerical simulation is needed to gain understanding of deployment and capture dynamics. In this paper, a lumped-parameter approach for modeling the net, and different models of contact dynamics are presented. A continuous compliant approach for the normal contact force and a modified damped bristle model for the friction force are chosen. The capability of the developed simulation tool to represent multiple dynamic conditions is demonstrated in this paper, and the results of a deployment dynamics simulation are presented; this reveals a snapping behavior of tension. Simulation of net-based capture of a cylindrical debris in microgravity and vacuum conditions is performed with the presented tool. The effect of employing different contact force models on the overall results of capture is evaluated, both from a dynamical and a computational point of view, and reasons to prefer non-linear models are discussed.

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1. Introduction

Space debris jeopardizes spacecraft operations in the most exploited Low Earth Orbits (LEOs), and collisions are recognized to be the leading cause of the increase in the number of space debris in orbit \[1\]. Since research has shown that the risk of accidental collisions can be reduced by actively removing a few massive objects per year \[2\], Active Debris Removal (ADR) is being studied by several researchers.

A promising technology to collect large non-operational spacecraft is tether-nets, devices that are thrown towards a debris, entangle it or close around it, and provide a link to tug it to a disposal orbit. This type of capturing device is lightweight and not bulky; it promises higher safety compared to the use of rigid links such as robotic arms, is likely to introduce distributed loads, and is compliant to the shape of the debris. The price to pay, in addition to the lower technological readiness of the system, is the non-repeatability of the capture and the flexibility of the net, which entails a more complicated dynamical system and requires high modeling, simulation and experimentation efforts. Before employing tether-net devices in ADR missions, it is necessary to accurately simulate the dynamics of deployment and capture.

Deployment and capture dynamics are highly non-linear, due to the inability of the material of the net to withstand compression, the inherently non-linear contact dynamics, the large deformations of the net, and the snapping tensions in the threads. This, together with the complex geometry of the system, entails high computational cost for dynamics simulation, which remains an open and challenging problem.

In this paper, a stand-alone tool for the numerical simulation of the dynamics of a net during deployment and contact with a massive object is presented. In Section [II] the state-of-the-art in the simulation of tether-nets for space debris capture is presented, focusing in particular on the modeling of the net and of contact dynamics. Section [III] presents the geometry of the net used for this work, the model implemented for the net deployment phase, and that for contact dynamics. Section [IV] deals with the results of numerical simulations: first the outcome of a simulation of the deployment phase is analyzed; then a debris capture scenario is proposed, and the simulation results are discussed, highlighting in particular the effect of using different contact models on the overall
capture dynamics. The paper is concluded by summarizing the achievements and limitations of the presented research.

II. State-of-the-art

Multiple methods to actively remove debris have been proposed in the last few years: some of them require establishing a link with the debris, through capture by means of robotic arms [3], tether-nets [4], or harpoons [5]; some others rely on contactless methods, such as lasers or ion beams [6]. Since the focus of this paper is the tethered-net concept, in the following we review works on mechanical (i.e., excluding electro-dynamic) tethers for ADR; the second part of the review deals more specifically with the state-of-the-art in the modeling and simulation of tether-nets.

In the framework of ADR, tethers can provide a link between the chaser spacecraft and the target debris, once the debris has been captured, for example, by means of a net or of a harpoon. Previous works on mechanical tethers have dealt primarily with the disposal phase of the mission, analyzing the dynamics of the system composed of chaser, tether, and debris during the tug. Among others, the dynamics of tug of a large space debris with a tether was studied by Aslanov and Yudintsev [7, 8], who demonstrated that safe transportation of debris can be achieved through tether tensioning and chaser-tether-target alignment; Aslanov recently explored the chaotic behaviour of such a system during tow [9]. Jasper and Shaub analyzed discrete input-shaping techniques for modulating the de-orbit burn and reducing the risk of collisions between chaser and target [10], while Sabatini et al. challenged the effectiveness of input shaping when the tether is initially slack and proposed different solutions to eliminate this problematic condition [11]. Linskens and Mooij used a sliding-mode controller to stabilize the chaser-tether-target system in the presence of perturbations [12], whereas Cleary and O’Connor proposed using wave-based control as a robust and generic method for maneuvering the chaser while maintaining control of the target at the end of the tether [13]. Hovell and Ulrich focused on the de-tumbling phase of an ADR mission, considering attitude stabilization of captured debris using multiple tethers [14].

Among methods to capture debris, the focus is here on nets. Interest in net-based systems to capture debris is relatively recent and much of the research is being carried out through the support
of the European Space Agency (ESA), with the aim of deorbiting the defunct Envisat satellite. The first work to give credit to tether-net systems for capture of uncooperative objects was ROGER preliminary study by Astrium [4]. Wormnes et al. performed multiple preliminary simulations of debris capture with a tether-net device by using a lumped-parameter approach with the 3D modeling environment Blender, and Bullet physics engine [15]. Parallel to that, Lavagna et al. implemented a lumped-parameter-based simulator in Matlab [16]. These two research activities have recently been merged, with the participation of some European industries: a hybrid simulator has been assembled [17], whose validation is reportedly under way, following a parabolic flight experiment [18]. Another research group collaborating with ESA created a tether-net simulator based on Cosserat rod theory for modeling the net and is validating it against results of a parabolic flight experiment as well [19]. Additionally, an in-orbit demonstration mission is being planned by Forslau et al. [20]. Casting and deployment performance of flexible nets in space was studied by Liu et al. [21] and by Shan et al. [22] with other simulators based on the lumped-parameter approach. The standard lumped-parameter model was used by Botta et al. in an early work [23], and was augmented by including representation of the bending stiffness of the net threads in subsequent research [24]. A sensitivity study on the effectiveness of capture was performed by Botta et al. exploiting a simulator built in Vortex Dynamics [25]. Benvenuto et al. chose the standard lumped-parameter approach and performed different studies on the capture and disposal phases [26].

Although some works exist on the simulation of net-based devices for ADR, much remains to be done, especially with regards to the contact dynamics modeling between net and target debris in space, and among different parts of the net: to the best of the authors' knowledge, only a few groups simulated the capture phase, and the contact dynamics models they employed have limitations. Moreover, no research on the effect of contact models on the capture dynamics was found in the literature to date. In the work by Wormnes et al. [15], contact is dealt with by imposing linearized velocity constraints, and friction force is accounted for with an extremely simplified model, in which the tangential force does not depend on the normal force. As recognized by the authors themselves, even though this approach is computationally affordable, it comes at a cost of lower physical relevance [15]. Benvenuto et al. implemented a linear continuous compliant model (i.e., a
spring-dashpot model) for the normal contact force and Coulomb's friction model [27]; recently, they improved the contact modeling using a non-linear continuous compliant model and a regularized version of Coulomb's friction model [26]. The simulator implemented by Botta et al. within Vortex Dynamics framework exploited a modified linear continuous compliant model for the normal contact force, a scaled box friction model, and some regularization in the sticking regime [24]. The effect of linear and non-linear compliant contact force models was evaluated in a frictionless and simplified scenario involving a net in our early work, in which a net is released under the effect of gravity from a fully deployed configuration and impacts a ground [23]; this brought to light that, although the trends were similar, conspicuous differences existed in the response to contact already after the first impact of the net with the ground. In this paper, we build upon that research, by extending the continuous compliant approach to account for contacts between net and debris as well as within the net itself, by introducing micro-slip frictional models and by applying the model to the simulation of capture of a debris of cylindrical shape in microgravity and vacuum conditions. Several contact force models, based on the recent reviews [28, 29] are selected for a detailed comparison of the response during the envelopment of the aforementioned debris.

III. Modeling of the Net and of Contact Dynamics

During the deployment and capture phases of a tether-net ADR mission, it is opportune to know how the net configuration evolves, how the threads impact on the debris and whether the debris is correctly entangled; hence, a reasonably detailed representation of the net and of contact dynamics is necessary. In this section, the focus is on the models implemented for simulating the dynamics of deployment of the net and its contact with the debris in space. The configuration of the net used in all the simulations and its representation in the software is first introduced. Then, the typical lumped-parameter approach is reviewed, as implemented in our simulator. Finally, collision detection and contact dynamics issues are addressed according to considerations of the system at hand.
A. Net Geometry

In the envisaged system, the net would be pulled out of a canister by several weights, which in turn are launched in the direction of the debris by a dedicated mechanism [4]. Different net geometries have been proposed, among which are planar, pyramidal, conical, and hemispherical [15, 16, 26, 30]. With the aim of deorbiting large debris, such as Envisat spacecraft, a 10×10 m² to 20×20 m² plane square net with a mesh length of 20 to 25 cm was recommended [4]. The configuration considered in this paper consists of a plane square net with square mesh and four corner masses, which is the primary candidate configuration for this mission: for the sake of clarity in the presentation of the results, we will analyze in the sequel the deployment dynamics of a 5×5 m² net with a mesh of 1 m, which results in 40 nodes and 64 threads. Simulations of larger and more complicated nets were accomplished in the course of research and performed consistently.

Paramount to the simulation of deployment and contact dynamics is the representation of the net geometry in the software environment. The nodes and threads of the net are numbered and a connectivity table is created in an automated way for a square net of any size with any square mesh and with four corner masses. As an example and as reference for understanding the plots of

Figure 1: Numbering of nodes and threads [23].
the simulation results, the numbering of nodes and threads, and part of the resulting connectivity table are shown in Fig. 1 and Table 1 for the net used in the simulations that will be discussed in the next sections. Here, the inertial reference frame used in the simulations is shown: in the derivation of the equations of motion, it is assumed that this inertial reference frame coincides with the local orbital reference frame. It may be noted that the different sizes of the nodes in Fig. 1 are indicative of the different node masses (as per mass lumping presented in Section III.B). Also, the notation introduced in the connectivity table above is used throughout the paper: index \( p \) will specify a thread linking \( i \)-th and \( j \)-th nodes, the direction of the \( p \)-th link being defined from the \( i \)-th to the \( j \)-th node. Additionally, the number of nodes on a side of the net will be indicated by \( N_s \) (\( N_s = 6 \) in the example), the phrase corner mass will refer to nodes 37, 38, 39, 40 in Fig. 1 while the phrase corner thread will refer to threads 61, 62, 63, 64 in Fig. 1[23].

In the automated creation of the connectivity table, a link is added and numbered with sequential \( p \) when, for all \( i = 1, ..., N_s^2 \) and for all \( j = i + 1, ..., N_s^2 \), the following conditions are verified:

\[
(j = i + 1 \land i \mod N_s \neq 0) \lor (j = i + N_s)
\]

Once all the inner threads are defined the four corner threads are appended to the table.

**B. Lumped-Parameter Model for the Net and Dynamics Equations**

One of the ways to model the net and account for its flexibility and elasticity is to take a lumped-parameter approach, which offers a good compromise between accuracy and computational efficiency. Much work on the application of this modeling technique to flexible nets was carried out for fishing nets [31, 32]; although the loads acting on an object immersed in water are obviously different from those acting on a body in space, the models developed for inertia and stiffness properties of nets remain applicable in either case [23]. As noted in Section II, most of the existing works on

<table>
<thead>
<tr>
<th>Link ( p )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>...</th>
<th>60</th>
<th>61</th>
<th>62</th>
<th>63</th>
<th>64</th>
</tr>
</thead>
<tbody>
<tr>
<td>Node ( i )</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>...</td>
<td>35</td>
<td>1</td>
<td>6</td>
<td>31</td>
<td>36</td>
</tr>
<tr>
<td>Node ( j )</td>
<td>2</td>
<td>7</td>
<td>3</td>
<td>8</td>
<td>4</td>
<td>...</td>
<td>36</td>
<td>37</td>
<td>38</td>
<td>39</td>
<td>40</td>
</tr>
</tbody>
</table>

Table 1: Part of the connectivity table relative to the net in Fig. 1.
the simulation of nets for ADR consider only axial dynamics. In a previous work, we included representation of the bending stiffness of the threads and found that, although this has a discernible effect on the deployment dynamics, the effectiveness of capture remains similar whether the bending stiffness of the threads is represented or not [24]. For this reason, bending stiffness of the net threads is not represented in this work.

According to the lumped-parameter method, the mass of the net is lumped at its physical knots (nodes), while the threads are represented by massless springs and dampers in parallel. The mass lumped in the \( i \)-th node is computed as:

\[
m_i = \begin{cases} 
\frac{\sum_{p \in P_i} m_p}{2} & \text{if } i \leq N_s^2 \\
\frac{\sum_{p \in P_i} m_p}{2} + m_{CM} & \text{otherwise}
\end{cases}
\]

where \( m_p \) is the mass of each of the adjacent threads (said to belong to the set \( P_i \)) and \( m_{CM} \) is the mass of a corner mass. As mentioned before, the tension in a thread is thought to be generated by a spring and a dashpot in parallel. Moreover, the nature of the material of the net (unable to withstand compression) suggests that a force within a thread should exist only when that thread is elongated and in tension. Mathematically, the tension in the \( p \)-th thread can be formulated as:

\[
T_p = \begin{cases} 
T_p \hat{e}_p & \text{if } (l_p > l_{p,0}) \land (T_p > 0) \\
0 & \text{if } (l_p \leq l_{p,0}) \lor (T_p \leq 0)
\end{cases}
\]

with \( T_p = k_p(l_p - l_{p,0}) + \beta k_p v_{r,p} \). In this expression, \( l_p \) and \( l_{p,0} \) are the length of the thread at the current time and without extension, respectively, \( v_{r,p} \) is the projection of the relative velocity of the end nodes of the thread in its axial direction, and \( \hat{e}_p \) the unit vector defining this direction (from the \( i \)-th to the \( j \)-th node in the connectivity table). Also, \( k_p = (EA)_p/l_{p,0} \) represents the axial stiffness of the thread, with \( E \) the Young’s modulus of the net material and \( A \) the cross section of the thread; \( \beta \) is a damping coefficient dependent on the material and geometry of the net: it is proportional to the desired damping ratio and to the first natural frequency of the system.

In an inertial reference frame that we assume coincident with the local orbital reference frame, the equations of motion of the system are obtained by writing Newton’s second law for each of the \( N_s^2 + 4 \) lumped masses, subject to external forces and tensions of the threads. For the \( i \)-th lumped
mass, we obtain:

\[ m_i a_i = \sum_{p \in P_i} (\pm T_p) + \sum_{s=1}^{S_i} F_{ext,s} \]  \hspace{1cm} (4)

where \( a_i \) is the absolute acceleration of the \( i \)-th node, \( T_p \) is each of the tension forces in the adjacent threads (belonging to the set \( P_i \)), and \( F_{ext,s} \) is each of the \( S_i \) external forces on the \( i \)-th mass. External forces might include gravity, when present, applied forces (for example at the time of ejection of the net, or as produced by a closing mechanism), and contact forces. In particular, for the net coming into contact with a debris, external forces will be contact forces to be defined in Section III C. In general, node \( i \) can impact the debris and other nodes at the same instant of time; in that case, the total contact force on that node is found by summing the different contributions.

In Eq. (4), it should also be noted that the tension in the \( p \)-th thread is defined as acting in the direction of the \( p \)-th thread \( \hat{e}_p \), i.e., from the \( i \)-th to the \( j \)-th node in the connectivity table as per Eq. (3), so it is taken with a positive sign in Eq. (4) for node \( i \); the corresponding tension on the \( j \)-th mass will be taken with a negative sign in the \( j \)-th equation [23].

For simulation purposes, the set of \( N^2 + 4 \) vectorial ordinary differential equations (ODEs) (4), is transformed into state-space form and solved numerically for the positions and velocities of the nodes. The simulator at hand has been implemented in Matlab, and the system of equations (4) is solved by exploiting the software’s built-in ODE integration capabilities, with Runge-Kutta methods.

C. Contact Dynamics Modeling

Multiple contact events are expected to occur during the capture phase of a tether-net ADR mission, both among different parts of the net and between the net and the target debris. As a result of this, the representation of the effect of contacts between the bodies in the system under consideration is key to the fidelity of the simulation to reality [23].

Many approaches are available for modeling contact dynamics: for a comprehensive literature review the reader may refer to the work by Gilardi and Sharf [33]. Continuous models, as opposed to impulsive models, consider the contact phase to have finite duration and are well-suited to describe the evolution of multibody systems, as the one under examination here. Among continuous
models, two approaches can be further distinguished: complementarity formulations and compliant formulations. In the first type of models, the contact forces are calculated by imposing contact conditions and solving a linear complementarity problem or a quadratic programming problem [34]. On the other hand, in compliant models the contact constraints are relaxed and a certain penetration is allowed: the contact forces are explicitly computed as functions of local deformations and their rates. It is foreseen that the contact scenario will be particularly complex for the problem under study. Since it is interesting to know how the system evolves during contact, and considering that the solution of a complementarity problem is NP-hard, a continuous compliant model was chosen for the implementation of contact dynamics for net-based debris capture [23].

Core issues in the simulation of debris capture with a net are collision detection and the modeling of the contact response, which are addressed in the following. A reasonable scenario to be simulated in Matlab is the capture of a cylindrical debris approximating the shape of launcher upper stages, which represent the majority of high priority targets for ADR missions [35]. The solution of such a complex contact problem as the capture of an object with a net is challenging from a computational point of view, because of contact detection issues, geometry considerations, and the inherent non-linearities. The chosen geometry of the debris allows to simplify the contact detection phase.

In a previous work we had studied the impact of a net on a ground, without considering self-collisions of the net, nor friction [23]. The dynamics of contact of the net nodes with individual faces of the cylinder is very similar to that case; nonetheless, the definition of penetration and rate of penetration, as well as the direction of the normal contact force, depend on the cylinder face involved in the contact. For the sake of fidelity to the physics of the problem, self-collisions among the net nodes as well as friction forces are taken into account in this work. Sections III C 1 address in detail the issues of collision detection and contact geometry for the system under consideration; Sections III C 2 and III C 3 provide discussion of different normal and friction force models respectively, and explanation of their implementation in the simulation tool at hand.
Collision Detection and Contact Geometry

Collision detection is performed at every time step of the simulation by checking if any of the net nodes and of the corner masses intersect the cylinder or any other net node or corner mass; if this happens, a contact force is applied to the bodies involved in the contact. The debris is assumed to be infinitely massive; this is justified by the ratio between the mass of a realistic debris (thousands of kilograms) and of a realistic net system (on the order of kilograms). As a result, forces and moments transmitted to the debris upon impact of the net have negligible effect on its motion.

For contact geometry purposes only, each node of the net and corner mass is modeled as a sphere of radius $R_i$ proportional to its mass, and dependent on its material density. The cylindrical debris is defined by specifying its radius $R_{cyl}$, its height $h_{cyl}$, and the position of its center in the inertial reference frame $(x_{cyl}, y_{cyl}, z_{cyl})$; it is oriented so as to have its axis parallel to the inertial $z$-axis. The distance of the $i$-th mass, with position $(x_i, y_i, z_i)$ in the inertial reference frame, from the axis of the cylinder is computed at every time step as:

$$d_i = \sqrt{(x_i - x_{cyl})^2 + (y_i - y_{cyl})^2}$$  \hspace{1cm} (5)

Each of the three surfaces of the cylinder is assigned an index: 1 indicates the top circular area, 2 the lateral surface, and 3 the bottom circular area. Conditions for contact of a node with each of the faces, as well as the direction normal to the plane of contact $\hat{u}_{n,i}$, are collected in Table 2. In Fig. 2 are reported the definitions of penetration and rate of penetration in the impacts between a node of the net and a face of the cylinder. The normal direction $\hat{u}_{n,i,2}$ in case of contact with face 2 is defined as shown in Fig. 2(b) and per the following equation:

$$\hat{u}_{n,i,2} = [(x_i - x_{cyl})/d_i, (y_i - y_{cyl})/d_i, 0]^T$$  \hspace{1cm} (6)

In Table 2 the second condition for contact determines whether penetration $\delta_i$ between a node or corner mass and a face of the cylinder occurs, i.e., whether $\delta_i > 0$. The penetration $\delta_i$ is therefore defined for each face as expressed in that condition; for example, for face 1, $\delta_i = R_i - (z_i - z_{cyl} - h_{cyl}/2)$. The rate of penetration is given by the derivative of $\delta_i$, or by $\dot{\delta}_i = -v_i \cdot \hat{u}_{n,i}$ with $v_i$ the velocity of the $i$-th mass in the inertial reference frame; for face 1, it is $\dot{\delta}_i = -v_i \cdot \hat{k} = -v_{z,i}$.

Due to the different contact geometry, the definition of penetration and rate of penetration in
Figure 2: Collision detection and definition of penetration and rate of penetration for impact between node $i$ and the cylinder. (a): face 1 and face 3. (b): face 2.

| Face # | Conditions for contact                                                                 | Normal direction | Table 2: Conditions for contact and definition of normal direction for each face of the cylinder.
<table>
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<tbody>
<tr>
<td>1</td>
<td>$(d_i &lt; R_{cyl}) \land (z_i \leq z_{cyl} + h_{cyl}/2 + R_i)$</td>
<td>$\hat{k}$</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$(z_{cyl} - h_{cyl}/2 &lt; z_i &lt; z_{cyl} + h_{cyl}/2) \land (d_i \leq R_{cyl} + R_i)$</td>
<td>$\hat{u}_{n,i,2}$</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>$(d_i &lt; R_{cyl}) \land (z_i \geq z_{cyl} - h_{cyl}/2 - R_i)$</td>
<td>$-\hat{k}$</td>
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</table>

The two spheres are said to intersect when the distance between their centers of mass becomes smaller than the sum of their radii: $||r_{2,i} - r_{1,i}|| < R_{2,i} + R_{1,i}$. If this happens, the two spheres are
Figure 3: Collision detection and definition of penetration and rate of penetration for impact between two nodes in contact pair $i$.

colliding with velocities $v_{1,i}$ and $v_{2,i}$, and the penetration and rate of penetration are found as:

$$\delta_i = R_{1,i} + R_{2,i} - ||r_{2,i} - r_{1,i}||$$  \hfill (8)

$$\dot{\delta}_i = -(v_{2,i} - v_{1,i}) \cdot \hat{u}_{n,i}$$  \hfill (9)

Accurate collision detection is critical to the accuracy of the simulation of contact, for example to avoid unrealistically high contact forces due to artificially large penetrations. In order to accurately detect collisions in the numerical simulation, a $1 \times 10^{-6}$ tolerance was selected for the employed Runge-Kutta integration scheme. It is anticipated that the need for such accuracy in the detection of collisions will have an important impact on the efficiency of the simulation.

2. Normal Contact Force

The normal contact force, by definition, acts in the direction normal to the plane of contact: $F_{n,i} = F_{n,i} \hat{u}_{n,i}$. Note that, due to the definition given for $\hat{u}_{n,i}$ in contact pair $i$ between two spheres, the force vectors applied to the two masses are $F_{n,2,i} = F_{n,i} \hat{u}_{n,i}$ and $F_{n,1,i} = -F_{n,i} \hat{u}_{n,i}$. In Section III.C.1 the direction normal to the contact plane was defined for each type of impact, together with the penetration and rate of penetration between the two impacting bodies. The magnitude of the normal contact force remains to be determined and is the topic of the current section.

As discussed before, a continuous compliant approach for modeling the contact dynamics was selected due to the complexity of the problem. Over the years, many continuous compliant models
Figure 4: Continuous compliant normal contact force model for contact pair $i$. (a): sphere-plane. (b): sphere-sphere.

have been proposed for the contact force generated in the direction normal to the plane of contact; interesting recent comparative studies can be found in references [28] and [29]. Typically, a spring simulates the elastic behavior, while a damper dissipates part of the energy during contact. The linear model has been widely used because of its simplicity: the normal contact force is thought to arise from a linear spring and a linear damper placed in parallel between the two contacting bodies (see the conceptual representation in Fig. 4). If, for the $i$-th contact, $k_{c,i}$ is the contact stiffness coefficient and $c_i$ is the contact damping coefficient (these coefficients being dependent on material and geometrical properties), the normal contact force can be simply computed as:

$$F_{n,i} = k_{c,i} \delta_i + c_i \dot{\delta}_i$$  \hspace{1cm} (10)$$

A better representation of the normal contact force is the non-linear spring-damper model introduced by Hunt and Crossley [36], which overcomes the problems of the linear model, namely the discontinuity of the force at the beginning of impact and a non-physical 'sticking' force keeping the bodies together at the end of impact [33]. In this formulation, if $n$ is a constant dependent on the contact scenario, $\bar{k}_{c,i}$ the non-linear stiffness coefficient, and $\lambda_i$ the non-linear damping coefficient, then the normal force arising at the $i$-th contact is found as:

$$F_{n,i} = \bar{k}_{c,i} \delta_i^n + \lambda_i \delta_i \dot{\delta}_i$$ \hspace{1cm} (11)$$

For this work, both the linear model (10) and a family of non-linear models (11) with different expressions for $\lambda_i$ have been implemented. In order to define the stiffness and damping coefficients
of the models, a few reasonable assumptions and considerations are made. First of all, we assume all contacts to be elastic. Second, and for contact geometry purposes only, the nodes of the net are approximated to spheres of radii proportional to their mass, as already mentioned. Moreover, the debris is expected to be much bigger than each node of the net, whence we can study the contact between net and debris as the result of multiple contacts among a sphere and a plane. As a consequence, apart from some special cases (e.g., a node impacting an edge of the debris), impact happens between two continuous and non-conforming surfaces, which make first contact at a point and for which the resulting stresses are highly concentrated. For computing contact stiffnesses, each contacting body can also be approximated to an elastic half-space loaded over a small elliptical region of its surface: in fact, the dimension of the contact area is expected to be much smaller than the characteristic dimensions of the bodies and than the radii of curvature of their surfaces at the impact point [23]. Within these assumptions, Hertzian contact theory is valid and it is possible to use the well-known results summarized below; the reader is referred to Johnson’s work for a detailed treatment of Hertzian theory [37].

For the reasons discussed above, the contact between the net and a debris is here studied as the result of the contact between the nodes of the net and a plane (see Fig. 4(a)), ignoring singular cases such as contacts between a node of the net and an edge of the debris. According to Hertzian contact theory for this scenario, \( n \) appearing in Eq. (11) is 1.5 and the contact stiffness \( \bar{k}_{c,i} \) relative to the impact of the \( i \)-th node with the debris can be computed with:

\[
\bar{k}_{c,i} = \frac{4}{3} \sqrt{R_i E_i^*} \\
E_i^* = \frac{E_{1,i}}{1 - \nu_{1,i}^2} + \frac{E_{2,i}}{1 - \nu_{2,i}^2}
\]

In these expressions, \( E_{j,i} \) and \( \nu_{j,i} \) for \( j = 1, 2 \) are the Young’s modulus and the Poisson’s ratio of the material of each body impacting at contact \( i \). For collisions between two nodes of radii \( R_{1,i} \) and \( R_{2,i} \) forming the \( i \)-th contact pair (see Fig. 4(b)), the contact stiffness is found as:

\[
\bar{k}_{c,i} = \frac{4}{3} \sqrt{R_{eq,i} E_i^*} \\
R_{eq,i} = \frac{R_{1,i} R_{2,i}}{R_{1,i} + R_{2,i}}
\]

The contact stiffness employed in the linear normal contact force model is estimated as \( k_{c,i} = \)
\[ \bar{k}_{c,i}\left(2\delta_{\text{max}}/3\right)^{n-1}, \] with \( \bar{k}_{c,i} \) from Eq. (12) or (14), once the order of magnitude of the penetration occurring between the bodies \( \delta_{\text{max}} \) is assessed. This estimation is justified by the fact that, if damping is temporarily neglected, \( F_{n,i} = k_{c,i}\delta_i \) should approximate linearly \( F_{n,i} = \bar{k}_{c,i}\delta_i^n \) while passing through zero.

The damping coefficient in the linear model is computed as \( c_i = 2\xi m_i k_{c,i} \) with \( \xi \) as the chosen damping ratio. On the other hand, the damping coefficient in the non-linear models \( \lambda_i \) has attracted the attention of many researchers, who have proposed different expressions, among which are those reported in Table 3. In these models there appears \( v_{0,i} \), the relative velocity in the normal direction at the beginning of the contact event, and \( \alpha_c \), a coefficient that relates the coefficient of restitution \( e \) to the initial impact velocity as \( e = 1 - \alpha_c v_0 \). For this work, we have chosen to implement the most traditional non-linear models as well as Zhang and Sharf’s model, all of which were thoroughly discussed in reference [38]. Alves et al. recommend employing Gonthier et al.’s model [39], or the equivalent model by Zhang and Sharf’s, because of its ability to model the dissipation of energy correctly for any value of the coefficient of restitution, including more plastic impacts [28]. Banerjee et al. suggest using two other models, one of which is linear, and the other one is piecewise non-linear [29]. Because of the non-physical force issues related to linear models and of the additional discontinuities that would be introduced by piecewise functions, we decided not to consider these two models in the current study. We just included the traditional Kelvin-Voigt model — which was used in previous works on the simulation of net-based capture of debris, such as [27] — in order to highlight its problems. It is also worth mentioning that the non-linear models, relying on Hertz theory, can make use of the previously described relationships to evaluate the contact stiffness and damping coefficients; this makes them preferable with respect to the linear models. A drawback is the introduction of additional parameters, that are not always known, such as \( \alpha_c \).

3. Friction Force

Strictly speaking, Hertzian theory is valid only for frictionless contact. However, it is demonstrated that normal compression and tangential displacement can be treated separately [37]. In particular, tangential traction does not affect normal motion if the two materials are similar; in any
<table>
<thead>
<tr>
<th>Authors</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hunt-Crossley [36]</td>
<td>$\lambda_{HC,i} = \frac{3}{2} \alpha_c k_{c,i}$</td>
</tr>
<tr>
<td>Herbert-McWhannel [10]</td>
<td>$\lambda_{HM,i} = \frac{3}{2} \alpha_c k_{c,i} \left(1 - \frac{1}{1 - \alpha_c v_{0,i} + \alpha_c v_{0,1}}\right)$</td>
</tr>
<tr>
<td>Lee-Wang [11]</td>
<td>$\lambda_{LM,i} = \frac{3}{4} \alpha_c k_{c,i}$</td>
</tr>
<tr>
<td>Lankarani-Nikravesh [12]</td>
<td>$\lambda_{LN,i} = \frac{3}{2} \alpha_c k_{c,i} \left(1 - \frac{\alpha_c v_{0,i}}{2}\right)$</td>
</tr>
<tr>
<td>Zhang-Sharf [38]</td>
<td>$\lambda_{LS,i} = k_{c,i} \ln \left[\frac{\lambda_i v_{0,i} + k_{c,i}}{-\lambda_i(1 - \alpha_c v_{0,i})v_{0,i} + k_{c,i}}\right] - 2\lambda_i v_{0,i} + \alpha_c \lambda_i v_{0,i} = 0$</td>
</tr>
</tbody>
</table>

Table 3: Different expressions for damping coefficient in non-linear models.

...case the effect is small and can reasonably be neglected (about 5% variation on total normal load) [37]. Therefore the normal force can always be found with Hertz theory.

Since the nodes are modelled as point masses, friction force is expected to have a significant effect on motion just in case of continued contact. Since this was found to happen only between the net and the debris, and for the sake of computational efficiency, the contact among nodes of the net and corner masses is assumed to be frictionless. For contacts between the net nodes and the debris, friction force is modelled; in this case, contact is complicated by different possible conditions: slipping and sticking. It is well-known that, in the slipping regime (i.e., when there exists relative velocity between the contacting bodies), the friction force is proportional to the normal contact force: $F_t = \mu_k F_n$, with $\mu_k$ the kinetic friction coefficient. Modeling of friction force in the sticking regime is more uncertain and has been the topic of multiple studies. In the traditional Coulomb’s model, the value of friction force in sticking is not uniquely defined: $F_t \leq \mu_s F_n$, with $\mu_s$ the static friction coefficient; multiple efforts to uniquely define this force, as well as to smoothen the transition from sticking to sliding friction have been made, for example in references [37, 43-48]. Among the various friction models available in the literature, our efforts focused on Johnson’s model and bristle model because of their capability to explicitly and uniquely define the friction force both in sticking and slipping conditions.

Johnson studied the sticking regime as incipient sliding: while there is no sliding motion between the two contacting bodies as a whole, the friction force causes a small relative motion over a portion of the interface (this is called micro-slip) [37]. For two spheres in sticking contact, Johnson found
that the relative tangential displacement $\delta_{t,i}$ can be related to the friction force with:

$$\delta_{t,i} = \frac{F_{t,i}}{8a_i} \left( \frac{2 - \nu_{1,i}}{G_{1,i}} + \frac{2 - \nu_{2,i}}{G_{2,i}} \right)$$

(16)

$$a_i = \left( \frac{3F_{n,i}R_{eq,i}}{4E_i^*} \right)^{1/3}$$

(17)

where $G_{j,i}$ for $j = 1, 2$ is the shear modulus of the materials of the contacting bodies. Using the expression for the normal contact force by Hertz $F_{n,i} = \tilde{k}_{c,i}\delta_i^n$ with $n = 1.5$, and extending the expression to 3D, the force of friction can be written as:

$$F_{t,i} = k_{t,i}\delta_{t,i}$$

(18)

$$k_{t,i} = 8\sqrt{R_{eq,i}}G_i^*$$

(19)

$$G_i^* = \frac{G_{1,i}}{2 - \nu_{1,i}} + \frac{G_{2,i}}{2 - \nu_{2,i}}$$

(20)

In these equations, $\delta_{t,i}$ is the vector of relative tangential displacement and $k_i$ is the tangential stiffness. Observe that the tangential stiffness varies during a contact, depending on the instantaneous penetration.

The bristle model imitates the presence of irregularities of the surfaces using bristles. In the original undamped model by Haessig and Friedland, the friction force is defined in 1D as [43]: $F_t = k_b s$ where $k_b$ is the equivalent bristle stiffness and $s$ is the bristle deflection. This idea was extended to 3D by Ma et al. [47], and a damped version of the 3D model was proposed by Liang et al. in order to stabilize the oscillations during the transition between sliding and sticking regimes [44]. The undamped 3D model reads [44]:

$$F_t(t) = -k_b s(t)$$

(21)

$$s(t) = \begin{cases} 
    s(t_0) + \int_{t_0}^{t} v_t(t) dt & \text{if } ||s(t)|| < s_{max}(t) \\
    s_{max}(t) & \text{if } ||s(t)|| \geq s_{max}(t) 
\end{cases}$$

(22)

$$s_{max}(t) = \frac{\mu F_{n}(t)}{k_b}$$

(23)

$$\mu = \begin{cases} 
    \mu_k & \text{if } v_t(t) > v_d \\
    \mu_s & \text{if } v_t(t) \leq v_d 
\end{cases}$$

(24)
In Eq. \(t_0\) is the starting time of contact, \(v_t\) is the vector of relative tangential velocity between the two contact bodies at the contact point, \(\hat{v}_t\) its direction and \(v_t\) its magnitude. The maximum bristle deflection \(s_{\text{max}}\) is defined as per Eq. \(23\) and depends on the sticking or slipping regime as indicated by Eq. \(24\); the boundary between sticking and slipping conditions is dictated by a threshold magnitude of relative velocity \(v_d\).

The damped bristle friction model reads \([44]\):

\[
F_t(t) = -k_b s(t) - c_b \dot{s}(t) \tag{25}
\]

with \(c_b\) the bristle damping coefficient, and \(\dot{s}(t)\) the derivative of the bristle deflection at time \(t\). The related algorithm is not thoroughly explained in \([44]\), but our work with this model has revealed that the inclusion of the damping term undermines the physical consistency of the model, if Eqs. \(22\) and \(23\) are not revised. In particular, any micro-slip friction model is expected to be consistent with Coulomb’s model of friction, that is \(||F_t(t)|| \leq \mu F_n(t)\). In the context of bristle model \(21\), this is achieved through the appropriate definition of \(s_{\text{max}}\). However, for the damped case \(25\), the conditions for switching between sticking and slipping (i.e., Eqs. \(22\)-\(24\)) must be modified to ensure \(||-k_b s(t) - c_b \dot{s}(t)|| \leq \mu F_n(t)\). This is achieved if we observe that \(\dot{s} = v_t\) if \(||s|| < s_{\text{max}}\) and we declare that \(\dot{s} = 0\) when \(||s|| = s_{\text{max}}\). In addition, as far as recovering the direction of the friction force in sliding, it is necessary to impose that the bristle deflection vector \(s\) is in the direction of \(v_t\).

The modified definitions for the damped bristle friction model of Eq. \(25\) are proposed as:

\[
s(t) = \begin{cases} 
  s(t_0) + \int_{t_0}^{t} v_t(t) dt & \text{if } ||F(t)|| \leq \mu F_n(t) \\
  s_{\text{max}}(t) \hat{v}_t(t) & \text{if } ||F(t)|| > \mu F_n(t)
\end{cases} \tag{26}
\]

\[
\dot{s}(t) = \begin{cases} 
  v_t(t) & \text{if } ||F(t)|| \leq \mu F_n(t) \\
  0 & \text{if } ||F(t)|| > \mu F_n(t)
\end{cases} \tag{27}
\]

where \(s_{\text{max}}\) is defined with Eq. \(23\) and \(\mu\) is selected with Eq. \(24\). For this model, initial conditions for both \(s(t_0)\) and \(\dot{s}(t_0)\) must be consistently set. In particular, initial conditions are computed as: \(s_0 = (\mu k F_n/k_b)\hat{v}_t\) and \(\dot{s}_0 = 0\) if the contact point is sliding; \(s_0 = 0\) and \(\dot{s}_0 = v_t\) if the contact point is sticking. For the problem at hand, \(v_{t,i}\) for each contact pair is found at every time step by projecting the relative velocity vector \(v_{r,i}\) on the tangent plane: \(v_{t,i} = (1 - \hat{u}_{n,i} \hat{u}_{n,i}^T)v_{r,i}\). 1 being a
3-by-3 identity matrix. If contact begins for pair \( i \), initial conditions are computed; otherwise, the bristle deflection is found from its value at the previous step - if contact is sticking - or from its maximum - if slipping.

Our implementations of the Johnson’s model and of the bristle model were validated by simulating the dynamics of a point mass dragged on the ground with a linearly increasing force. Upon comparison of the results and performance of the two models, it was noticed the damped bristle friction model has a few advantages over Johnson’s friction model: fewer transitions between sticking and slipping regimes, a more continuous friction force magnitude, and - consequently - a much lower computational time. For these reasons, the damped bristle friction model was chosen for simulation of the contact between the net and the debris system studied in this paper.

Nonetheless, Johnson’s model proved helpful in selecting the value for the bristle stiffness coefficient \( k_{b,i} \) by making direct correspondence between Eqs. (18) and (21). In particular, the constant \( k_{b,i} \) was estimated by fitting the curve \( k_{t,i} = k_{t,i}(\delta_i) \) for one contact pair with a polynomial of degree zero, from \( \delta_i = 0 \) to the maximum occurring \( \delta_i \). The damping coefficient was then estimated as \( c_{b,i} = 2 \xi m_i k_{b,i} \), with \( m_i \) the mass of a net node. In the simulation results presented in this manuscript, these coefficients are assumed to be independent of the contacting node ID.

IV. Deployment and Capture Simulation Results

With the models presented in Section III, numerical simulations have been performed for the net dynamics in various gravity conditions, subject to diverse initial conditions, and for differently scaled net geometries [23]. The simulation tool proved able to deal with disparate dynamic scenarios, among which are free-fall, rotation of the net, fall with forces applied to corner nodes, deployment, simple impact conditions, and capture of debris. In this section we present the results of two particularly interesting simulations: the deployment of a net in space, and the capture of a still debris of cylindrical shape.

Table 4 presents the dimensions and mass properties of the net used in the simulations, where \( L \) is the net side length, whereas \( l_0 \) and \( l_{CT,0} \) are the lengths of the net mesh and of the corner threads; the radii of the net threads and of the corner threads are indicated with \( R_{net} \) and \( R_{CT} \); the
<table>
<thead>
<tr>
<th>Parameter</th>
<th>$L$</th>
<th>$l_0$</th>
<th>$l_{CT,0}$</th>
<th>$m_{CM}$</th>
<th>$R_{net}$</th>
<th>$R_{CT}$</th>
<th>$\xi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit</td>
<td>(m)</td>
<td>(m)</td>
<td>(m)</td>
<td>(kg)</td>
<td>(m)</td>
<td>(m)</td>
<td>(-)</td>
</tr>
<tr>
<td>Value</td>
<td>5</td>
<td>1</td>
<td>1.4142</td>
<td>0.5</td>
<td>0.001</td>
<td>0.002</td>
<td>0.106</td>
</tr>
</tbody>
</table>

Table 4: Geometry, mass and material properties of the simulated nets.

<table>
<thead>
<tr>
<th>Material</th>
<th>Property</th>
<th>Value 1</th>
<th>Value 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kevlar (Net)</td>
<td>$\rho$ (kg/m$^3$)</td>
<td>1440</td>
<td>1440</td>
</tr>
<tr>
<td></td>
<td>$E$ (GPa)</td>
<td>70</td>
<td>69</td>
</tr>
<tr>
<td>Aluminum (Corner masses, debris)</td>
<td>$G$ (GPa)</td>
<td>2.9</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>$\nu$ (-)</td>
<td>0.36</td>
<td>0.33</td>
</tr>
</tbody>
</table>

Table 5: Material properties used in simulations.

employed damping ratio is $\xi$. For coherent simulations, it is necessary to specify a plausible material for the net, the corner masses, and the debris: we chose to use Kevlar for the net and Aluminum for the corner masses and the debris. The applicable material properties are collected in Table 5. For contact dynamics simulation purposes, further coefficients need to be defined and are indicated in Table 6. For the static friction coefficients $\mu_s$, Kevlar-Kevlar pairs were assumed comparable to Nylon-Nylon pairs and Aluminum-Kevlar pairs were assumed comparable to Aluminum-Teflon pairs; the kinetic friction coefficients were estimated by dividing $\mu_s$ by 1.25. The threshold magnitude of the tangential relative velocity for sticking-slipping transition was set to $v_d = 0.001$ m/s. Also, the contact stiffness of the net nodes was divided by 10 in order to balance the fact that the mass is lumped in the nodes.

A. Deployment Simulation Results

The results of a deployment simulation in 0g environment are discussed here. In the simulated scenario, the net starts in a partially closed configuration on a plane parallel to the $X$-$Y$ plane: the initial distance between two connected nodes is set to be half of the unstretched length of the threads. Although this choice of initial conditions does not represent a realistic folding pattern,
Table 6: Contact properties used in simulations.

<table>
<thead>
<tr>
<th></th>
<th>Al - Al</th>
<th>Al - Kevlar</th>
<th>Kevlar - Kevlar</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_c$ (°)</td>
<td>0.25</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>$\mu_s$ (-)</td>
<td>0.42</td>
<td>0.19</td>
<td>0.2</td>
</tr>
<tr>
<td>$k_b$ (N/m)</td>
<td>$1.05 \times 10^8$</td>
<td>$1.05 \times 10^8$</td>
<td>$1.05 \times 10^8$</td>
</tr>
<tr>
<td>$c_b$ (N/ms)</td>
<td>206</td>
<td>206</td>
<td>206</td>
</tr>
</tbody>
</table>

Figure 5: Deployment sequence.

It allows to have some deployment dynamics so that the debris can be captured. Deployment is achieved by imparting initial velocities to the four corner masses so as to give symmetric initial conditions to the net:

$$\begin{align*}
  \mathbf{v}_{N^2+1,0} &= [-1 \quad -1 \quad -2]^T \text{ m/s} \\
  \mathbf{v}_{N^2+2,0} &= [1 \quad 1 \quad -2]^T \text{ m/s} \\
  \mathbf{v}_{N^2+3,0} &= [-1 \quad 1 \quad -2]^T \text{ m/s} \\
  \mathbf{v}_{N^2+4,0} &= [1 \quad 1 \quad -2]^T \text{ m/s}
\end{align*}$$

The $6(N^2 + 4)$ first order ODEs obtained by converting Eq. (4) into state-space form were integrated for 2.5 s, starting from the abovementioned initial conditions, which took 44 s to integrate with an Intel® Core™ i7-4712HQ CPU @ 2.30 GHz processor. Several snapshots of the net during...
its deployment are included in Fig. 3 starting with partially deployed initial configuration at \( t = 0.0 \text{ s} \) to the fully deployed configuration at \( t = 2.5 \text{ s} \). As time passes, the outer threads start elongating (see \( t = 1.0 \text{ s} \)), followed by innermost threads (see \( t = 2.0 \text{ s} \)). The net configuration at all times remains symmetric: this is expected, since the initial conditions, the applied loads, as well as the geometry and physical properties of the net are symmetric. The symmetry check in multiple dynamics simulations, together with the work-energy balance and the conservation of linear momentum checks, served as validation of the simulator.

In Fig. 6 the \( x \)- and \( z \)-coordinates of a subset of nodes are shown, to confirm that the symmetry among some groups of nodes is indeed preserved, other than for the accumulation of numerical error towards the end of the simulation period. The symmetry of the considered groups of nodes, whose indexes are in the legend, can be verified in Fig. 1. The \( y \)-coordinates of nodes are not reported in this document for the sake of conciseness, but exhibit a behavior analogous to that of \( x \)-coordinates.

Fig. 7 displays the magnitude of the force due to tension within the threads of the net on nodes 37, 1, 8 and 15, which lie on a diagonal of the net, as a function of time. The first characteristic emerging from the analysis of this graph is that the net is subject to snap loads: the threads are stretched and subject to fairly high loads for small periods of time only. The abrupt changes in tension is one of the main dynamic characteristics that make it computationally expensive to simulate the deployment of tether-net devices in space, especially due to the fact that the computational
cost increases rapidly with the increase in the number of nodes. From this figure it is also evident that tension is propagated from outer to inner nodes of the net. Node 37 is a corner mass, which is subject to non-zero initial velocity in the simulation. As node 37 moves, the thread that links it to the rest of the net straightens and eventually stretches, creating a tension force as per Eq. (3): as a consequence of this, a peak in tension can be observed at time 0.36 s on nodes 37 and 1. As time passes, tension propagates in the net: a second peak is visible for node 1, and corresponds to the onset of tension in the threads number 1 and 2. The delay between the peaks can be explained by the time it takes for the next innermost thread to straighten and reach its nominal length. Also, it can be observed that nodes 8 and 15 are subject to a force arising from tension at sequentially later instants [23].

B. Capture Simulation Results

In this section are presented the results of net-based capture of a stationary cylindrical debris ($R_{cyl} = 1 \text{ m}, h_{cyl} = 2 \text{ m}$), centered with respect to the net, and with the net released from a distance of 2 m in the $z$-direction. As was discussed in Section III.C2 multiple models for the normal force
of contact are implemented in our Matlab simulator. Simulation of debris capture with the net described in Table 5 was performed with each of the models presented, by integrating the dynamics evolution for 8 seconds. This allowed us to evaluate the effect of employing different models on the overall capture dynamics.

The numerical simulation proved itself able to represent the contacts between the net and the debris, and among the nodes of the net. Fig. 8 shows some snapshots of the system during capture, comparing the results of simulation with the linear and Hunt and Crossley’s non-linear normal contact force models. The net is deployed with the same initial conditions as before (see $t = 0.0$ s), and contact with the cylinder is seen at $t = 2.0$ s. Due to contact with the cylinder, the net envelops it, and discrepancies in the positions of nodes and corner masses become appreciable between the two models (e.g., at $t = 4.0$ s). The corner masses impact each other at $t \approx 5$ s below the debris because of symmetry, and open again afterwards (e.g., at $t = 6.0$ s). In Fig. 9 are compared the responses with Hunt and Crossley’s model with and without friction. In this case, the discrepancies in the net configuration at corresponding times are less important, and become appreciable just after impact of the corner masses.

The times needed for integration of the five non-linear models with friction, of the linear model with friction, as well as Hunt and Crossley’s model without friction, are compared in Fig. 10(a); all simulations were performed on a laptop with Intel® Core™ i7-4712HQ CPU @ 2.30 GHz. It is noticed that simulation with Lankarani and Nikravesh non-linear model took the longest time to integrate ($\approx 55$ min), whereas simulation with Hunt and Crossley’s and Lee and Wang’s models took approximately half the time (between 24 and 26 min); as could be foreseen, a shorter integration time (of $\approx 11$ min) was sufficient to perform a simulation without friction. In order to evaluate the effect of employing the different contact models on the overall dynamics, three significant events in the capture maneuver were identified: first impact with the debris, closure defined as the instant when the four corner masses enter a circle of radius $R_{cyl}/2$, and subsequent re-opening of the net defined as the instant when the four corner masses exit a circle of radius $R_{cyl}$. These noteworthy instants of time are compared in Fig. 10(b). The time of first contact, as expected, is exactly the same ($t \approx 1.8$ s) in any simulation. Closure happens at $t \approx 4.7$ s in every simulation; the earliest
closure is obtained at $t = 4.72 \text{ s}$ with Lee and Wang’s model, the latest one is found at $t = 4.75 \text{ s}$ with the linear model. The simulated responses become more differentiated with the time, as more contacts occur. Thus, re-opening of the corner masses happens between $t = 6.0 \text{ s}$ with the model without friction and $t = 6.8 \text{ s}$ with Hunt and Crossley’s model. Overall, the effect of employing different normal contact force models or of neglecting friction is not fundamental to simulating the capture dynamics, especially until net closure. However, the choice of the contact force model
Figure 9: Capture sequence with Hunt and Crossley's model with friction (black net) and without friction (grey net).

does appear to influence substantially the computational cost. It was observed that the non-linear models that express the damping coefficient in terms of the coefficient of restitution sometimes incur in numerical difficulties; considering the differences in computational efficiency and similarities in the overall dynamics, the more simple non-linear models (such as Hunt and Crossley’s or Lee and Wang’s) are recommended for simulating this particular complex dynamic scenario. From the results of simulation with Hunt and Crossley’s model, it was also observed that the restitution coefficient
characterizing a new contact is lower than 0.7 in only approximately 4% of occurrences. Although Alves et al. recommend models that are able to recover the restitution coefficient correctly for the whole span of its values (i.e., from 0 to 1), they notice that all the proposed models provide similar damping factors when the restitution coefficient is higher than 0.7 [28]. This corroborates the finding that all the models employed in this work provide similar overall dynamics in this capture scenario. Moreover, from this analysis, it appeared that simulation of friction force does not have a significant effect on the response, but doubles the computational cost. Thus, a model without friction is deemed to be sufficient for describing the overall dynamics of capture in this particular scenario. However, it is expected that friction will have a more important effect in scenarios where the rotation of the debris is modelled.

Fig. 11 presents the contact force obtained with the linear model as well as with Hunt and Crossley’s and Zhang and Sharf’s non-linear models on two of the masses impacting the cylinder: node 15, that impacts the top surface, and node 9, that impacts the lateral surface. $F_n$ is the normal contact force, whereas $F_{t1}$ and $F_{t2}$ indicate two orthogonal components of the friction force. For node 15, $(F_{t1}, F_{t2}, F_n)$ are aligned with the inertial reference frame. For node 9, $F_n$ depends instantaneously on the point of contact, $F_{t1}$ is aligned with the inertial Z-axis, and $F_{t2}$ completes the orthogonal triad. Multiple peaks are observed; these correspond to multiple impact occurrences between the node and the cylinder, which are due both to the overall net dynamics and to the
elasticity of impacts. Comparing the results of simulations with the three models, it can be noticed that the trends of the contact forces are similar, especially until around \( t = 5 \) s; however, it is clear that the normal contact force with the linear model can become unrealistically negative (i.e., a force that tends to keep the bodies together), whereas this does not happen with the non-linear model. Quantitatively, these results confirm that the orders of magnitude of the contact forces are consistent among the three models. However, the linear model yields normal contact forces noticeably lower than the non-linear models at the beginning of contact (e.g., approximately 25% decrease with respect to Hunt and Crossley’s model at the first impact occurrence. From these figures, it is also evident that the forces of contact at first impact and during later sustained contact are characterized by different orders of magnitude: for node 15, the first impact force is \( F_n \approx 1200 \) N, whereas the contact forces experienced later are \( O(1) \) N.

At a first glance, from Fig. 11 it may seem that the contact forces are discontinuous, contrary to the nature of the continuous compliant model. A zoom on the first peak of the normal contact force experienced by node 15, in Fig. 12(a), reveals that it does vary continuously during the very short impact period, of 0.1 ms duration. Figure 12(a) reports the results obtained with all the implemented models: it shows that the first impact happens at exactly the same instant with all the models, and that little variation in the normal contact force exists among the non-linear models. On the other hand, it makes the differences between the linear model and the non-linear models even more visible. It can be appreciated how, at onset of contact, the linear model causes a contact force that becomes instantaneously different from zero (although at that instant the penetration in null): this is due to the linear damping term. Also, the non-physical sticking behavior as the objects are separating is clearly visible. In Fig. 12(b), the normal contact force at three subsequent impact events experienced by node 15 is depicted. This highlights that the dynamics of node 15 is influenced by the small differences observed during its first impact with the debris: for example, the second peak is higher in magnitude and occurs later in simulations characterized by higher normal contact forces during the first impact (see 12(a)).

The number of contacts experienced by the net during capture is represented in Fig. 13 for the simulation with Hunt and Crossley’s model. Since first impact with the top surface occurs, i.e., at
$t \approx 1.8 \text{ s}$, four contacts are observed. The four contacts are sustained for certain intervals, such as from 1.85 s to 1.95 s; this was also ascertained by verifying that $F_n$ of node 15 oscillates after the second peak and settles at $F_n \approx 2 \text{ N}$ until the third peak (see Fig. 12(b)). At around $t = 1.95 \text{ s}$, 20 contacts are observed, which consist of the 4 nodes impacting the top and 16 new impacts of nodes with the lateral surface. At the corresponding time, the first impact force on node 9 is found in Fig. 11(b). Additionally, from Fig. 13 it is noticed that contact is often sustained and that multiple simultaneous contacts occur in the later part of the simulation: often more than 8, and up to 20. Considering that the net and debris in this set of simulations are smaller than those envisaged for
actual ADR missions, and that as a result many more simultaneous contacts are expected to occur in the real scenario, the continuous compliant modeling approach for the normal contact force seems well-suited to the problem. This formulation is also able to solve both impact and sustained contact situations, both of which were observed during net-based capture of debris.

In Fig. 14 the friction force on node 15 and some related quantities are depicted for a short interval in order to validate the implemented friction model and highlight some of its features. The threshold velocity $v_d$ determines the velocity at which transitions between sticking and slipping
regimes occur; in the shown time span, five transitions are visible in the first subplot. At all transitions, a discontinuity in $\mu F_n$ is observed in the second plot, due to the change in the coefficient of friction; also, from this diagram it is verified that $F_t \leq \mu F_n$ always, and that $F_t = \mu F_n$ during slipping. Finally, the third diagram compares the direction of the relative velocity and of the friction force, measured with $\theta$ from the $X$-axis: as expected, the two vectors are always aligned during this time span. This verifies the recovery of Coulomb’s friction in slipping, demonstrates that the magnitude of the friction force is continuous, and confirms that transitions happen when they are expected to.

V. Conclusion

Simulation of net-based space debris capture entails multiple modeling issues and numerical difficulties. Numerical simulation of the deployment of a net in microgravity conditions revealed the snapping behavior of tension, which is likely to cause numerical problems as the size of the system increases. Moreover, contact dynamics is inherently non-linear and requires a modeling approach
that grants both a reasonable numerical efficiency and physical accuracy.

The study of a capture scenario involving a target of shape similar to a realistic debris has confirmed the ability of our simulator to identify contacts and their effect on the system. To the authors' knowledge, this work is the first effort to evaluate the effect of employing different contact models on the capture dynamics, and to use micro-slip friction modeling for this application.

The continuous compliant approach for the normal contact force is well-suited to this problem, because of the high number of simultaneous contacts and the fact that both impacts and sustained contacts occur. Non-linear models are better suited to represent the physics of the problem with respect to the linear model and, being based on Hertzian theory, allow to make use of theory to evaluate the contact stiffness coefficient, otherwise hard to estimate; on the other hand, they introduce additional coefficients that are not always known.

The effect of using different continuous compliant models is moderate from the time of first contact of the net with the debris to net closure, and important after closure; the overall performance of the capture maneuver is not significantly affected. On the other hand, modeling choices have an appreciable effect on the computational time. With a view to achieve simulation of debris capture with a realistically sized net, simple non-linear models such as Hunt and Crossley's and Lee and Wang's are recommended. In case of more plastic impacts with respect to those observed in this research, a more precise model would be Zhang and Sharf's, which performed well in our simulations.

The employed micro-slip friction modeling allows to uniquely define the friction force and to smoothen the transition from sticking to sliding friction. In the considered capture scenario, friction had very little influence on the response of the system and its omission improved noticeably the numerical efficiency. Although this may suggest that a model without friction is sufficient for simulation of net-based capture, it should be taken into account that, as the size of the net increases, the discrepancies are expected to become more important. Also, in a scenario with a rotating debris, friction would play a more important role.

The employed contact modeling is appropriate for representing contact with a smooth debris, but other modeling techniques may need to be devised in order to take into account contacts happening on edges or corners of debris of more complex shape. Contact detection would also be more realistic.
if the threads of the net were modeled in a different way, for example as a series of lumped masses.

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References


doi:10.1016/j.aquaeng.2006.03.003.


[38] Zhang, Y. and Sharf, I., “Compliant Force Modelling for Impact Analysis,” in “International Design


props.html 2006. Composite Materials Design course online material, Department of Materials Science and Engineering, Michigan Technological University, [retrieved 27 April 2016].
