Empirical Analysis of Horizontal Ground Displacement Generated by Liquefaction-Induced Lateral Spreads

by

S.F. Bartlett and T.L. Youd

Technical Report NCEER-92-0021
August 17, 1992

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PREFACE

The National Center for Earthquake Engineering Research (NCEER) was established to expand and disseminate knowledge about earthquakes, improve earthquake-resistant design, and implement seismic hazard mitigation procedures to minimize loss of lives and property. The emphasis is on structures in the eastern and central United States and lifelines throughout the country that are found in zones of low, moderate, and high seismicity.

NCEER’s research and implementation plan in years six through ten (1991-1996) comprises four interlocked elements, as shown in the figure below. Element I, Basic Research, is carried out to support projects in the Applied Research area. Element II, Applied Research, is the major focus of work for years six through ten. Element III, Demonstration Projects, have been planned to support Applied Research projects, and will be either case studies or regional studies. Element IV, Implementation, will result from activity in the four Applied Research projects, and from Demonstration Projects.

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Tasks in Element I, Basic Research, include research in seismic hazard and ground motion; soils and geotechnical engineering; structures and systems; risk and reliability; protective and intelligent systems; and societal and economic impact.

The soils and geotechnical engineering program constitutes one of the important areas of research in Element I, Basic Research. Major tasks are described as follows:
1. Perform site response studies for code development.
2. Develop a better understanding of large lateral and vertical permanent ground deformations associated with liquefaction, and develop corresponding simplified engineering methods.
3. Continue U.S. - Japan cooperative research in liquefaction, large ground deformation, and effects on buried pipelines.
4. Perform soil-structure interaction studies on soil-pile-structure interaction and bridge foundations and abutments, with the main focus on large deformations and the effect of ground failure on structures.
5. Study small earth dams and embankments.

This report describes an empirical model for estimating the horizontal ground displacement caused by liquefaction-induced lateral spreads. The model was developed from multiple linear regression analyses of data pertaining to earthquake, topographical, and geological variables for Japanese and U.S. earthquakes. Two types of lateral spreads are distinguished in the model: lateral spread toward a free face; and lateral spread down gentle ground slopes. Horizontal movement associated with free face lateral spreads was found to correlate with the logarithm of the free face ratio, which is the height of the free face divided by horizontal distance from the free face. In contrast, displacement associated with ground slope failure is strongly correlated with the steepness of the ground slope. The model is expressed as a multiple linear regression equation linking lateral movement with moment magnitude of the earthquake, distance from the seismic source, free face ratio, ground slope, thickness of saturated granular soil with a modified standard penetration value \((N_{1})_{50}\) less than or equal to 15, \((N_{1})_{60}\) of the soil with lowest factor of safety against liquefaction, and depth to the soil with lowest safety factor against liquefaction. Because the model was developed for a wider range of seismic and site conditions than utilized in previously proposed empirical models, it is more general and will result in better estimates. The model appears to give the best predictions for earthquakes with moment magnitudes of 6.5 to 8.0 at sites underlain by sands and silty sand layers with \((N_{1})_{50} \leq 15\) and thickness greater than 0.3 m at depths less than 15 m. The model does not appear to work well for gravels with mean grain sizes greater than 2 mm. Because the model was primarily developed from western U.S. and Japanese data, it is best suited to regions that have high to moderate ground motion attenuation.
ABSTRACT

Liquefaction-induced ground failure is responsible for considerable damage to engineered structures during major earthquakes. Presently, few empirical techniques exist for estimating the amount of horizontal ground displacement resulting from liquefaction-induced lateral spread. None of these techniques fully addresses all the earthquake and site conditions known to influence ground displacement.

This study compiles earthquake, geological, topographical, and soil factors that affect ground displacement and develops empirical models from these factors. Case histories of lateral spread are gathered from the 1906 San Francisco, 1964 Alaska, 1964 Niigata, 1971 San Fernando, 1979 Imperial Valley, 1983 Nihonkai-Chubu, 1983 Borah Peak, Idaho, and 1987 Superstition Hills earthquakes. Multiple linear regression (MLR) is used to develop empirical models from the compiled data. Two general models are derived herein, one for free face failures and one for ground slope failures. The predictive performance of the proposed empirical models is determined by comparing predicted displacements with those actually measured at the case history sites.
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SECTION 1
INTRODUCTION

Lateral spread on gently sloping ground is generally the most pervasive and damaging type of liquefaction-induced ground failure generated by earthquakes (NRC, 1985). Lateral spread generated by the 1906 San Francisco earthquake damaged several buildings, bridges, roads, and pipelines (Youd and Hoose, 1978). Most notably, lateral spread along Valencia Street between 17th and 18th Street severed water lines to downtown San Francisco. The resulting loss of water greatly hampered fire fighting efforts during the ensuing fire. Lateral spread during the 1964 Alaska earthquake caused $80 million damage (1964 value) to 266 bridges and numerous sections of embankment along the Alaska Railroad and Highway (McCulloch and Bonilla, 1970; Kachadoorian, 1968). Liquefaction and lateral spread also produced widespread damage during the 1964 Niigata, Japan earthquake (Hamada et al., 1986). In Niigata, liquefaction of loose, channel deposits caused the banks of the Shinano River to displace as much as 10 meters toward the center of the channel.

Two general questions must be answered when evaluating the liquefaction hazard for a given site: (1) "Are the sediments susceptible to liquefaction?"; and (2) "If liquefaction does occur, what will be the ensuing amount of ground deformation?" Generally accepted empirical and analytical criteria have been developed to evaluate liquefaction susceptibility (Seed and Idriss, 1971; Seed et al., 1983, 1985; NRC, 1985; Liao, 1986). However, little progress has been made in developing methods for estimating the amount of horizontal ground displacement. This need was noted by the National Research Council in outlining new initiatives in liquefaction research: "Methods of evaluating the magnitude of permanent soil deformations induced by earthquake shaking, while considered in the past, have emerged as a pressing need to understand the dynamic behavior of structures and soil deposits. Both triggering and dynamic soil strength must be considered in studying the effect of liquefaction or high pore pressure on deformations. Calculations based on realistic constitutive models are needed to help comprehend the development of permanent deformations and progressive failure (NRC, 1985, p. 217)."

This paper presents a statistical analysis of liquefaction-induced, ground displacement resulting from lateral spread on gently sloping ground. Multiple linear regression (MLR) is used to develop empirical models from earthquake, topographical, geological, and soil conditions associated with lateral spreads from 8 major earthquakes. The MLR models developed herein provide a means of predicting the amount of horizontal ground displacement at potentially liquefiable sites in earthquake-prone regions.
SECTION 2
LIQUEFACTION, LATERAL SPREAD, AND DISPLACEMENT MODELS

2.1 Liquefaction

In this study, we use "liquefaction" to describe any significant loss of shear strength in a saturated, cohesionless soil due to a transient rise in excess pore pressure generated by strong ground motion. Flow failure, lateral spread, ground oscillation, differential settlement, loss of bearing strength, ground fissures, and sand boils are evidences of excess pore pressure generation and liquefaction.

Unconsolidated fluvioglacial, deltaic, loess, flood plain, fan delta, lacustrine, playa, colluvial, dune, sebkha, estuarine, and lagoon sediments may be moderately to highly susceptible to liquefaction (Youd and Perkins, 1978). Saturated, granular soils found in these depositional environments consist mainly of interbedded layers of loose, sand, silt, and fine gravel. Saturated, poorly compacted, artificial fills are also moderately to highly susceptible to liquefaction (Youd and Perkins, 1978).

Dense, consolidated, or well-cemented, granular soils are not usually susceptible to liquefaction. These soils do not incur significant collapse and pore pressure generation during strong ground motion. Consequently, the loss of shear strength in these soils is negligible. Similarly, liquefaction does not usually occur in nonsensitive, clayey soils. Seed and Idriss (1982) give the following criteria to identify clayey soils that are not normally susceptible to liquefaction: (1) soils with a clay content greater than 15 percent, (2) soils with a liquid limit greater than 35 percent, and (3) soils with a moisture content less than 0.9 times the liquid limit.

2.2 Lateral Spread

Lateral spread is the most common type of liquefaction-induced, ground failure. During lateral spread, blocks of intact, surficial soil displace along a shear zone that has formed within the liquefied layer (Figure 2-1). Upon reaching mobilization, the surficial blocks are transported downslope or in the direction of a free face (i.e., channel or abrupt topographical depression) by earthquake and gravitational forces. Horizontal ground displacement resulting from lateral spread ranges from a few centimeters to several meters. This displacement typically occurs on gentle slopes that range from 0.3 to 5 percent (Youd, 1978). Although, ground displacements as large as 5 to 6 m have occurred on a 0.2 percent slope during the 1964 Niigata, Japan earthquake (Hamada, 1992) and displacements as large as 0.3 m occurred on 0.05 to 0.10 percent slopes during the 1964 Alaska earthquake (Hartlett and Youd, 1992).

Ground deformation resulting from lateral spread typically forms a graben or extensional fissures at the head of the failure, shear deformations along the side margins, and buckling or compression of the soil at the toe. Rigid structures at the head of the failure are commonly pulled apart; those at the toe are compressed or buckled. Buried objects, such as pipelines and piles, are often sheared by differential movement within or at the side margins of the lateral spread.

2.3 Modeling Lateral Spread Displacement

Predicting the amount of ground displacement resulting from dynamic and static forces acting upon a composite system of liquefied and non-liquefied soil is a challenging problem. Ultimately, horizontal ground displacement is controlled by: (1) the degree of shear strength loss in the liquefied layer, (2) the continuity and boundary conditions surrounding the failure, (3) the magnitude and direction of the dynamic and static shear forces acting upon the mobilized soil, and (4) the time interval that these forces exceed the shear strength of the liquefied soil. A rigorous solution of this problem requires a dynamic, 3-D,
analysis of a nonlinear, anisotropic, heterogeneous material.

Several analytical and numerical models have been proposed for calculating liquefaction-induced, ground displacement. However, these models have not been applied to a wide range of earthquake and site conditions. More validation and calibration studies are needed before the practicing engineer can place a high degree of confidence in these techniques.

2.4 Static Models from Elastic Theory

Hamada et al. (1987), Towhata et al. (1990), and Yasuda et al. (1990) have used static, 2-D, elastic models to estimate the amount of lateral spread displacement resulting from the 1964 Niigata and 1983 Nihonkai-Chubu earthquakes. Hamada et al. (1987) propose that upon reaching liquefaction, the frictional resistance between the liquefied, subsurface layer and the nonliquefied, surface layer approaches zero. The nonliquefied, surface layer is then modeled as a 2-D, elastic beam that is deformed by pre-earthquake, static shear stresses. Likewise, the model proposed by Towhata et al. (1990) treats the nonliquefied, surface layer as a 2-D, elastic beam that is deformed by static shear stresses. The resulting strain is approximated as a sinusoidal curve with zero displacement assigned to the base of the liquefied layer and the maximum displacement assigned at the ground surface. An analytical, closed-form solution is used to calculate the displacement at the ground surface by minimizing the potential energy of the system.

In the first step of a static, 2-D, elastic, finite element procedure proposed by Yasuda et al. (1990), the pre-earthquake, static shear stresses and pre-liquefaction strains are calculated for each element using the elastic modulus of the nonliquefied soil. In the next step, the post-liquefaction strains are calculated for the mesh by holding the pre-earthquake static stresses constant and by reducing the shear modulus of the soil to represent liquefaction. Finally, the strains from the second analysis are subtracted from those of the first to calculate ground displacement vectors.

2.5 Dynamic Models

Prevost (1981), Finn and Yogendrakumar (1989), and Finn (1990) have developed dynamic, 2-D and 3-D models that can be used to predict liquefaction-induced ground displacement. For example in a finite element model, TARA-3FL, proposed by Finn and Yogendrakumar (1989), the pre-earthquake, static, stress-strain state is calculated for each element in the finite-element mesh. In the subsequent, dynamic part of the analysis, as liquefaction is triggered in specific elements according to the criteria developed by Seed (1983) and Seed et al. (1985), the shear strength of these elements is allowed to drop to its steady-state value.

Figure 2-1 Block diagram of a lateral spread before and after failure. Liquefaction occurs in the cross-hatched zone. Surface layer displaces laterally downslope (after Youd, 1984).
The post-liquefaction shear strength of these elements is unable to sustain the imposed static and dynamic shear stresses, and the mesh is allowed to progressively deform until equilibrium is restored between the final stress state and the steady-state strength of the liquefied soil. Because deformation can become large, the finite element mesh has to be progressively updated. Calculation of the incremental deformation is done on the current shape of the soil mass and not on the initial shape as in conventional finite element analysis. Finn (1990) also recommends that an static stability analysis be performed on the final soil configuration to verify that the factor of safety is equal to or greater than unity.

2.6 Sliding Block Analysis

Newmark (1965), Goodman and Seed (1966) and Makdisi and Seed (1978), Dobry and Baziar (1990), Byrne (1990), Yegian, et al. (1991), and Mabey (1992) have developed various methods of estimating liquefaction-induced, ground failure displacement from sliding block analysis. In these dynamic, 1-D models, a rigid, soil block is allowed to displace along a planar failure surface during time intervals when the earthquake inertial force, \( P_d \), exceeds the yield coefficient of the soil, \( K_y \). The value of \( P_d \) is calculated by multiplying the horizontal ground acceleration by the mass of the block. Also, \( P_d \) is assumed to act parallel to the base of the block and is not allowed to vary along the failure surface. \( K_y \) is normalized for the weight of the block and is calculated from:

\[
K_y = \frac{(F_n - P_d)}{W} \tag{2.6.1}
\]

where:
- \( F_n \) = resisting force due to the dynamic shear strength of the soil
- \( P_d \) = force acting on the block due to active earth pressure
- \( W \) = weight of the block.

Typically, resisting forces due to irregularities along the failure surface are assumed to be negligible and are omitted from the analysis. Additionally, upslope translation of the block is usually small because the magnitude of the inertial force in the upslope direction seldom greatly exceeds the weight of the downslope component of the soil block.

The net downslope displacement of the block, \( D_s \), is governed by the magnitude and time interval that \( P_d \) exceeds \( K_y \). Several researchers have proposed solutions for \( D_s \), using various shapes for the base input motion. Yegian et al. (1988) have presented solutions for triangular, sinusoidal, and rectangular, base input motions. Franklin and Chang (1977), Makdisi and Seed (1978), Sarma (1979), and Whitman and Liao (1985) have proposed similar solutions for earthquakes of various magnitudes.

As originally introduced by Newmark (1965), sliding block analysis is based on the assumption that the soil within the failure plane deforms as a perfectly plastic material (i.e., \( F_n \) remains constant with strain). Many researchers have incorporated the residual strength of the liquefied soil to represent \( F_n \) in sliding block analysis. However, the use of a constant value, such as residual strength, to represent the nonlinear, stress-strain behavior of a liquefied soil is a simplification that remains to be validated. In addition, current laboratory and field methods for estimating residual strength are based on limited data and exhibit a great deal of scatter as noted by Marcuson et al. (1990). Thus, the ability of sliding block analysis to accurately predict liquefaction-induced ground displacement may be limited by the uncertainty associated with obtaining representative values of residual strength for the failure surface.

Byrne (1990) has proposed a more sophisticated sliding block model that incorporates shear strength degradation during cyclic loading. Instead of the
rigid-plastic spring proposed by Newmark (1965), a nonlinear spring is used to account for shear strength degradation as liquefaction develops. In Byrne's model, the stiffness of the spring is expressed as a function of both the residual strength and the limiting strain of the liquefied soil. Both of these factors are in turn strongly influenced by the relative density of the cohesionless soil (Seed and Harder, 1950; Seed et al., 1984).

2.7 Empirical Models

Hamada et al. (1986) and Youd and Perkins (1987) have proposed empirical models to predict lateral spread displacement. We will briefly review these models to provide ideas on how earthquake and site conditions can be quantified and used as predictor variables in empirical models.

2.7.1 Liquefaction Severity Index

Youd and Perkins (1987) evaluated cases of liquefaction-induced ground failure that occurred in a very specific geologic setting. They limited their study to lateral spreads that occurred on gentle slopes or into river channels having widths greater than 10 meters. Their study was also restricted to lateral spreads which occurred in saturated, cohesionless, Holocene fluvial or deltaic deposits with estimated standard penetration resistances ranging from 2 to 10 blows per foot. By restricting their study to these site conditions, Youd and Perkins postulated that ground displacement, S, becomes primarily a function of the amplitude, A, and duration of strong ground motion, D:

$$ S = f(A, D). $$  \hspace{1cm} (2.7.1.1)

Other researchers have shown that A and D are functions of earthquake magnitude, M, and distance from the seismic energy source, R, (Joyner and Boore, 1981; 1988; Krinitzsky and Chang, 1988b). In general, A attenuates logarithmically with R, and D shows a slight increase with increasing R. For many of the lateral spreads in the Youd–Perkins study, strong motion records were not available; thus, they chose to express S as a function of M and log R:

$$ S = f(M, \log R). $$  \hspace{1cm} (2.7.1.2)

The moment magnitude, \( M_w \), was chosen to represent \( M \), because it generally provides a better estimate of the total energy released during a seismic event than other measures of earthquake magnitude (Kanamori, 1978).

Youd and Perkins (1987) introduced the "Liquefaction Severity Index" or LSI as a convenient scale to represent the general maximum value of \( S \) for a given lateral spread occurring within the defined geological setting. Localities where the reported horizontal ground displacement had obviously exceeded 100 inches (2.5 m) were excluded from the formulation of the LSI equation. Youd and Perkins considered these large displacements to be so damaging and erratic in nature that extending the LSI beyond 100 inches (2.5 m) was not meaningful; thus, the LSI was chosen to range between 0 and 100 inches.

Least squares regression was used to develop the following equation:

$$ \log \text{LSI} = -3.49 - 1.86 \log R + 0.98 M_w $$  \hspace{1cm} (2.7.1.3)

where:
- LSI = maximum, permanent, horizontal displacement in inches (i.e., mm divided by 25)
- \( R \) = horizontal distance from the energy source in kilometers
- \( M_w \) = moment magnitude.
2.7.2 Hamada et al. (1986) Equation for Lateral Spread Displacement

Based on pre- and post-earthquake aerial photographs, Hamada et al. (1986) published horizontal ground displacement vector maps for many areas damaged by lateral spreads in Niigata and Noshiro, Japan, during the 1964 Niigata and 1983 Nihonkai-Chubu earthquakes, respectively. Using borehole logs, they also constructed subsurface cross-sections along the longitudinal axis of many of the lateral spreads in these two cities. Guided by changes in the surface topography and breaks in the vector displacement pattern, Hamada et al. divided each cross-section into segments or blocks that appeared to have displaced as a discrete unit (Figure 2-2a and 2-2b). They averaged the displacement vectors, the thickness of the liquefied layer(s), and the slope measurements for each block and used these averages in their correlative analyses.

Hamada et al. used the Factor of Liquefaction Resistance, $F_L$, to estimate the thickness of the liquefied layer(s), $H$ (Appendix III of Hamada et al., 1986; Iwasaki et al., 1978). $F_L$ is a factor of safety that compares the liquefaction resistance of the soil to the dynamic shear stresses generated by the earthquake:

$$F_L = R/L.$$  \hspace{1cm} (2.7.2.1)

Layer(s) having $F_L$ values less than 1.0 were considered to have liquefied. The in-situ resistance of the soil, $R$, was calculated from empirical curves adopted by the Japanese Code of Bridge Design. $R$ is a function of the standard penetration resistance of the soil, the effective overburden stress, $\sigma_v'$, and the mean grain size, $D_{50}$:

$$R = 0.0882(N/\sigma_v' + 0.7)^{1/2} + 0.19$$ \hspace{1cm} (2.7.2.2)

where:
- $N =$ SPT blow count in blows per foot
- $\sigma_v'$ = effective overburden stress in kg/cm$^2$.

Equation 2.7.2.2 is used for soils with 0.02 mm $\leq D_{50} \leq 0.05$ mm and Equation 2.7.2.3,

$$R = 0.0882(N/\sigma_v' + 0.7)^{1/2} + 0.225 \log (0.35/D_{50})$$ \hspace{1cm} (2.7.2.3)

is used for soils with 0.05 mm $\leq D_{50} \leq 0.6$ mm and Equation 2.7.2.4,
is used for soils with $0.6 \text{ mm} \leq D_{90} \leq 1.5 \text{ mm}$. The dynamic earthquake shear stress, $L$, is normalized for $\sigma'$ and is given by:

$$L = 0.65(\sigma/\sigma')r_d$$  \hspace{1cm} (2.7.2.5)

where:

$r_d$ = a stress reduction factor that reduces $L$ with depth.

Hamada et al. identified the liquefied layer(s) in each cross-section by using $F_l$. Layers with $F_l$ values less than 1.0 were marked as having liquefied. Boundaries for liquefied layers were interpolated between bore holes to construct a continuous profile along the longitudinal axis of the lateral spread. If liquefaction was indicated in more than one layer, the total thickness of the liquefied layer, $H$, included the combined thickness of liquefiable layers plus the thickness of any intermediate nonliquefiable layer(s) (Figure 2-2a).

Hamada et al. also estimated the ground slope along the longitudinal axis of each displaced block and correlated it with ground displacement. The ground slope, $\theta$, ($\%$), was defined simply as:

$$\theta = 100(Y/X)$$  \hspace{1cm} (2.7.2.6)

where:

$Y$ = vertical change in surface elevation across the block
$X$ = length of the block (Figure 2-2a).

For lateral spreads that occurred near the banks of the Shinano River, the measurement of $\theta$ was artificially steepened to the bottom of the river channel (Figure 2-2b).

Hamada et al. also postulated that the slope of the bottom of the liquefied layer may have influenced horizontal displacement. This slope, $\theta_2$, ($\%$), was measured by scaling the slope of the bottom of the liquefied layer from the constructed cross-section (Figure 2-2a):

$$\theta_2 = 100(Y_2/X_2).$$  \hspace{1cm} (2.7.2.7)

where:

$Y_2$ = vertical change in subsurface elevation across the block
$X_2$ = length of the block.
Hamada et al. found that MLR models which included $H$ in conjunction with the larger value of either $\theta_1$ or $\theta_2$ yielded the best predictions of horizontal displacement. They proposed the following regression model:

$$D = 0.75 H^{0.50} \theta^{0.33}$$  \hspace{1cm} (2.7.2.8)

where:

- $D$ = horizontal ground displacement, (m)
- $H$ = thickness of the liquefied layer, (m)
- $\theta$ = the larger value of $\theta_1$ or $\theta_2$, for the block, (%).

Figure 2-3 is a plot of observed displacements from Niigata and Noshiro, Japan, plotted against displacements predicted by this model. Predictions that fall on or near the 45 degree solid line are closely approximated by the model. The dashed line below the 45 degree solid line represents a 100 percent overprediction bound (i.e., the predicted displacements are 2 times larger than observed displacements). The dashed line above the 45 degree solid line represents a 50 percent underprediction bound (i.e., the predicted displacements are one-half times larger than the observed displacements). Approximately 80 percent of the displacements predicted by the Hamada et al. model fall between these two prediction bounds.

### 2.7.3 Summary of Empirical Models

The LSI equation of Youd and Perkins (1987) is based primarily on earthquake factors and is intended to provide a conservative upper bound for estimating horizontal ground displacement at sites having a moderate to high liquefaction susceptibility. In contrast to the LSI equation, the model proposed by Hamada et al. (1986) emphasizes the thickness of the liquefiable layer and slope, but it does not address the importance of earthquake factors. This thickness-slope model appears to produce reasonable estimates for $M = 7.5$ earthquakes and for highly liquefiable sediments that are located approximately 20 to 30 km from the seismic source. However, it yields less reliable results for smaller or larger seismic events occurring at varying distances (Bartlett and Youd, 1990). Additionally, the characteristics of the liquefied deposits in Niigata and Noshiro cities are relatively homogeneous (i.e., uniform, medium-to fine-grained, clean sand). Extrapolation of the regression equation to gravelly and silty sediments yields poorer predictions (Bartlett and Youd, 1990).

### 2.7.4 Towards a More Comprehensive, Empirical Model

It is difficult to quantify and model all factors that contribute to liquefaction and ground displacement. Thus, the modeler is forced to select only a handful of the most influential factors and attempt to represent their complex interaction. Based on the studies by Hamada et al. (1986), Youd and Perkins
(1987), and Bartlett and Youd (1990), we believe that a more comprehensive empirical model should include, but not be restricted to the following: (1) earthquake factors (e.g., peak ground acceleration and duration of strong ground motion or earthquake magnitude and distance from the zone of seismic energy release), (2) topographical factors (e.g., ground slope and/or the distance to and height of a free face, if present), (3) geological factors (e.g., thickness of and depth to the liquefied layer), and (4) soil factors (e.g., residual strength, mean grain size, and the silt and clay content of the liquefied soil).
SECTION 3
EMPIRICAL ANALYSIS OF LATERAL SPREAD FOR NIIGATA AND NOSHIRO, JAPAN

3.1 Multiple Linear Regression

Multiple linear regression (MLR) is often used to predict the behavior of complex phenomena that are influenced by several factors (Draper and Smith, 1981). In applying MLR analysis, it is assumed that changes in the independent variables, \( X(s) \), are accompanied by a corresponding change in the response of the dependent variable, \( Y \). The true response, \( \eta \), is expressed in terms of an unknown function, \( \phi \), which contains the \( X(s) \), and the unknown parameters, \( B(s) \), that accompany the \( X(s) \).

\[
\eta = \phi(X_1, X_2, \ldots, X_p; B_1, B_2, \ldots, B_p) \tag{3.1.1}
\]

For example, in this study we postulate that horizontal ground displacement is a function of several independent variables.

\[
\eta = \phi(E, G, T, S; B_E, B_G, B_T, B_S) \tag{3.1.2}
\]

where:
- \( E \) = Earthquake factors
- \( G \) = Geological factors
- \( T \) = Topographical factors
- \( S \) = Soil factors
- \( B_E, B_G, B_T, B_S \) = unknown parameters corresponding to \( E, G, T, \) and \( S \)

Ideally, the value of \( \eta \) for a given set of \( X(s) \) is the same each time the experiment is performed. But, in reality, \( \eta \) is seldom observed due to the presence of many uncontrolled and unmeasured variables that affect \( \eta \). The deviation of the observed response, \( Y \), from \( \eta \) is called experimental error, \( \epsilon \).

\[
\eta - Y = \epsilon \tag{3.1.3}
\]

In MLR analysis, \( \phi \) is approximated by an additive, linear model. The values of \( Y \) and the \( X(s) \) are often transformed (e.g., \( 1/X \), log \( X \), \( e^X \), etc.) in order to produce a linear form.

\[
Y = b_0 + b_1X_1 + b_2X_2 + \ldots + b_pX_p + \epsilon \tag{3.1.4}
\]

The regression coefficients, \( b_0, \ldots, b_p \), are best-fit estimates of the \( B(s) \), and \( \epsilon \) is a best-fit estimate of \( \epsilon \). The method of ordinary least squares is commonly used to estimate the \( B(s) \) by minimizing the error sum of squares:

\[
S_e = \Sigma(\epsilon)^2 \tag{3.1.5}
\]

where:
- \( \epsilon \) = difference between the measured response, \( Y \), and the response predicted by the regression equation, \( Y_{\text{hat}} \), i.e.,

\[
\epsilon = Y - Y_{\text{hat}} \tag{3.1.6}
\]

To provide these best-fit estimates, it is assumed: (1) the \( \epsilon(s) \) are random variables with an expected value of zero (i.e., \( E(\epsilon) = 0 \)), (2) the variance of the \( \epsilon(s) \) is constant for all values of the \( X(s) \) (i.e., \( V(\epsilon) \) is constant), (3) the \( \epsilon(s) \) are not correlated, and (4) the values of the can be measured without error. Also, if the \( \epsilon(s) \) appear to be normally distributed, then partial and sequential t and F tests can be performed on each of the \( B(s) \) to verify that these coefficients are statistically significant (i.e., \( b_0, \ldots, b_p \) are not equal to
zero) (Draper and Smith, 1981). Standardized residual plots are commonly used to evaluate the validity of these assumptions. Also, these plots give the investigator valuable information about the general performance of the MLR model. The standardized residual, \( e^* \), for each observation is calculated from:

\[
\text{e}_i = \frac{e_i}{(\text{standard deviation of e})} \quad (3.1.7)
\]

where:
\[
e = Y - \hat{Y}_{\text{mat}}.
\]

Typically, the \( e_i(s) \) from the model are plotted against the corresponding \( X(s) \) and \( \hat{Y}_{\text{mat}}(s) \) to confirm that the \( e_i(s) \) are independent and have a constant variance with a mean of zero. An acceptable standard residual plot gives the impression of a horizontal band of data centered on zero. Approximately 95 percent of the \( e_i(s) \) should fall within \( \pm 2 \) standard deviations of zero, and almost all \( e_i(s) \) should fall within \( \pm 3 \) standard deviations of zero line shown in Figure 3-1a.

![Figure 3-1](image)

**Figure 3-1** Examples of standard residual plots. (a) Satisfactory residual plot gives overall impression of horizontal box centered on zero line. (b) A plot showing nonconstant variance. (c) A plot showing linear trend suggesting that the residuals are not independent and that another variable is needed in the model. (d) A plot illustrating the need for a transformation or a higher order term to alleviate curvature in the residuals.

Observations having \( e_i(s) \) that plot more than 2 to 3 standard deviations above or below the zero line are potential outliers. Figures 3-1b through 3-1d are
examples of unsatisfactory residual plots. The behavior shown in Figure 3-1b suggests that the variance of \( e_i(s) \) is not constant (i.e., the residual band widens). Data with a nonconstant variance are usually corrected by transforming \( Y \) or by using weighted, least-squares regression analysis (Draper and Smith, 1981). The linear trend shown in Figure 3-1c suggests that the \( e_i(s) \) are not independent or that some other X is needed in the model. A residual plot displaying curvature (Figure 3-1d) indicates that a higher order term, a cross-product, or a transformation of \( Y \), or of the \( X(s) \), is needed to produce a more linear form.

The performance of MLR models is judged by the coefficient of determination, \( R^2 \):

\[
R^2 = \frac{[\Sigma(Y')-\bar{Y}']^2/n}{[\Sigma(Y')-\bar{Y}]^2/n}
\]

(3.1.8)

where \( n \) is the sample size. The value of \( R^2 \) ranges from 0 to 1 and measures the proportion of the variability of \( Y \) being explained by the \( X(s) \). For example, a \( R^2 \) of 0.50 means that 50 percent of the variability in \( Y \) is being explained by the \( X(s) \).

In this study, we used a modified stepwise regression procedure to guide the development of our MLR models. In short, this procedure begins by searching the set of \( X(s) \) for the \( X \) with the highest correlation with \( Y \) and this \( X \) enters the model. In the next step, the remaining \( X(s) \) are re-examined to find the \( X \) that yields the next highest improvement in \( R^2 \) and this \( X \) is added to the model. The process of examining and adding \( X(s) \) to the model continues until no additional \( X \) can be found that significantly improves \( R^2 \). At the end of each step, partial t-tests are performed on all \( X(s) \) in the current model to verify that each \( X \) is still statistically significant. (Sometimes \( X(s) \) introduced during earlier steps become nonsignificant at later steps because they are correlated with other \( X(s) \) that are just entering the model). Variables that become nonsignificant during later steps are removed from the model prior to beginning the next step.

3.2 Compilation of Case History Data for MLR Analyses

Prior to beginning MLR analysis, 448 horizontal displacement vectors were compiled as the dependent variable, \( D_h \) (m) from 8 earthquakes and the listed lateral spread sites in Table 3-1. Tabulated values of \( D_h \) were measured or estimated by the respective investigators using various methods. The values of \( D_h \) for Niigata and Noshiro, Japan, and the Jensen Filtration Plant were calculated from pre- and post-earthquake photographs using photogrammetric techniques (Hamada et al., 1986; O'Rourke et al., 1990). These measurements have an accuracy of \( \pm 0.72 \), \( \pm 0.17 \), and \( \pm 0.47 \) m, respectively. The values of \( D_h \) for the Wildlife Instrument Array and Juvenile Hall were calculated from pre- and post-earthquake ground surveys (Youd, 1973b; Youd and Bartlett, 1988) and have an accuracy of approximately \( \pm 0.02 \) m. The estimates of \( D_h \) at other lateral spread sites were obtained from reports of dislocated or offset buildings, bridge components, fences, canals, etc. The accuracy of \( D_h \) at these sites is difficult to determine, but is approximately \( \pm 0.1 \) to \( \pm 0.5 \) m.

Because of the large size of these data, the MLR database has been tabulated in ASCII format on the computer disk labeled "Appendix 3" in the file MLR.DAT. This disk and two additional disks, which comprise "Appendix 4", are available from NCEER Information Service, care of Science and Engineering Library, 304 Capen Hall, State University of New York at Buffalo, Buffalo, New York, 14260.

Table 3-2 lists the earthquake, topographical, geological, and soil independent variables that were compiled and tested in our MLR analyses. These data were obtained from seismological reports, topographical maps and surveys, borehole logs, and soil grain-size analyses (Appendix 1, 3, and 4). We used liquefaction susceptibility analysis of SPT data to calculate many of the geological and soil independent variables (Seed and Idriss, 1971; Seed et al., 1983; 1985, NRC 1985,
Liao, 1986). In all, liquefaction susceptibility analysis was performed for 267 boreholes from the lateral spread sites listed in Table 3-1 (Appendix 1 and 4).

<table>
<thead>
<tr>
<th>TABLE 3-1</th>
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<tr>
<td><strong>EARTHQUAKES AND LATERAL SPREAD SITES USED IN THIS STUDY</strong></td>
</tr>
</tbody>
</table>

**1906 San Francisco Earthquake** (Youd and Hoose, 1978)
- Coyote Creek Bridge near Milpitas, California
- Mission Creek Zone in San Francisco, California
- Salinas River Bridge near Salinas, California
- South of Market Street Zone in San Francisco, California

**1964 Alaska Earthquake** (Bartlett & Youd, 1992; McCulloch & Bonilla 1970)
- Bridges 141.1, 147.4, 147.5, 148.3, Matanuska River, Alaska
- Bridges 63.0, 63.5, Portage Creek, Portage, Alaska
- Highway Bridge 629, Placer River, Alaska (Ross et al., 1973)
- Snow River Bridge 605A, Snow River, Alaska (Ross et al., 1973)
- Bridges 3.0, 3.2, 3.3, Resurrection River, Alaska

**1964 Niigata, Japan, Earthquake** (Hamada et al., 1986)
- Numerous lateral spreads in Niigata, Japan

**1971 San Fernando Earthquake**
- Jensen Filtration Plant, San Fernando, CA, (O'Rourke et al., 1990)
- Juvenile Hall, San Fernando, CA, (Bennett, 1989; Youd, 1973b)

**1979 Imperial Valley Earthquake** (Bennett et al., 1984)
- Heber Road near El Centro, California (Dobry et al., 1992)
- River Park near Brawley, California

**1983 Borah Peak Idaho, Earthquake**
- Whiskey Springs near Mackay, Idaho (Andrus and Youd, 1987)
- Pence Ranch near Mackay, Idaho (Andrus et al., 1991)

**1983 Nihonkai-Chubu Earthquake** (Hamada et al., 1986)
- Lateral spreads in the Northern Sector of Noshiro, Japan

**1987 Superstition Hills Earthquake** (Holzer et al., 1988; 1989)
- Wildlife Instrument Array, Brawley, CA, (Youd and Bartlett, 1988)

At many sites, there was more than one borehole drilled within the zone of ground deformation (for example, see Figure 3-2). For these sites, we used an inverse-distance, linearly-weighted average to interpolate all geological and soil independent variables between boreholes. This averaging scheme assigns the largest weight to the borehole located closest to the displacement vector.
\[ \bar{x}_{AVG} = W_1 x_1 + W_2 x_2 + \ldots + W_n x_n \]  
(3.2.1)

where \( \bar{x}_{AVG} \) is the weighted average, \( x_1, \ldots, x_n \) are the corresponding values of \( x \) to be averaged for \( n \) boreholes, and \( W_1, \ldots, W_n \) are the weights. These weights are calculated from:

\[ W_i = \frac{1/d_i}{\Sigma(1/d_i)} \]  
(3.2.2)

where \( d_i \) is the distance from the \( i^{th} \) borehole to the displacement vector of interest and \( \Sigma(1/d_i) \) is summed for \( n \) boreholes (Appendix 1, Section A1.4). These weighted averages were calculated for each \( D_R \) prior to performing the regression analyses.

3.3 Strategy for Development of MLR Models

Because the earthquakes that generated lateral spreads in Niigata and Noshiro, Japan were seismically similar, we initially ignored the effects that \( M, R, A, \) and \( D \) have on displacement during preliminary model development. (Niigata and Noshiro incurred 7.5 and 7.7 magnitude earthquakes and were situated approximately 21 and 27 km from the zone of seismic energy release, respectively (Hamada, 1986; Mogi et al., 1964; Hwang and Hammad, 1984). By restricting our initial analyses to these two earthquakes, we were able to develop site-specific MLR models based solely on topographical, geological, and soil factors. Also, the extensive displacement and subsurface data for these two cities provided us with a large MLR database amenable to statistical analyses. (Three hundred and seventy seven (377) of the 448 tabulated displacement vectors are from Niigata and Noshiro). After developing site-specific models for Japan, we added the U.S. data to the analyses and adjusted the site-specific MLR models for a wider range of earthquake, topographical, and soil conditions not present in the Japanese data.

We observed two general types of lateral spread in Niigata: (1) lateral spread towards a free face, and (2) lateral spread down gentle ground slopes where a free face was not present. For example, Figure 3-2 shows the pattern of ground displacement along the banks of the Shinano River near the northern abutment of the Echigo Railway Bridge. The large and erratic displacements near the river obviously resulted from a lack of lateral resistance to deformation created by the incised channel. In contrast, ground deformation occurring north of the railroad embankment was smaller (a maximum of 2 m), more uniform, and directed away from the channel. Lateral spread in this area was not impacted by the channel, but resulted from movement down a gentle gradient that slopes 0.2 percent to the northeast. Our preliminary regression models for Niigata showed that the topographical regression coefficients fitted for free face failures differed significantly from those fitted for ground slope failures. Thus, we developed a separate MLR model for each type of failure. In Section 3.4, we discuss the development of a site-specific, free face model using data exclusively from Niigata. (No free face failures were identified in the study of Noshiro, Japan by Hamada et al., 1986). In Section 3.5, we discuss the development of a site-specific, ground slope model using ground slope failures from both Niigata and Noshiro. In Section 4.0, the U.S. case studies are included in the analyses and the MLR models are adjusted for a wider range of earthquake and site conditions.
Figure 3-2 A section of working maps developed by Hamada et al. (1986), showing displacement vectors and locations of SPT boreholes from an area along the Shinano River near the Echigo Railway Bridge in Niigata, Japan (source: unpublished ground failure maps courtesy of M. Hamada).
<table>
<thead>
<tr>
<th>Earthquake Variables</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>M*</td>
<td>Earthquake moment magnitude, $M_w$.</td>
</tr>
<tr>
<td>R*</td>
<td>Nearest horizontal distance to seismic energy source or fault rupture, (km).</td>
</tr>
<tr>
<td>A</td>
<td>Peak horizontal ground acceleration, (g).</td>
</tr>
<tr>
<td>D</td>
<td>Duration of strong ground motion (&gt;0.05 g), (s).</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Topographical Variables</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>S*</td>
<td>Ground slope, (%).</td>
</tr>
<tr>
<td>L</td>
<td>Distance to the free face from the point of displacement, (m).</td>
</tr>
<tr>
<td>H</td>
<td>Height of free face, (m).</td>
</tr>
<tr>
<td>W*</td>
<td>Free face ratio, (%), (i.e., 100 H/L).</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Geological Variables</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_s$</td>
<td>Thickness of liquefied zone(s) (Simplified procedure), (m).</td>
</tr>
<tr>
<td>$T_{L1}$</td>
<td>Thickness of liquefied zone(s) (Liao’s 50% probability curve), (m).</td>
</tr>
<tr>
<td>$T_{10}$</td>
<td>Thickness of saturated cohesionless soils with $(N_1)_{90} \leq 10$, (m).</td>
</tr>
<tr>
<td>$T_{15}$</td>
<td>Thickness of saturated cohesionless soils with $(N_1)_{90} \leq 15$, (m).</td>
</tr>
<tr>
<td>$T_{20}$</td>
<td>Thickness of saturated cohesionless soils with $(N_1)_{90} \leq 20$, (m).</td>
</tr>
<tr>
<td>$I_s$</td>
<td>Index of Liquefaction Potential (Simplified procedure).</td>
</tr>
<tr>
<td>$I_{L}$</td>
<td>Index of Liquefaction Potential (Liao’s 50% probability curve).</td>
</tr>
<tr>
<td>$Z_{OA}$</td>
<td>Depth to top of liquefied zone (Simplified procedure), (m).</td>
</tr>
<tr>
<td>$Z_{OB}$</td>
<td>Depth to top of liquefied zone (Liao’s 50% prob. curve), (m).</td>
</tr>
<tr>
<td>$Z_{BO}$</td>
<td>Depth to bottom of liquefied zone (Simplified procedure), (m).</td>
</tr>
<tr>
<td>$Z_{BOB}$</td>
<td>Depth to bottom of liquefied zone (Liao’s 50% prob. curve), (m).</td>
</tr>
<tr>
<td>$Z_s$</td>
<td>Depth to the lowest factor of safety (Simplified procedure), (m).</td>
</tr>
<tr>
<td>$Z_L$</td>
<td>Depth to the lowest factor of safety (Liao’s 50% prob. curve), (m).</td>
</tr>
<tr>
<td>$Z_N$</td>
<td>Depth to lowest SPT N value in saturated cohesionless soil, (m).</td>
</tr>
<tr>
<td>$Z_{N_{10}}$</td>
<td>Depth to lowest SPT $(N_1)_{90}$ value in saturated cohesionless soil, (m).</td>
</tr>
<tr>
<td>$N$</td>
<td>Lowest SPT $N$ value in saturated cohesionless sediments.</td>
</tr>
<tr>
<td>$N_{10}$</td>
<td>Lowest SPT $(N_1)_{90}$ value in saturated cohesionless sediments.</td>
</tr>
<tr>
<td>$J_s$</td>
<td>Lowest factor of safety below water table (Simplified procedure).</td>
</tr>
<tr>
<td>$J_{L}$</td>
<td>Lowest factor of safety below water table (Liao 50% prob. curve).</td>
</tr>
<tr>
<td>$N_{10BS}$</td>
<td>$(N_1)_{90}$ value corresponding to $J_s$.</td>
</tr>
<tr>
<td>$N_{10BL}$</td>
<td>$(N_1)_{90}$ value corresponding to $J_L$.</td>
</tr>
<tr>
<td>$K_s$</td>
<td>Average factor of safety in $T_s$.</td>
</tr>
<tr>
<td>$K_L$</td>
<td>Average factor of safety in $T_L$.</td>
</tr>
<tr>
<td>$Q_s$</td>
<td>Average $(N_1)_{90}$ in $T_s$.</td>
</tr>
<tr>
<td>$Q_L$</td>
<td>Average factor of safety in $T_L$.</td>
</tr>
</tbody>
</table>

* Indicates independent variables used in the final MLR model.
### TABLE 3-2 (CONTINUED)

**SUMMARY OF INDEPENDENT VARIABLES CONSIDERED IN MLR ANALYSIS**

(For more information on these variables, see Section 4 and Appendix 1)

<table>
<thead>
<tr>
<th>Soil Variables</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>D50_s</td>
<td>Average D_50 in T_s, (mm).</td>
</tr>
<tr>
<td>D50_L</td>
<td>Average D_50 in T_L, (mm).</td>
</tr>
<tr>
<td>D50_H</td>
<td>Average D_50 in T_H, (mm).</td>
</tr>
<tr>
<td>D50_18</td>
<td>Average D_50 in T_18, (mm).</td>
</tr>
<tr>
<td>D50_20</td>
<td>Average D_50 IN T_20, (mm).</td>
</tr>
<tr>
<td>F_s</td>
<td>Average fines content in T_s, (particle size &lt;0.075 mm, in percent).</td>
</tr>
<tr>
<td>F_L</td>
<td>Average fines content in T_L, (particle size &lt;0.075 mm, in percent).</td>
</tr>
<tr>
<td>F_H</td>
<td>Average fines content in T_H, (particle size &lt;0.075 mm, in percent).</td>
</tr>
<tr>
<td>F_18</td>
<td>Average fines content in T_18, (particle size &lt;0.075 mm, in percent).</td>
</tr>
<tr>
<td>F_20</td>
<td>Average fines content in T_20, (particle size &lt;0.075 mm, in percent).</td>
</tr>
</tbody>
</table>

* Indicates independent variables used in final MLR model.

---

### 3.4 Free Face MLR Model for Lateral Spreads in Niigata, Japan

Stepwise regression analyses of lateral spreads in Niigata show that the proximity of the channel is the most important site factor affecting ground displacement for free face failures. Figure 3-3 is a plot of horizontal displacement, \( D_H \) (m), versus the horizontal distance from the channel, \( L \) (m), for several lateral spreads along the Shinano River. This plot suggests that \( D_H \) decays logarithmically with increasing \( L \). A regression of this single factor yields the following equation:

\[
D_H = 11.6 - 4.38 \times \log L \quad (3.4.1)
\]

The \( R^2 \) for this model is 31.7 percent. Partial t-tests for the intercept, \( D_H \), and the partial slope for \( \log L \) are significant at the 99.9 percent confidence level (i.e., 99.9% probability that these coefficients are not equal to zero).

In analyzing the ground deformation near the Shinano River, we noted that \( D_H \) near the bridge abutments appears to have been impeded by the structures. For example, Figure 3-2 shows that the displacement vectors near the north abutment of the Echigo railroad decreased from approximately 8 m (at a locality 75 m east of the bridge) to 3 m (at 10 m east of the bridge). In order to minimize the variability in \( D_H \) resulting from
bridge interference, we did not compile displacement vectors found within approximately 50 m of the bridges.

In addition to L, the height of the free face (i.e., the depth of the channel), H, (m), is also correlated with \( D_h \). To normalize \( L \) for the effect of \( H \), we combined these two topographic measures into one independent variable called the free face ratio, \( W \), (%) (Figure 3-4):

\[
W = 100 \frac{H}{L} \tag{3.4.2}
\]

Although there is considerable scatter due to other geological and soil factors not accounted for in the model, \( D_h \) appears to increase in a nonlinear fashion with \( W \) (Figure 3-4). To fit this nonlinearity, we tried the following models:

\[
D_h = b_1 \sin \left( \frac{H}{L} \right) \tag{3.4.3a}
\]

\[
D_h = b_0 + b_1 \log W \tag{3.4.3b}
\]

\[
\log D_h = b_0 + b_1 \log W \tag{3.4.3c}
\]

Equation 3.4.3a is a plausible model for planar failure surfaces that intersect the free face. This model presupposes that \( D_h \) is proportional to the gravitational shear force acting along the base of the mobilized soil block (Figure 3-5). However, for the Niigata data, Equation 3.4.3a yielded poorer predictions compared with Equations 3.4.3b and 3.4.3c (R' equals 28.6 percent versus 39.1 and 38.0 percent, respectively).

A fit of model 3.4.3b yielded the highest \( R^2 \) value, but a residual plot of \( e_i(s) \) versus \( D_{\text{inf}} \) from this model showed evidence of nonconstant variance. Figures 3-2 and 3-3 shows that \( D_h \) is more variable near the free face. This extra variability is most likely due to changes in the subsurface geology or is a result of impediments to displacement such as retaining walls, piles, and other buried structures found along the margins of the channel. Because MLR analysis assumes that the variance of \( e_i \) is constant throughout the ranges of the independent variables, nonconstant variance is undesirable and may pose problems in estimating the prediction limits for the true value of \( D_h \). A log transformation of the dependent variable, \( Y \), is a standard technique used to reduce nonconstant variance (Draper and Smith, 1981). Also, Hamada et al., 1986 and Youd and Perkins (1987) used log-log transformations similar to Equation 3.4.3c in their MLR models. Thus, we ultimately decided to transform \( D_h \) to \( \log D_h \) (Figure 3-5) and fitted model 3.4.3c with the following coefficients:

![Figure 3-4 Plot of ground displacement, \( D_h \), versus free face ratio, \( W \), for lateral spreads along the Shinano River, Niigata, Japan.](image1)

![Figure 3-5 Diagram showing that the shear force acting along the base of slide block is proportional to \( \sin (H/L) \).](image2)
\[ \log D_h = -0.138 + 0.660 \log W. \] (3.4.4)

A residual plot of \( e_i(s) \) versus \( D_{\text{max}} \) for Equation 3.4.4 indicates that the log-log model eliminates the nonconstant variance (Figure 3-7). The intercept and slope for this equation are highly significant. (The intercept is significant at the 97 percent confidence level and the slope for \( \log W \) is significant at the 99.9 percent confidence level.)

In analyzing free face failures near the Shinano River in Niigata, we postulated that the slope of the river bank into or away from the channel may have had an effect on \( D_h \) and we tabulated and tested a second variable, \( S, \) (\%), to represent that possible effect. The value of \( S \) was assigned a positive value for cases where the ground sloped toward the channel (Figure 3-8, Case 1) and a negative value for cases where the ground sloped away from the channel (Figure 3-8, Case 2). The inclusion of \( S \) in the free face model did not improve \( R^2 \) significantly; hence, we concluded that the slope of the floodplain near the Shinano River does not vary enough to have markedly affected \( D_h \). In general, we found this conclusion to be true for other case history sites where free face failures occurred near major river channels.

We also adjusted the free face model for the effects of subsurface geology and soil conditions. Stepwise regression indicated that the cumulative thickness of the liquefied layer, \( T, (m) \), is the next variable that should enter the model. Some modelers have used liquefaction analyses based on empirical curves and SPT \( (N_{16}) \) values to estimate \( T \) (Seed and Idriss, 1971; Seed et al., 1983, 1985; NRC, 1985; Hamada, 1986; Liao, 1986). However, these techniques require an estimate of the earthquake magnitude, \( M \), and peak ground acceleration, \( A \), as input into the analyses. Thus, \( T \) determined from these methods will be correlated with the earthquake factors \( M \) and \( A \). To minimize the correlation between earthquake and site factors, we defined and tested three estimates of \( T \) that are calculated without performing

Figure 3-6 Plot of \( \log D_h \) versus \( \log W \) for lateral spread displacements along the Shinano River, Niigata, Japan showing an approximately linear relationship.

Figure 3-7 Standardized residuals, \( e_i's \), from model 3.4.4 plotted against \( \log D_h \), showing no evidence of nonconstant variance.
liquefaction analyses. We defined \( T_{10}, T_{15}, \) and \( T_{20} \) as the cumulative thickness, (m), of saturated cohesionless sediments with SPT \((N1)_{60}\) values \(\leq 10, 15,\) and \(20\), respectively. Saturated soils with a clay content \(\geq 15\) percent were not added to these cumulative thickness. Also, because most boreholes included in our study were drilled to a maximum depth of 20 meters, \( T_{10}, T_{15}, \) and \( T_{20} \) were generally accumulated in the upper \(20\) m of the soil profile. The substitution of \( T_{15}, T_{15}, \) and \( T_{20} \) for \( T \) into the free face model:

\[
\log D_H = b_0 + b_1 \log W + b_2 T. \tag{3.4.5}
\]

yields \(R^2\) values of 50.9, 52.4, and 63.8 percent, respectively. We ultimately chose to use \( T_{15} \) instead of \( T_{20} \) in all subsequent models because our case history data suggests that lateral spreads are generally restricted to deposits having \((N1)_{60}\) values \(\leq 15\) for \( M \leq 8.0 \) earthquakes. A plot of the \( e_i(s) \) from model 3.4.4 versus \( T_{15} \) shows an approximately linear relationship between \( \log D_H \) and \( T_{15} \) (Figure 3-9a), thus we formed the model:

\[
\log D_H = -0.537 + 0.568 \log W + 0.0458 T_{15}. \tag{3.4.6}
\]

All regression coefficients for this model are significant at the 99.9 percent confidence level.

After adjusting the free face model for the influence of \( W \) and \( T \), stepwise regression indicated that the percentage of fines, \( F_s \) (particle size \(\leq 0.075\) mm) of the liquefied layer is the next variable that should enter the free face model. In Figure 3-9b, the \( e_i(s) \) from Equation 3.4.6 are plotted against the average fines content in \( T_{15} \). The linear trend implies that horizontal displacement decreases with increasing fines content. The free face model adjusted for \( F \) is:

\[
\log D_H = 0.355 + 0.594 \log W + 0.0369 T_{15} - 0.0102 F_{15}. \tag{3.4.7}
\]

where:

\( F_{15} \) = average fines content in \( T_{15} \), in percent.

The \(R^2\) for this model is 66.0 percent and all regression coefficients are significant at the 99.9 percent confidence level.

The mean grain size, \( D_{50} \) (mm), of the channel deposits along the Shinano River also had a minor influence on displacement. In Figure 3-9c, the \( e_i(s) \) from model 3.4.7 are plotted against the average \( D_{50} \) in \( T_{15} \). Although showing considerable scatter, this plot suggests that displacement decreases as the average \( D_{50} \) value in \( T_{15} \) increases. Model 3.4.7 adjusted for \( D_{50} \) is:

\[
\log D_H = 0.301 + 0.563 \log W + 0.0338 T_{15} - 0.0244 F_{15} - 1.50 D_{50_{15}}. \tag{3.4.8}
\]

where:

\( D_{50_{15}} \) = average \( D_{50} \) in \( T_{15} \), in millimeters.

The \(R^2\) for this model is 70.0 percent and partial t-tests show that intercept is significant at the 90 percent confidence level and all other regression coefficients are significant at the 99.9 percent confidence level.

In addition to \( D_{50_{15}} \), the \((N1)_{60}\) value associated with the lowest factor of safety against liquefaction in the liquefied profile, \( N_{1_{opt}} \), makes a minor contribution to improving the performance of the free face model. The value of \(R^2\) increased from 70.0 to 72.4 percent as \( N_{1_{opt}} \) was included. To determine \( N_{1_{opt}} \), a factor of safety against liquefaction, \( FS \), was calculated for each \((N1)_{60}\) value in the profile by applying the "simplified procedure" for liquefaction analysis (Seed.
Figure 3-8 Definition of free face factors, L and H, and ground slope, S, for free face failures.

and Idriss, 1971; Seed et al., 1983; 1985; see also Appendix 1): 

$$FS = \frac{CSRL}{CSRQ}$$  \hfill (3.4.9)

where:

CSRL = cyclic stress ratio required for liquefaction
CSRQ = cyclic stress ratio induced in the profile by the earthquake.

The $(N1)_{60}$ value corresponding to the lowest $FS$ in the profile was assigned to $N1_{60}$. The $e_i(s)$ from Equation 3.4.8 plotted against $N1_{60}$ indicate that displacement tends to decrease with increasing values of $N1_{60}$ (Figure 3-9d).
Figure 3.9 (a) Standardized residuals from model 3.4.4 plotted against the thickness of saturated cohesionless sediments with SPT (N160) values ≤ 15, T15, showing a linear relationship between T15 and e1. (b) Standardized residuals from model 3.4.6 plotted against the average fines content, F15, in T15, showing a linear relationship between F15 and e1. (c) Standardized residuals from model 3.4.7 plotted against the average mean grain size, D5015, in T15, showing a linear relationship between D5015 and e1. (d) Standardized residuals from model 3.4.8 plotted against the SPT (N1) value corresponding to the lowest factor of safety in the liquefied profile, N1ref, showing a linear relationship between N1ref and e1.

The addition of N1ref in the free face model yields:

\[
\text{LOG } D_H = 0.610 + 0.572 \text{ LOG } W + 0.0247 \text{ T}_{15} - 0.0278 \text{ F}_{15} - 1.61 \text{ D50}_{15} - 0.0315 \text{ N1}_{\text{ref}} \tag{3.4.10}
\]

The inclusion of other possible geological and soil factors from Table 3-2 in Equation 3.4.10 did not appreciably improve the performance of the model; thus, this equation was adopted as the final model for free face failures in Niigata.

We did obtain a slightly higher \( R^2 \) values by including interaction terms in the model (i.e., cross-products of LOG \( W, T_{15}, F_{15}, D50_{15}, \) and \( N1_{\text{ref}} \)), but the physical
meanings of these interactions were difficult to interpret. Thus, we do not believe that the slight improvement in $R^2$ warrants the addition of higher order terms and we formulated all of our subsequent models with first order terms only.

All regression coefficients for Equation 3.4.10 are significant at 99 percent confidence level. Appendix 2 contains the MINITAB printout for this model (Minitab, 1989; Ryan et al., 1985). This output lists the regression coefficients, their standard deviations, partial t-tests for significance, an analysis of variance (ANOVA) table, and a list of potential outliers. The standardized residual plots for this model are also given in Appendix 2.

In addition to evaluating $R^2$, a plot of $D_h$ versus $D_{h\text{me}}$ provides a simple way to view the predictive performance of this model (Figure 3-10). The solid "MEASURED = PREDICTED" line represents a perfect prediction line. Observations plotting near this line are closely approximated by the regression model. The dashed line below the "MEASURED = PREDICTED" line represents a 100 percent overprediction bound. Observations plotting below this line are being overpredicted by a factor of two or greater. The dashed line above the "MEASURED = PREDICTED" line is a 50 percent underprediction bound. Observations falling above this line are being underpredicted by a factor of two or greater. In summary, 92 percent (128 out of 135) of the displacements predicted by Equation 3.4.10 fall between these upper and lower prediction bounds.

3.5 Ground Slope Model for Niigata and Noshiro, Japan

Stepwise regression indicated that ground slope, $S$, is highly correlated with $D_h$ for ground slope failures in Niigata and Noshiro, Japan. In Niigata, lateral spread occurred on very gentle, uniform slopes ($S \leq 1$ percent); whereas, in Noshiro, lateral spread developed on undulating, dune deposits with slopes that are as steep as 5 percent in some locales. Because of the undulating topography found in Noshiro, we used slightly different techniques to measure $S$ for uniform and nonuniform slopes. Figure 3-11, Case 1 shows the technique we used to measure $S$ for the long, uniform slopes that were typical of Niigata. The value of $S$, (%) for these cases was calculated simply as:

$$S = 100\ Y/I$$  \hspace{1cm} (3.5.1)

However, in Noshiro, the amount of ground displacement was strongly influenced by undulations in the sand dunes. For example, Figure 3-12 shows that the ground displacement tended to mirror the topography, increasing near the steeper part of the undulating dune and decreasing in more gentle reaches. From the observed

\hspace{1cm} 3-14
As was discovered in developing the free face model, regression analyses of ground slope failures in Niigata and Noshiro indicate that a model comprised of \( \log D_h \) and \( \log S \) produces an approximately linear form (Figure 3-13):

\[
\log D_h = 0.430 + 0.442 \log S. \tag{3.5.2}
\]

The \( R^2 \) for this model is 42.1 percent. The regression coefficients for this model are significant at the 99.9 percent confidence level.
A slightly better fit ($R^2 = 45.6\%$) was obtained by forming the model: 

$$D_H = b_0 + b_1 \log S,$$

but like the free face model, a plot of $e_i(s)$ versus $D_{\text{fixed}}$ suggested a slight problem with nonconstant variance. Also, because we transformed $D_H$ to $\log D_H$ in developing the free face model, it was beneficial to maintain the same functional form in developing the ground slope model. This will allow us to combine both models into a single regression operation as the earthquake factors are brought into the analyses (for further discussion, see the next section).

As was discovered in developing the free face model, the variables $T_{IS}$, $D_{50,IS}$, and $F_{IS}$ also contribute to improving the performance of the ground slope model. Furthermore, like the free face model, standardized residuals from model 3.5.2 plotted against $T_{IS}$, $D_{50,IS}$, and $F_{IS}$ suggest that the relationships between displacement and these variables are approximately linear, thus these factors were also included in the ground slope model:

$$\log D_H = 0.698 + 0.378 \log S + 0.0362 T_{IS} - 0.0326 F_{IS} + 0.929 D_{50,IS}. \quad (3.5.3)$$

No additional geological and soil factors substantially improved this model, thus it was adopted as the final model for predicting ground slope failures in Niigata and Noshiro. The $R^2$ for Equation 3.5.3 is 54.2 percent and the intercept and regression coefficients are significant at the 99 percent confidence level (see MINITAB output in Appendix 2). A plot of $D_H$ versus $D_{\text{predicted}}$ for this model shows that 97 percent (224 out of 232) of the predicted displacements fall between the 100 percent overprediction and 50 percent underprediction bounds (Figure 3-14).

**Figure 3-13** Plot of log of displacement, $\log D_H$, versus log of ground slope, $\log S$, for ground slope failures indicating an approximately linear trend.

**Figure 3-14** Plot of measured displacements, $D_H$, versus predicted displacements, $D_{\text{predicted}}$, for Equation 3.5.3, ground slope failures, Niigata and Noshiro.
SECTION 4
COMBINED MLR MODEL FOR JAPANESE AND U.S. CASE HISTORIES

4.1 Earthquake Factors

The site-specific models developed for Niigata and Noshiro, Japan, were adjusted for a wider range of seismic and site conditions by including the U.S. data in the analyses. Youd and Perkins (1987), in developing the LSI model, proposed that displacement is a function of the amplitude, \( A \), and duration, \( D \), of strong ground motion.

\[
D_H = f(A, D)
\]  

(4.1.1)

where:
\( A \) = peak horizontal ground acceleration (in decimal fraction of g).
\( D \) = time interval between the first horizontal 0.05 g peak to the last 0.05 g peak recorded by a strong motion instrument (in seconds).

Unfortunately, strong motion records were not available for many of the lateral spread sites listed in Table 3-1. For these uninstrumented sites, \( A \) and \( D \) were estimated from empirical relationships based on earthquake magnitude, \( M \), and the log of the distance to the seismic energy source, \( \text{LOG} R \) (Joyner and Boore, 1988; Youd and Perkins, 1987; Krinitzsky and Chang, 1988b; Appendix I).

In addition to \( A \) and \( D \), Youd and Perkins showed that \( D_H \) is a function of \( M \) and attenuates logarithmically with increasing \( R \).

\[
D_H = f(M, \text{LOG} R)
\]  

(4.1.2)

where:
\( M \) = moment magnitude, \( M_w \).
\( R \) = horizontal distance from the seismic source (in km).

The moment magnitude, \( M_w \), is commonly used to represent \( M \) for these type of analyses because \( M_w \) is a better estimate of the amount of seismic energy released by a given earthquake than other measures of earthquake magnitude, especially for \( M > 8.0 \) events (Kanamori, 1978). Other earthquake magnitude measures such as the local magnitude, \( M_l \), and the surface wave magnitude, \( M_s \), are approximately equivalent to \( M_w \) for \( 6 \leq M \leq 8 \) earthquakes (Krinitzsky and Chang, 1988b).

The distance from the seismic source, \( R \), is measured as the horizontal distance from the site in question to the nearest point on a surface projection of the fault rupture zone. Epicentral distances may be adequate estimates of \( R \) for \( M \leq 6 \) earthquakes, but should not be used for larger earthquakes. Earthquakes with \( M > 6 \) are generally associated with large fault rupture zones that are not adequately characterized by a single point, such as the epicenter. Source zones for strike-slip and normal faults are usually delineated by a band that incorporates surface ruptures associated with recent (i.e., Holocene) faulting events. For these type of faults, which are common in the western U.S., source distances are measured horizontally from the nearest edge of the surface rupture zone to the site in question. For reverse faults, shallow-angle thrusts, and subduction-zone earthquakes, the associated zone of tectonic crustal uplift generally delineates the surface projection of the seismic source. For these type of faults, the source distance is measured from the nearest point of the tectonic uplift zone to the site in question.

Our preliminary regression analyses of the combined U.S. and Japanese data indicated that MLR models based on \( M \) and \( \text{LOG} R \) yield \( R^2 \) values that are about 10 to 15 percent higher than models based on \( A \) and \( D \). Thus, we chose to use \( M \) and \( \text{LOG} R \) in subsequent models. However, we do not wish to imply that \( M \) and \( \text{LOG} R \) are better measures of seismic energy than instrumentally obtained values of \( A \).
and D. Because A and D are more fundamental measures of the seismic energy delivered to a given site than M and LOG R, in general A-D models should yield equivalent, or slightly superior performance, when compared with M-LOG R models. Unfortunately, our MLR database contains many estimated values of A and D, and the poorer quality of these data appears to be hampering our ability to develop satisfactory A-D models.

The LSI model proposed by Yold and Perkins (1987) and the site-specific models developed in the previous section suggest that a more comprehensive MLR model(s) for predicting ground displacement should include, but not be restricted to the following factors:

\[ \text{LOG } D_h = f(M, \text{LOG } R, \text{LOG } W, \text{LOG } S, T_{15}, F_{10}, D_{50}. N_{1:0.0}) \]  

(4.1.3)

In developing preliminary MLR models from this function, we divided the MLR database into two databases, one for free face failures and one for ground slope failures and attempted fitting separate regression coefficients for M and LOG R for each type of failure. However, this attempt yielded unsatisfactory results. We concluded that the U.S. database does not contain a sufficient number of ground slope failures to independently adjust the ground slope model for the effects of M and LOG R. To overcome this limitation, we combined the free face and ground slope databases and formulated the MLR model to fit common earthquake regression coefficients for each type of failure. The same model was formulated to fit separate topographical, geological, and soil parameters for free face and ground slope failures.

\[ \text{LOG } D_h = f(M, \text{LOG } R, \text{LOG } W_{10}, T_{15}, F_{10}, D_{50}. N_{1:0.0}. \text{LOG } S_{10}, T_{15}, F_{10}, D_{50}. N_{1:0.0}) \]  

(4.1.4)

The subscripts ff and gs in Equation 4.1.4 indicate those variables that were assigned to the free face and ground slope components of the model, respectively. Inherent in this formulation is the assumption that M and LOG R influence free face failures in the same way that they influence ground slope failures. This appears to be a reasonable assumption because the amount of seismic energy delivered to a free face and a ground slope failure is the same for a particular seismic event and liquefaction locality. Given that we separately adjust each type of failure for the effects of topographical, geological and soil conditions (i.e., W, S, T, F, D50, and N1:0), it seems reasonable to fit common earthquake parameters for free face and ground slope failures.

Based on the function expressed in Equation 4.1.4, we formulated the following MLR model:

\[ \text{LOG}(D_h + 0.01) = b_0 + b_{w0} + b_1 M + b_2 \text{LOG } R + b_3 \text{LOG } W_{10} + b_4 T_{15} + b_5 F_{10} + b_6 D_{50} + b_7 N_{1:0.0} + b_{w0} \text{LOG } S_{10} + b_{w1} T_{15} + b_{w2} F_{10} + b_{w3} D_{50} \]  

(4.1.5)

The fitted parameter \( b_3 \) is the intercept of the combined free face and ground slope components of the model. The regression coefficient \( b_{w0} \) is used to adjust \( b_3 \) for any difference that may exist between the intercepts of the free face and ground slope components (i.e., the intercept for the free face component of the model is calculated by adding \( b_3 \) and \( b_{w0} \)). Because \( \text{LOG}(0) \) is undefined, we expediently added 0.01 m to all values of \( D_h \) prior to performing the regression. This expedient enabled us to calculate \( \text{LOG}(D_h) \) for the zero displacement observations that are included in our MLR database.

A least squares fit of Equation 4.1.5 yields these regression coefficients: \( b_0 = -5.085, b_{w0} = -0.559, b_1 = 0.976, b_2 = -1.053, b_3 = 0.693, b_4 = 0.0272, b_5 = -0.0328, b_6 = -1.124, b_7 = -0.0118, b_8 = 0.356, b_9 = 0.0403, b_{10} = -0.0336, b_{11} = -1.535 \). The \( R^2 \) for this equation is 74.9 percent and all regression coefficients, except for \( b_7 \), are significant at the 99 percent confidence level. The fitted value for \( b_7 \) is significant at the 92 percent confidence level. Figure 4-1 shows that 89 percent (i.e., 399 out of 448) of the predicted displacement values fall
between the 100 percent overprediction and 50 percent underprediction bounds. The free face component of equation 4.1.5 is:

\[
\log(D_a + 0.01) = -5.644 + 0.976 M - 1.053 \log R + 0.693 \log W + 0.0272 T_{1S}
- 0.0028 F_{1S} - 1.124 D50_{1S} - 0.0118 N_{les}
\]  

(4.1.5a)

and the ground slope component is:

\[
\log(D_a + 0.01) = -5.085 + 0.976 M - 1.053 \log R + 0.356 \log S + 0.0403 T_{1S}
- 0.0336 F_{1S} - 1.535 D50_{1S}
\]  

(4.1.5b)

Based on our data, Equation 4.1.5 appears to be performing reasonably well for \( M = 6.5 \) to 7.5 earthquakes and for liquefied sites within a 30 km radius of the seismic source. However, a comparison with data published by Ambraseys (1988) shows that Equation 4.1.5 will tend to overpredict \( D_a \) at liquefied sites with \( R > 30 \) km. In his study, Ambraseys compiled values of \( M_a \) and the farthest distance to observed liquefaction effects, \( R_f \), (km), for several earthquakes and bounded these data with the equation:
\[ M = 0.18 + 9.2 \times 10^8 R_e + 0.90 \log R_e. \]

(4.1.6)

(See solid, curved bound shown on Figure 4-2). Ambraseys' study suggests that liquefaction (i.e., ground displacement, fissures, sand boils, etc.) are almost always located within this bound and for \( R > R_e \), liquefaction effects are usually not observed. We used Equation 4.1.5 to back-calculate the distance, \( R \), corresponding to the inception of lateral spread by using \( D_H = 0.05 \) m and mean values of \( W, S, T_{15r}, F_{15r}, D50_{15r}, N1_{15r} \) from our MLR database (Table 4-1). Figure 4-2 shows the results superimposed upon Ambraseys' data and \( R_e \) bound. This plot shows that the free face component of Equation 4.1.5 continues to predict ground displacement beyond Ambraseys' \( R_e \) bound beginning at \( M > 6.25 \) and \( R > 30 \) km and the ground slope component of Equation 4.1.5 does likewise for \( M > 7.0 \) and \( R > 70 \) km. Because Equation 4.1.5 does not correctly attenuate \( D_H \) with increasing \( R \), we concluded that it should not be applied at sites with \( R > 30 \) km. This is a serious limitation to the application of Equation 4.1.5, especially for evaluating large earthquakes that typically produce significant lateral spread displacement beyond 30 km. Unfortunately, most of our case history sites are from \( R \leq 30 \) km; thus, there is very limited information for adjusting Equation 4.1.5 based solely on our compiled data.

The data from Ambraseys' study, however, offer a means of adjusting Equation 4.1.5 so that it more properly attenuates \( \log D_H \) as a function of \( R \). To this end, we included 19 observations from Ambraseys' study (Table 4-2) in our analysis to strengthen the MLR database for \( R > 30 \) km. Because the majority of our case history sites are from earthquakes with \( 6.4 \leq M \leq 6.6 \) and \( 7.4 \leq M \leq 7.8 \), we selected only those observations from Ambraseys' study that fall within these same ranges. Also, prior to incorporating Ambraseys' data into the regression analysis, we needed reasonable estimates of the topographical, geological, and soil conditions at these sites. Because these factors were not compiled by Ambraseys, we decided to use average values of \( \log W, \log S, T_{15r}, F_{15r}, D50_{15r}, N1_{15r} \) from our database to approximate average site conditions at Ambraseys' sites (Table 4-1). In addition, because Ambraseys' sites represent the maximum \( R \) to observable liquefaction effects, we assumed that a minimal amount of lateral spread occurred at these localities and assigned \( D_H = 0.05 \) m to the observations listed in Table 4-2. Also, these observations were randomly assigned to either the free face or ground slope component of our MLR model prior to performing the regression analyses.

The functional form of Ambraseys' equation suggests that, in addition to the earthquake factors, \( M \) and \( \log R \), we need to include a \( R \) term in Equation 4.1.5. Therefore, we postulate that:

\[
\begin{align*}
\log(D_H+0.01) &= b_0 + b_{30} + b_1 M + b_2 \log R + b_3 R + b_4 \log W_H + b_5 T_{15r} + b_6 F_{15r} + b_7 D50_{15r} \\
&+ b_8 N1_{15r} + b_9 \log(S)_H + b_{10} T_{15p} + b_{11} F_{15p} + b_{12} D50_{15p}
\end{align*}
\]

(4.1.7)
A regression of this equation yields the following coefficients: $b_0 = -6.086$, $b_{\text{off}} = -0.483$, $b_1 = 1.106$, $b_2 = -0.978$, $b_3 = -0.0101$, $b_4 = 0.703$, $b_5 = -0.0308$, $b_6 = -0.983$, $b_7 = -0.0118$, $b_8 = 0.373$, $b_9 = 0.0384$, $b_{10} = -0.0304$, $b_{11} = -1.096$. All regression coefficients are significant at the 99 percent confidence level, except for $b_9$ which is only significant at the 93 percent confidence level, respectively. The $R^2$ for this equation is 83.6 percent.

Once again we used Equation 4.1.7 to back-calculate $R$ for the inception of lateral spread by inputting $D_h = 0.05$ m and using the average site conditions listed in Table 4-1. Figure 4-3 shows the results compared with Ambraseys' data and $R_h$ bound. The functional form of Equation 4.1.7 mimics Ambraseys' $R_h$ bound quite well and the free face and ground slope components of the model provide a reasonable fit to Ambraseys' data. Thus, we concluded that the functional form of Equation 4.1.7 appears to more correctly attenuate $D_h$ as a function of $M$ and $R$ than Equation 4.1.5.

A further examination of Equation 4.1.7 shows that $b_3 = b_{\text{off}}$, and $b_6 = b_{10}$, and $b_7 = b_{11}$, suggesting that common regression coefficients can be fitted for $T_{15\sigma}$ and $T_{15\sigma}^d$, and for $F_{15\sigma}$ and $F_{15\sigma}^d$, and for $D_{50_{15\sigma}}$ and $D_{50_{15\sigma}}$. Also, because the regression coefficient for $N_{15\sigma}$, i.e., $b_8$, is significant only at the 93 percent confidence level, it was dropped from the analysis. Hence, we simplified the model to:

$$\text{LOG}(D_h + 0.01) = b_0 + b_{\text{off}} + b_1 M + b_2 \text{LOG} R + b_3 \text{LOG} W_{15} + b_4 \text{LOG} S_{\mu} + b_5 T_{15} + b_6 F_{15} + b_7 D_{50\sigma}$$ \hspace{1cm} (4.1.8)

After fitting Equation 4.1.8, we performed a sensitivity analysis and found that the transformation of $T_{15}$ to LOG $T_{15}$ and the transformation of $F_{15}$ to LOG $(100-F_{15})$ yielded predicted displacements that are more credible for small values of $T_{15}$ and $F_{15}$. Thus, we modified the model to:

$$\text{LOG}(D_h + 0.01) = b_0 + b_{\text{off}} + b_1 M + b_2 \text{LOG} R + b_3 \text{LOG} W_{15} + b_4 \text{LOG} S_{\mu} + b_5 T_{15} + b_6 F_{15} + b_7 D_{50\sigma}$$ \hspace{1cm} (4.1.9)

A least squares fit of Equation 4.1.9 yields the following regression coefficients: $b_0 = -15.787$, $b_{\text{off}} = -0.579$, $b_1 = 1.178$, $b_2 = -0.927$, $b_3 = -0.013$, $b_4 = 0.657$, $b_5 = 0.429$, $b_6 = 0.348$, $b_7 = 4.527$, $b_8 = -0.922$. All coefficients are significant at the 99.9 percent confidence level and the $R^2$ for Equation 4.1.9 is 82.6 percent. Figure 4-4 shows that 90 percent (421 out of 467) of the predicted displacement values, $D_{\text{inhar}}$, fall between the 100 percent overprediction and 50 percent underprediction bounds. The free face component of Equation 4.1.9 is:

$$\text{LOG}(D_h + 0.01) = -16.366 + 1.178 M - 0.927 \text{LOG} R - 0.013 R$$
$$+ 0.657 \text{LOG} W + 0.348 \text{LOG} T_{15} + 4.527 \text{LOG}(100-F_{15}) - 0.922 D_{50\sigma}$$ \hspace{1cm} (4.1.9a)
and the ground slope component is:

\[ \text{LOG}(D_h + 0.01) = -15.787 + 1.178 M - 0.927 \text{LOG} R - 0.013 R \\
+ 0.429 \text{LOG} S + 0.348 \text{LOG} T_{15} + 4.527 \text{LOG}(100 - P_{15}) - 0.922 D_{50} \]

\[ (4.1.9b) \]

![Figure 4-4 Plot of measured displacements, $D_h$, versus predicted displacements, $D_{\text{hat}}$, for Equation 4.1.9 using Japanese, U.S., and Ambroseys' data.](image)

After fitting this model, we re-examined all independent variables listed in Table 3-2 for any linear trends that might greatly improve the performance of the model and found none. Thus, this is our final MLR model.

Although $D_{\text{hat}}$ is an estimate of the average displacement, $D_h$, for a set of inputted $X(s)$, it is often desirable for engineering purposes to determine an upper bound or limit to the value of $D_h$ that can be reasonably expected at a given site. Figure 4-4 shows that most values of $D_{\text{hat}}$ predicted from the model fall below the "MEASURED = 2 X PREDICTED LINE." This suggests that if $D_{\text{hat}}$ is increased by a factor of 2, then this result provides a conservative estimate of $D_h$ that is not likely to be greatly exceeded. Also, because the relationship between $D_h$ and the $X(s)$ may be strongly nonlinear outside the ranges of the $X(s)$ used in developing the model, extrapolation of Equation 4.1.9 may yield less reliable predictions. In short, it appears that this equation yields good results for $6.0 \leq M \leq 8.0$ earthquakes and at sites underlain by continuous layers of sands and silty sands having $N_{15} \leq 15; 0.075 \leq D_{50} \leq 1.0$ mm, $0 \leq P_{15} \leq 50 \%$, $1 \leq T_{15} \leq 15$ m, $Z_{\text{RLS}} \leq 20$ m, $1 \leq W \leq 20 \%$, and $0.1 \leq S \leq 6 \%$ (See Table 3-2 for definitions of these factors and Section 5 for a discussion of their...
application.) Also, because this model was developed from Japanese and western U.S. data, it is most applicable to regions having high to moderate ground motion attenuation. Extrapolation of the model beyond these conditions may be warranted in some cases, if the inputted factors are reasonably close to these ranges and the extrapolation is deemed to yield conservative results (i.e., overly-predicted estimates of $D_n$). This model should not be applied to metastable soils (e.g., loess deposits, sensitive clays, and collapsible silts). These metastable soils were not analyzed by this study and may produce large ground displacements, or even flow failure.
### TABLE 4-1
AVERAGE SITE CONDITIONS FOR CASE STUDIES TABULATED BY EARTHQUAKE

<table>
<thead>
<tr>
<th>Earthquake</th>
<th>LOG W&lt;sub&gt;ω&lt;/sub&gt;</th>
<th>LOG S&lt;sub&gt;ω&lt;/sub&gt;</th>
<th>T&lt;sub&gt;1/5&lt;/sub&gt;</th>
<th>F&lt;sub&gt;1/5&lt;/sub&gt;</th>
<th>D50&lt;sub&gt;1/5&lt;/sub&gt;</th>
<th>N1&lt;sub&gt;surf&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>1906 San Francisco</td>
<td>1.2981</td>
<td>-0.1549</td>
<td>4.6</td>
<td>18</td>
<td>0.227</td>
<td>6.4</td>
</tr>
<tr>
<td>1964 Alaska</td>
<td>1.3832</td>
<td>-1.0969</td>
<td>8.9</td>
<td>32</td>
<td>0.828</td>
<td>9.9</td>
</tr>
<tr>
<td>1964 Niigata</td>
<td>0.9244</td>
<td>-0.3188</td>
<td>9.5</td>
<td>10</td>
<td>0.311</td>
<td>4.8</td>
</tr>
<tr>
<td>1971 San Fernando</td>
<td>1.1427</td>
<td>0.0899</td>
<td>3.9</td>
<td>50</td>
<td>0.076</td>
<td>8.0</td>
</tr>
<tr>
<td>1979 Imperial Valley</td>
<td>0.8244</td>
<td>-0.2518</td>
<td>2.7</td>
<td>27</td>
<td>0.106</td>
<td>4.3</td>
</tr>
<tr>
<td>1983 Nihonkai-Chubu</td>
<td>—</td>
<td>0.1847</td>
<td>2.1</td>
<td>1</td>
<td>0.350</td>
<td>—</td>
</tr>
<tr>
<td>1987 Superstition Hills</td>
<td>1.3642</td>
<td>—</td>
<td>3.0</td>
<td>33</td>
<td>0.081</td>
<td>4.0</td>
</tr>
<tr>
<td>mean</td>
<td>1.1562</td>
<td>-0.2580</td>
<td>5.0</td>
<td>24</td>
<td>0.283</td>
<td>6.2</td>
</tr>
</tbody>
</table>

1 The combined free face and ground slope data were used to calculate the means for T<sub>1/5</sub>, F<sub>1/5</sub>, and D50<sub>1/5</sub>.

### TABLE 4-2
OBSERVATIONS FROM AMBRASEYS' STUDY USED TO ADJUST MLR MODELS

<table>
<thead>
<tr>
<th>M&lt;sup&gt;1&lt;/sup&gt;</th>
<th>R&lt;sup&gt;1&lt;/sup&gt;</th>
<th>LOG W&lt;sub&gt;ω&lt;/sub&gt;</th>
<th>LOG S&lt;sub&gt;ω&lt;/sub&gt;</th>
<th>T&lt;sub&gt;1/5&lt;/sub&gt;</th>
<th>F&lt;sub&gt;1/5&lt;/sub&gt;</th>
<th>D50&lt;sub&gt;1/5&lt;/sub&gt;</th>
<th>N1&lt;sub&gt;surf&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.8</td>
<td>80</td>
<td>1.1562</td>
<td>—</td>
<td>5.0</td>
<td>24</td>
<td>0.283</td>
<td>6.2</td>
</tr>
<tr>
<td>7.5</td>
<td>77</td>
<td>—</td>
<td>-0.2580</td>
<td>5.0</td>
<td>24</td>
<td>0.283</td>
<td>—</td>
</tr>
<tr>
<td>6.5</td>
<td>28</td>
<td>1.1562</td>
<td>—</td>
<td>5.0</td>
<td>24</td>
<td>0.283</td>
<td>6.2</td>
</tr>
<tr>
<td>6.4</td>
<td>15</td>
<td>—</td>
<td>-0.2580</td>
<td>5.0</td>
<td>24</td>
<td>0.283</td>
<td>—</td>
</tr>
<tr>
<td>7.4</td>
<td>80</td>
<td>1.1562</td>
<td>—</td>
<td>5.0</td>
<td>24</td>
<td>0.283</td>
<td>6.2</td>
</tr>
<tr>
<td>6.6</td>
<td>18</td>
<td>—</td>
<td>-0.2580</td>
<td>5.0</td>
<td>24</td>
<td>0.283</td>
<td>—</td>
</tr>
<tr>
<td>7.6</td>
<td>92</td>
<td>1.1562</td>
<td>—</td>
<td>5.0</td>
<td>24</td>
<td>0.283</td>
<td>6.2</td>
</tr>
<tr>
<td>7.7</td>
<td>85</td>
<td>—</td>
<td>-0.2580</td>
<td>5.0</td>
<td>24</td>
<td>0.283</td>
<td>—</td>
</tr>
<tr>
<td>7.5</td>
<td>72</td>
<td>—</td>
<td>-0.2580</td>
<td>5.0</td>
<td>24</td>
<td>0.283</td>
<td>—</td>
</tr>
<tr>
<td>6.4</td>
<td>17</td>
<td>—</td>
<td>-0.2580</td>
<td>5.0</td>
<td>24</td>
<td>0.283</td>
<td>—</td>
</tr>
<tr>
<td>6.6</td>
<td>37</td>
<td>—</td>
<td>-0.2580</td>
<td>5.0</td>
<td>24</td>
<td>0.283</td>
<td>—</td>
</tr>
<tr>
<td>6.6</td>
<td>21</td>
<td>1.1562</td>
<td>—</td>
<td>5.0</td>
<td>24</td>
<td>0.283</td>
<td>6.2</td>
</tr>
<tr>
<td>6.5</td>
<td>30</td>
<td>1.1562</td>
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<td>5.0</td>
<td>24</td>
<td>0.283</td>
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<td>-0.2580</td>
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<tr>
<td>7.6</td>
<td>95</td>
<td>1.1562</td>
<td>—</td>
<td>5.0</td>
<td>24</td>
<td>0.283</td>
<td>6.2</td>
</tr>
</tbody>
</table>

1 M and LOG R values are from Ambraseys' case studies and values for LOG W, LOG S, T<sub>1/5</sub>, F<sub>1/5</sub>, D50<sub>1/5</sub>, and N1<sub>surf</sub> are averages from our MLR database (see Table 4-1).

2 The same averages for T<sub>1/5</sub>, F<sub>1/5</sub>, and D50<sub>1/5</sub> were used in both the free face and ground slope components of the regression model.
SECTION 5
APPLICATION AND LIMITATIONS OF MLR MODEL

Figures 5-1 to 5-3 are histograms of the independent variables used in developing Equation 4.1.9. These graphs provide a general guide to the range of conditions for which this equation is applicable. This section further discusses the application and limitations of the MLR model.

5.1 Ground Motion Attenuation

Equation 4.1.9 was developed mainly from stiff-soil sites in the western U.S. and from stiff soil sites in Japan that were within 30 km of the seismic source. For these regions and conditions, Equation 4.1.9 should be directly applied to predict lateral spread displacement. For other regions of the world, such as the eastern U.S. where ground motion attenuates more slowly with distance, and for other site conditions, such as liquefiable deposits over soft clay layers where ground motion may be strongly amplified (i.e., soft soil sites), a correction must be applied to Equation 4.1.9 to account for these different seismic and site conditions. The preferred method to adjust the model would be to directly regress $D_n$ on the earthquake factors, $M$ and $R$. However, because $A$ was measured only at a few of the case history sites, the direct development of a $M$--$A$ model was not possible with the limited data. We attempted to estimate $A$ from empirical $M$--$R$ relationships (Appendix 1, Section A1.1.3) and use those estimates to develop a $M$--$A$ model, but this attempt yielded poorer results than models based on $M$, LOG $R$, and $A$.

Until better case histories are assembled to adequately develop $M$--$A$ and $A$--$D$ models, we propose the following procedure to adjust the values of $R$ that are used into Equation 4.1.9 for other regions of the world or for soft soil sites. Figure 5-4 is a plot of $A$ estimated for stiff soil sites in the western U.S. using attenuation relationships and soil amplification factors published by Idriss (Idriss, in press; Idriss, 1990; Equation A1.1.3.2). These accelerations should roughly represent those incurred at our case history sites, which are primarily from stiff soil sites in the western U.S. and Japanese sites found within 30 km of the seismic source. The values of $A$ plotted on this figure were calculated for their respective values of $M$ and $R$ by applying a peak-acceleration attenuation equation developed by Idriss for bedrock sites and then correcting those bedrock accelerations for stiff soil conditions. To adjust the bedrock accelerations to stiff soil conditions, we multiplied the peak bedrock accelerations by a correction factor that was estimated from an acceleration amplification curve for soft soils published by Idriss (1990) (Figure 5-5). The stiff soil acceleration curve was approximated by fitting a series of points that were positioned midway between the non-amplification curve for rock (i.e., 45 degree line) and the high-amplification curve for soft soils (Figure 5-5). The procedure for using the curves shown in Figure 5-4 to correct the R inputted into Equation 4.1.9 for soft soil sites or non-western U.S. or non-Japanese sites is as follows. (1) Using standard procedures, the design earthquake magnitude, $M$, and peak ground acceleration, $A$, are determined for the candidate site. (2) That magnitude and acceleration are then plotted on Figure 5-4. (3) From that plotted point, an equivalent source distance, $R_{eq}$, is interpolated from the R-curves given in Figure 5-4. (4) That $R_{eq}$ is then entered into Equation 4.1.9 instead of the actual $R$ to calculate $D_n$. For example, during the 1989 Loma Prieta, California earthquake ($M_w = 6.9$), liquefaction and minor lateral spreading occurred on Treasure Island, at a distance of about 80 km from the seismic energy source. Application of that distance in Equation 4.1.9 along with appropriate site properties indicates that an insignificant amount of displacement should have occurred on the island. (This site also falls outside of Ambroseys' $R_n$ bound, suggesting that liquefaction should have not occurred.) However, considerable ground motion amplification was measured at Treasure Island, which was constructed by placing hydraulic fill over thick deposits of soft, San Francisco Bay mud.
Figure 5-1 Histograms for (1) displacement, $D_H$, (2) magnitude, $M$, (3) distance to seismic energy source, $R$, (4) peak ground acceleration, $A$. 
Figure 5-2 Histograms for (1) duration, D, (2) free face ratio, W, (3) ground slope, S, and, (4) thickness with $N_{10} < 15$, $T_{15}$. 

5-3
Figure 5-3 Histograms for (1) fines content, $F_{15}$, (2) mean grain size, $D_{50_{15}}$, (3) depth to low FS, $Z_s$, (4) lowest $N_{160s}$. 

5-4
Although maximum bedrock accelerations measured just a few hundred meters away on nearby Yerba Buena Island were roughly 0.07 g, and accelerations measured on nearby stiff soil sites were generally about 0.10 g, the instrumented value of $A$ on Treasure Island was 0.16 g. Thus, this measured $A$ was more than twice the bedrock acceleration and was also significantly higher than that expected for stiff soil sites at that distance. However, if $M = 6.9$ and $A = 0.16$ is plotted on Figure 5-4, the resulting $R_{eq}$ is about 50 km (compared to the actual source distance of 80 km). Entering a $R_{eq}$ of 50 km into Equation 4.1.9 with the appropriate soil properties predicts that a few tenths of a meter of lateral spread displacement should occur near the free face edges of the island. This prediction roughly corresponds with that measured on Treasure Island after the earthquake.

5.2 Earthquake Magnitude

The bulk of our data are from $6 \leq M \leq 8$ earthquakes and extrapolation of Equation 4.1.9 beyond this range increases the uncertainty in the predicted displacements (Figure 5-1). However, because lateral spread displacement appears to decrease markedly for $M < 6$ earthquakes, extrapolation of Equation 4.1.9 to $M < 6$ earthquakes appears to yield predicted displacements, which with conservative allowance for the greater uncertainty, appear to be usable for engineering analyses. Extrapolation of the equation to earthquakes with $M > 8$ also appears to give reasonable predictions for fine to coarse grain sands and silty sands based on the limited data available from extremely large earthquakes. (Seven
observations from the 1964 Alaska earthquake (M = 9.2) were fitted and an examination of the \( e_i(s) \) for these data shows no unusual residual behavior (see \( e_i(s) \) versus M plot in Appendix 2 for Equation 4.1.9). Nonetheless, the addition of more case studies for \( M > 8 \) earthquakes would strengthen the MLR database and improve its reliability for extremely large events.

5.3 Distance to the Fault Rupture or Zone of Seismic Energy Release

Equation 4.1.9 appears to attenuate \( D_M \) with increasing \( R \) in a manner that is consistent with our case history data and with Ambraseys' \( R \) bound. On the other hand, if the inputted \( R \) is allowed to decrease to a distance that is approaching zero, the predicted displacements from Equation 4.1.9 can become quite large, especially for \( M > 7.5 \). Based on a sensitivity analysis, in which we allowed \( M \) to vary from 6.5 to 9.5 and using the average site conditions given in Table 4-1, we noted that \( D_{\text{max}} \) becomes unreasonably large (e.g., larger than 5 to 10 m) when the inputted \( R \) is allowed to decrease below the values listed in Table 5-1. Because Equation 4.1.9 appears to yield predicted displacements that are larger than those normally expected for lateral spread and because only a few case histories are available for lateral spreads located very near the seismic source, extrapolation of Equation 4.1.9 to distances less than those listed in Table 5-1 may yield unreliable estimates of \( D_M \).
<table>
<thead>
<tr>
<th>M</th>
<th>R (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.5</td>
<td>0.25</td>
</tr>
<tr>
<td>7.0</td>
<td>1</td>
</tr>
<tr>
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<td>8.5</td>
<td>25</td>
</tr>
<tr>
<td>9.0</td>
<td>50</td>
</tr>
</tbody>
</table>

5.4 Free Face Ratio and Ground Slope

Most of the free face failures analyzed by this study are for $W \leq 20$ percent and caution should be used when extrapolating the MLR model beyond this value. However, some extrapolation may be warranted at sites where the liquefiable sediments are not deemed to be prone to slumping or flow failure. For example, an important problem for engineers is the estimation of the potential lateral spread displacement near bridge crossings where $W$ typically exceeds 20 percent. In this study, six observations having $20 < W \leq 55$ percent were fitted and Equation 4.1.9a appears to yield credible predictions for this range. Nonetheless, field observations of free face failures along river channels reveals that the displacement may have a significant vertical component due to rotation of slump blocks. Also, gravitational shear forces near the free face may be large enough to induce flow failure in highly susceptible soils. If slumping or flow failure is a potential concern, Equation 4.1.9 is not applicable and more sophisticated 2-D models, such as dynamic, finite-element analyses should be used (Prevost, 1981; Finn and Yogendrakumar, 1989).

In formulating Equation 4.1.9a, we attempted a MLR model that included both $W$ and $S$ as topographical factors, but our analyses suggested that the inclusion of $S$ does not significantly improve the performance of the free face model. Thus, we concluded that the slope of the river banks, either into or away from the channel, does not vary enough to have markedly affect displacement when compared with the influence exerted by $W$. However, most of the free face failures in our database had values of $S < 0.5$ percent. If additional conservatism is desired at sites where $S > 0.5$ percent, the results from the ground slope component, (i.e., Equation 4.1.9b) could be added to the results of the free face component, (i.e., Equation 4.1.9a); but in most cases, we do not believe that this degree of conservatism is required.

In applying Equation 4.1.9 to sites which are farther removed from the free face, questions may arise about whether to apply Equation 4.1.9a or Equation 4.1.9b. In highly liquefiable sediments, like those found in Niigata, movement of the river banks towards the channel initiated at a maximum distance of 100 times the height of the channel (i.e., 100 $H$). Thus, in highly susceptible soils, the free face appears to influence $D_n$ for values of $W \geq 1$ percent. (Note that 100 $H$ is equal to a $W$ value of 1 percent). However, we do not recommend the exclusive use of the Equation 4.1.9a at all sites with $W \geq 1$ percent. Figure 3-4 shows that at some places in Niigata, free face failure was not initiated until $W$ was about 5 percent. Thus, for sites with $1 < W \leq 5$ percent, it is possible that $D_n$ may be also influenced by $S$, and the ground slope model may be just as applicable as the free face model. For ambiguous cases, we suggest estimating $D_n$ from both Equation 4.1.9a and 4.1.9b and applying the larger value for design purposes. As previously mentioned, if the designer believes that both the free face and ground slope will contribute to produce displacement, then both components of Equation 4.1.9 could be added to produce a conservative estimate of $D_n$. 

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5-7
For the ground slope failures evaluated herein, $S$ ranged from about 0.1 percent to 6 percent. Extrapolation of Equation 4.1.9b beyond this range may lead to uncertain predictions. For $S < 0.1$ percent, chaotic displacements due to ground oscillation are likely to exceed those from lateral spreading and Equation 4.1.9b may give uncertain estimates of $D_{H}$ for flat ground conditions. Also, ground slopes that exceed 6 percent may cause flow failure in highly susceptible soils and consequently produce large displacements. Equation 4.1.9b is not valid for estimating $D_{H}$ for such conditions.

5.5 Gravelly Soils

During preliminary analyses, we noted that our MLR models performed poorly at predicting lateral spread displacements measured at gravelly sites from the 1964 Alaska and 1983 Borah Peak, Idaho earthquakes. Due to the high number of outliers for gravelly sites, it appears that gravel has a different displacement behavior than sand and silt. For example, Figure 5-6 shows the $e_{i}$'s plotted against $D_{50g}$ for one of our preliminary MLR models. Four of 6 observations with $D_{50g}$ values > 2 mm are potential outliers. These outliers are from alluvial gravels that underwent lateral spread at Whiskey Spring and Pence Ranch during the 1983 Borah Peak, Idaho, earthquake (Andrus and Yould, 1987).

Because our data contains only a few examples of lateral spread at gravelly sites, we did not have an adequate number of observations to properly fit the displacement behavior of gravelly sediments. Thus, we removed observations with $D_{50g} > 2$ mm from the MLR database prior to fitting Equation 4.1.9. (Case histories at Whiskey Springs, Idaho; Pence Ranch, Idaho; and some gravelly sites in Alaska were removed). Figure 5-6 also shows that there are very few observations for $1 < D_{50g} \leq 2$ mm. Consequently, for verified predictions, we restrict the use of Equation 4.1.9 to saturated cohesionless sediments with $D_{50g}$ values ≤ 1 mm.

5.6 Fines Content and Layered Profiles

In addition to $D_{50g}$, there are limits on the range of the average fines content, $F_{IS}$, for which Equation 4.1.9 has been verified. Figure 5-3 shows that most of $F_{IS}$ values in the MLR database are from soils with $F_{IS}$ values ≤ 50 percent; thus, we limit the use of Equation 4.1.9 to this range. Also, because $F_{IS}$ is strongly correlated with $D_{50g}$, there are limits on the allowable range for the combination of these factors. Figure 5-7 is a plot of $F_{IS}$ versus $D_{50g}$ for the 267 boreholes included in this study. This plot shows the envelope of $F_{IS}$ and $D_{50g}$ values for which Equation 4.1.9 has been verified. Extrapolation of the model to soils with textures beyond these limits introduces extra uncertainty into the predicted displacement.

Figure 5-6 Standardized residuals, $e_{i}$'s, plotted against the average mean grain size, $D_{50g}$, in $T_{16}$, showing outliers in gravelly soils.
Equation 4.1.9 suggests that fines content has a major influence on lateral spread displacement. All other factors remaining constant, predicted displacements diminish rapidly with increasing fines content. For example, lateral spread at Juvenile Hall during the 1971 San Fernando Valley, California earthquake illustrates the marked affect that the inputted fines content has on predicted values of displacement. Lateral spread at Juvenile Hall ranged from 0.5 to 1.68 m and occurred on a gentle ground slope ($S = 1.2\%$) (Bennett et al., 1989; Youd, 1973b). Liquefaction appears to have occurred primarily in a sandy silt (ML) layer that is interbedded with thinner, silty sand (SM) layers. The $T_s$ layer at Juvenile Hall has an average fines content of about 59 percent and an average mean grain size of 0.06 mm. Equation 4.1.9 predicts a maximum displacement of about 0.7 m for Juvenile Hall, which underestimates the maximum observed value by a factor slightly greater than 2 (Figure 5-8). However, further examination of the borehole data and watertable elevations taken soon after the earthquake suggests that a relatively thin, continuous silty sand (SM) layer may have liquefied just below the watertable. If this SM layer is analyzed separately, and not averaged with the thicker, underlying ML layer, the inputted factors are: $T_s = 0.6$ m, $F_s = 41\%$, $D_{50_s} = 0.131$ mm and Equation 4.1.9b predicts an average displacement of about 1.8 m for this soil, which is in good agreement with the observed displacement. Thus, in analyzing a layered system with two potentially liquefiable layers that have distinctly different textures, averaging $F_s$ and $D_{50_s}$ throughout the entire $T_s$ layer may produce smaller predicted displacements than if the individual layers are analyzed separately. This is especially true if the thickest layer has a high fines content. Hence, for
conservative design in layered profiles, we recommend that distinctly different \( T_H \) layers be analyzed separately by calculating \( T_H \), \( F_H \), and \( D50_H \) for each distinct soil type. The total predicted displacement may then be conservatively estimated as the sum of the displacements predicted for the individual layers. However, to divide the profile into individual layers, there must be a distinct textural difference between the layers.

Also, Figure 5-8 shows that a small number of Japanese observations are underpredicted by a factor greater than 2. The poorer quality of the subsurface data available for these observation errors in the measured displacements at these locales (Section 3.2) may be the reason for the slight underprediction of these smaller displacements.

5.7 Soils with \((N1)_w\) Values Greater than 15

In almost all cases reviewed herein, significant ground displacement was restricted to saturated cohesionless soils having \((N1)_w\) values \( \leq 15 \). This finding does not appear to be coincidental, nor does it appear to represent a deficiency in our MLR database. The case studies reviewed herein do include boreholes where all \((N1)_w\) values in the profile exceed 15 (e.g., boreholes from Juvenile Hall, Heber Road, Niigata, and Noshiro, Japan), but these boreholes were generally located near the margins of the lateral spreads where no appreciable amount of displacement was reported. Thus, in general, cohesionless materials with \((N1)_w\) values > 15 appear to be resistant to lateral spread displacement for
M < 8 earthquakes and we limit the application of Equation 4.1.9 to saturated, cohesionless soils having \((N_l)_{o_0}\) values ≤ 15.

However, it is possible during the 1964 Alaska earthquake that fluvioglacial deposits with \((N_l)_{o_0}\) values > 15 underwent lateral spread. During this extremely large and long-lived earthquake (M = 9.2), gravelly, channel deposits (15 ≤ \((N_l)_{o_0}\) ≤ 20) displaced a maximum of 1 m at the Resurrection River and Placer Rivers (Bartlett and Youd, 1992). However, the quality of the subsurface data for these Alaskan sites is poor. The penetration tests at the Resurrection River were performed with a non-standard hammer and have questionable validity. Also, N values recorded in gravelly soils generally tend to be higher when compared with finer grained sediments of comparable relative densities. Thus, we found no conclusive evidence of significant displacement in sediments with \((N_l)_{o_0}\) values > 15 for M > 8 earthquakes.

5.8 Thickness and Depth of the Liquefiable Layer

Prior to applying Equation 4.1.9, however, standard liquefaction analyses (Seed and Idriss, 1971; Seed et al., 1983; 1985; NRC, 1985) should be performed to verify that liquefaction is expected in the layer for the inputted earthquake factors (Figure 5-9). Based on the compiled case history data, it appears that lateral spread occurs in relatively thick, \(T_{us}\) layers (\(T_{us}\) values for our database average 5.5 m (Table 4-1)). In general, \(T_{us}\) is thicker than 1 m at locales that underwent a significant amount of lateral spread. In a few instances, however, small displacements occurred along the margins of some lateral spreads where the thickness of the \(T_{us}\) layer appears to be less than 1 m. Thus, for conservative design, we suggest that continuous, \(T_{us}\) layers having a thickness less than 1 m be considered as potential candidates for lateral spread. However, because our MLR database contains only a few cases of \(T_{us} < 1 \text{ m}\), Equation 4.1.9 may yield less reliable results for these conditions. Equation 4.1.9 should not be applied at sites having thin, noncontinuous, \(T_{us}\) layers. The researched database is deficient for such conditions.

Our liquefaction analyses also suggest that the depth to the top of the liquefied layer, \(Z_{TLS}\), is usually found within the upper 10 m of the profile and that the depth to the bottom of the liquefied layer, \(Z_{MLS}\), is usually found within the upper 20 m of profile at sites that underwent a significant amount of lateral spread. These same analyses also suggest that the depth to the lowest factor of safety against liquefaction, \(Z_s\), generally occurs in the upper 15 m of the profile.

5.9 Residual Strength and SPT N Measures

Many analytical and numerical models use residual strength as a key input parameter for estimating liquefaction-induced ground displacement. Seed et al. (1988) have proposed an empirical curve relating residual strength with SPT \((N_l)_{o_0}\) values. In this study, we also postulated that lateral spread is a function of residual strength and devised several SPT N and \((N_l)_{o_0}\) variables to represent residual strength in our preliminary MLR models. We tested models that included the lowest N, lowest \((N_l)_{o_0}\), average N, and average \((N_l)_{o_0}\) values in the liquefied profile. Of these measures, \(N_{LMS}\) (defined in Table 3-2) yielded the best results when included in the free face model developed for Niigata and Noshiro, Japan. \(R^2\) values for the models increased from 2 to 9 percent depending upon which other independent variables were present.) However, as the U.S. data were added to the analyses, \(N_{LMS}\) made only a slight contribution to improving \(R^2\) (\(R^2\) increased only 0.1 percent when \(N_{LMS}\) was present in the final model). Thus, this variable was dropped from the final model.

Given that lateral displacement is correlated with residual strength and that residual strength is a function of the lowest and average SPT N and \((N_l)_{o_0}\) values in the liquefied profile, we offer the following explanations to why our SPT N
and \((N1)_{10}\) variables appear to be only modestly correlated with displacement. First, there probably is a certain amount of variability in the SPT N and \((N1)_{10}\) values tabulated in our MLR database due to the different types of hammers that were employed at the various case history sites. Second, perhaps the lowest and average N and \((N1)_{10}\) values in the MLR database do not vary enough to show a strong correlation with displacement. Tabulated values of \(N1_{065}\) range from 1.3 to 14.7, and have a mean of 6.1 (Figure 5-3). Approximately 75 percent of the compiled \(N1_{065}\) values are \(\leq 8\), and 90 percent are \(\leq 10\). Third, and most importantly, it appears that residual strength, and ground displacement are not solely a function of the lowest or average N and \((N1)_{10}\) values in the liquefied profile, but are strongly influenced by other subsurface and soil factors, such as the fines content, thickness, and mean-grain size of the liquefied layer. This study indicates that \(P_{50}, T_{50}\), and \(D_{50}\) are strongly to moderately correlated with displacement. Also, \(T_{50}\) correlates reasonably well with \(N1_{065}\) \((R = -0.59)\) suggesting that thick, \(T_{50}\) layers tend to have lower \(N1_{065}\) values. Hence, we suggest that relatively thick, clean, fine-grained, \(T_{50}\) layers appear to produce lower residual strengths and are consequently subjected to a larger amount of ground displacement.

5.10 Boundary Effects

Because the regression coefficients for Equation 4.1.9 are heavily dependent upon Japanese case studies, where liquefaction was widespread and lateral boundary effects were relatively minor, our model may overpredict ground displacements occurring near the margins of smaller lateral spreads. Figure 5-8 shows that a few of the U.S. and Japan observations are overpredicted by a factor greater than 2. Overpredicted observations at the Jensen Filtration Plant and Heber Road were measured at the head or along the side margins of the lateral spreads where ground was apparently inhibited by the nearby lateral boundary. We suspect that changes in the subsurface geology played a large role in limiting liquefaction and ground displacement at these locales. Also, errors the estimates of \(D_{50}\) for these smaller displacements may have contributed to the underpredictions (Section 3.2).

Equation 4.1.9 also significantly overestimates ground displacement measured at Mission Creek and the South of Market Zone following the 1906 San Francisco earthquake (Figure 4-4). At these sites approximately 1.5 m of lateral spread occurred on gentle slopes of 0.6 to 0.8 percent, respectively (Youd and Hoose, 1978; O'Rourke et al., 1991). Equation 4.1.9 predicts approximately 7.5 m of displacement for the South of Market Zone and 12 m for the Mission Creek Zone. We believe that this overestimation is largely due to: (1) the poor quality of available subsurface data at these two sites, and (2) local boundary effects. The penetration tests at Mission Creek and South of Market were performed with a non-standard hammer and no grain-size distribution data are available (O'Rourke et al., 1991). We converted the non-standard penetration resistances to SPT N values and used estimated soil properties for our liquefaction analyses, but these estimates are suspect. Also, ground displacement at these sites were inhibited by lateral boundary effects. The ground failure at Mission Creek formed in a narrow, sinuous, old, creek channel which caused the lateral spread to change directions at several junctures along its path (O'Rourke et al., 1991). These directional changes undoubtedly impeded the ground displacement.

5.11 Flow Chart for the Application of Equation 4.1.9

Figure 5-9 summarizes the suggested procedure for applying the MLR model. In summary, Equation 4.1.9 yields the best results for \(6 \leq N \leq 8.0\) earthquakes and at sites underlain by continuous layers of sands and silty sands having \(N1_{065} \leq 15, 0.075 \leq D_{50} \leq 1.0\,\text{mm}, 0 \leq P_{50} \leq 50\,\%, 1 \leq T_{50} \leq 15\,\text{m}, 1 \leq W \leq 20\,\%, \) and \(0.1 \leq S \leq 6\,\%.\) Also, the depth to the bottom of the liquefied layer, \(Z_{ML}\), should be found within the upper 20 m of the profile. In addition, because this model was developed from western U.S. and Japanese sites founded primarily on stiff soils,
Figure 5-9 Flow chart for the application of Equation 6.1.0
it is most applicable to these soil conditions and to seismic regions having high
to moderate ground motion attenuation. However, Figure 5-4 may be used to adjust
the value of \( R \) used in the model so that it can be applied to other regions with
different seismic attenuation or to sites where significant soft soil
amplification is expected.

5.12 Calculation of the Upper Prediction Limit for Displacement

Although \( D_{\text{lim}} \) is an estimate of the mean displacement for a given set of \( X(s) \),
it is often desirable to determine an upper bound or limit to the amount of
displacement that can be reasonably expected at a given site. Figure 4-4 shows
that almost all values of \( D_{\text{lim}} \) calculated from Equation 4.1.9 fall below the
"MEASURED = 2 X PREDICTED LINE." This suggests that if \( D_{\text{lim}} \) is multiplied by a
factor of 2, then that result will provide a conservative estimate of \( D_{\text{H}} \) that is
not likely to be exceeded. However, MLR models offer a more rigorous,
probabilistic approach to calculating the upper prediction limit or bound for the
true response. For example, a 90 percent prediction limit forms the bound where
it is expected that 90 percent of the observed displacements will be less than
the bound and 10 percent will exceed the bound. The predicted value,

\[
D_{\text{lim}} = b_0 + b_1X_1 + \ldots + b_pX_p, \tag{5.12.1}
\]

is a best-fit estimate of

\[
E(D_H) = b_0 + b_1X_1 + \ldots + b_pX_p, \tag{5.12.2}
\]

where:

\[
E(D_H) \text{ is the mean or expected value of } D_H.
\]

The variance of \( D_{\text{lim}} \), i.e., \( V(D_{\text{lim}}) \), is calculated from:

\[
V(b_0) + X_{12}V(b_1) + \ldots + X_{p1}^2V(b_p) + 2X_{1p} \text{ covar } (b_0, b_1) + \ldots + 2X_{p1}X_{p2} \text{ covar } (b_{p-1}, b_p) \tag{5.12.3}
\]

where: "covar" is the covariance.

This expression is solved in matrix notation as follows (Draper and Smith, 1981):

\[
V(D_{\text{lim}}) = \sigma^2(X_s'CX_s) \tag{5.12.4}
\]

where:

\[ \sigma = \text{standard deviation of } D_H \text{ and is estimated by the standard deviation of}
\text{the regression model, } s. \text{ The } s \text{ for Equation 4.1.9 is } 0.207 \text{ (i.e., } s = (\text{MS}
\text{ error})^{\frac{1}{2}} \text{ from the Analysis of Variance Table for Equation 4.1.9, see
Appendix 2}.\]

The matrix, \( X_s \), contains the site-specific values of the \( X(s) \) used to calculate
\( D_{\text{lim}} \):

\[
X_s' = [1, X_1, \ldots, X_p] \tag{5.12.5}
\]

and the \( C \) matrix is the variance-covariance matrix:

\[
\begin{bmatrix}
C_{00} & C_{01} & \ldots & C_{0p} \\
C_{10} & C_{11} & \ldots & C_{1p} \\
\ldots & \ldots & \ldots & \ldots \\
C_{p0} & C_{p1} & \ldots & C_{pp}
\end{bmatrix} \tag{5.12.6}
\]

The \( C \) matrix is calculated from the matrix operation:

\[
(X'X)^{-1} \tag{5.12.7}
\]
where:
- \( X \) is the matrix comprised of the set of \( X(s) \) used in fitting the regression model.

The \( C \) matrix for Equation 4.1.9 has been tabulated in the file C.DAT on the computer disk labelled Appendix 3 which is available from NCEER.

The \( 1-\alpha \) upper prediction limit for the true displacement is given by (Draper and Smith, 1981):

\[
D_{\text{hat}} + t_{(\alpha - \psi, \nu)} \times s \times (1 + X'_{a}C_{a}X_{a})^{1/2}. \quad (5.12.8)
\]

where:
- \( \alpha \) is selected by the evaluator and is called the significance level.

For example, to calculate the 90 percent upper prediction limit for the true displacement, then \( 1-\alpha \) equals 0.90 and \( \alpha \) equals 0.10. The critical t-value for the selected \( 1-\alpha \) value is determined from the t distribution for \( n-p-1 \) degrees of freedom (Table 5-2). Equation 4.1.9 has 457 degrees of freedom, \( n = 467, \ p = 10 \). Because t-values are not usually tabulated for 457 degrees of freedom, the critical t-values corresponding to 400 degrees of freedom have been listed in Table 5-2. (The use of \( t_{457} \) critical values has very little impact on the final answer because \( t_{457} \) is closely approximated by \( t_{400} \).)

As an example of the application of Equation 4.1.9 and Equation 5.12.8, we will calculate the 90 percent upper prediction limit for true value of \( D_{H} \) at borehole 5-42, in Niigata (see Figure 3-2). Because this is a ground slope failure, we will apply Equation 4.1.9b and the following values of the \( X(s) \) to form the \( X_{a} \) matrix: \( (1, 0, M = 7.5, \ \text{LOG} \ R = 1.32, \ R = 21, \ \text{LOG} \ W'_{f} = 0, \ \text{LOG} \ S_{p} = -0.699, \ \text{LOG} \ T_{15} = 0.477, \ \text{LOG}(100-F_{15}) = 1.978, \ \text{D50}_{h} = 0.433) \). The predicted value for \( \text{LOG}(D_{H} + 0.01) \) from Equation 4.1.9 is -0.03275 and:

\[
D_{\text{hat}} = 10^{-0.03275} = 0.01 - 0.917 \text{ m.} \quad (5.12.9)
\]

To calculate the 90 percent upper predict limit for \( D_{H} \), we first form the \( X_{a}' \) matrix:

\[
X_{a}' = \begin{bmatrix} 1 & 0 & 7.5 & 1.32 & 21 & 0 & -0.699 & 0.477 & 1.978 & 0.433 \end{bmatrix} \quad (5.12.10)
\]

The first two elements in this matrix, \( \{1,0\} \), are called dummy variables (Draper and Smith, 1981). The first element, \( \{1\} \), indicates that \( D_{\text{off}} \) applies to the ground slope component of the model, and the second element, \( \{0\} \), indicates that \( D_{\text{off}} \) does not apply. Next, we perform the matrix operation \( X_{a}'C_{a}X_{a} \):

\[
X_{a}'C_{a}X_{a} = 0.0213. \quad (5.12.11)
\]

The 90 percent upper prediction limit is calculated from equation 5.12.8:

\[
\text{LOG}(D_{H90}) = -0.03275 + 1.284 \times 0.207 \times (1+0.0213)^{1/2} = 0.236 \quad (5.12.12)
\]

or

\[
D_{H90} = 10^{0.236} - 0.01 = 1.712 \text{ m.} \quad (5.12.13)
\]

Thus, we conclude that we are 90 percent confident (i.e., 90 percent probability) that the true value of \( D_{H} \) at borehole 5-42 will not exceed 1.712 m.

Similarly, we can calculate the 90 percent upper prediction limit for true value of \( D_{H} \) at borehole G10-39 (Figure 3-2). Because this is a free face failure, we apply Equation 4.1.9a and the following values of \( X(s) \) to form the \( X_{a} \) matrix:

\[
(1, 1, M = 7.5, \ \text{LOG} \ R = 1.32, \ R = 21, \ \text{LOG} \ W_{f} = 0.477, \ \text{LOG} \ S_{p} = 0, \ \text{LOG} \ T_{15} = 1.114,
\]

5-15
LOG (100-F15) = 1.982, D5015 = 0.400. The predicted value of LOG(D_H + 0.01) from Equation 4.1.9a is 0.27241 and:

\[ D_{\text{predicted}} = 10^{0.27241} - 0.01 = 1.862 \, \text{m.} \]  
(5.12.14)

The \( x'_s \) matrix is:

\[ x'_s = [1, 1, 7.5, 1.32, 21, 0.477, 0, 1.114, 1.982, 0.400] \]  
(5.12.15)

The dummy variables for the first two elements in this matrix, \( \{1,1,\} \), indicates that both \( b_y \) and \( b_0 \) apply to the free face component of the model. The value of \( x'_s'Cx_s \) equals 0.0151, and the 90 percent upper prediction limit is:

\[ \text{LOG(D}_{\text{H90}}) = 0.27241 + 1.284 \times 0.207 \times (1+0.0151)^{1/2} = 0.540 \]  
(5.12.16)

or

\[ D_{\text{H90}} = 10^{0.540} - 0.01 = 3.459 \, \text{m.} \]  
(5.12.17)

Thus, we are 90 percent confident that the true value of \( D_H \) at borehole 5-42 will not exceed 3.459 m.

Other upper prediction limits, besides the 90% upper prediction limit used in the above example, can be calculated by simply selecting the desired confidence level from Table 5-2 and using that value in Equation 5.12.8 for \( t_{(\alpha/2, f-1)} \).

<table>
<thead>
<tr>
<th>TABLE 5-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>CRITICAL t VALUES FOR CONFIDENCE LIMITS</td>
</tr>
<tr>
<td>(based on 400 degrees of freedom, after Ostle and Malone, 1988)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Confidence level (1-( \alpha )) in percent</th>
<th>75%</th>
<th>80%</th>
<th>85%</th>
<th>90%</th>
<th>95%</th>
<th>97.5%</th>
<th>99.5%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.676</td>
<td>0.843</td>
<td>1.038</td>
<td>1.284</td>
<td>1.649</td>
<td>1.966</td>
<td>2.588</td>
</tr>
</tbody>
</table>

5.13 Comparison of Equation 4.1.9 with Other Empirical Models

We applied the LSI model proposed by Youd and Perkins (1987) and the slope-thickness model of Hamada et al. (1986) to our compiled data and compared the performance of these models with Equation 4.1.9 (Figure 5-10). The LSI model (Equation 2.7.1.3) conservatively bounds almost all of the U.S. data, but underestimates many of the displacement vectors measured in Niigata and Noshiro, Japan. There are a few plausible reasons for this underprediction of the Japanese data by the LSI. First, the LSI was primarily developed from U.S. case studies where subsurface conditions were generally less favorable to widespread liquefaction. Thus, the LSI does not adequately reflect the high liquefaction susceptibility of the soils found in Niigata and Noshiro, Japan. Second, the location of the seismic source for the 1964 Niigata and 1983 Nihonkai-Chubu earthquakes is not well known. Both of these subduction zone earthquakes occurred in the Japan Sea where the faulting is not well understood. We estimate that R was approximately 21 and 27 km, respectively from Niigata and Noshiro cities, based on studies of crustal warping in the Japan Sea (Mogi et al., 1964; Hwang and Hammack, 1984). However, if R was indeed closer than our estimates, then the LSI would bound much more of the Japanese data.
The thickness-slope model proposed by Hamada et al. (Equation 2.7.2.8) performs adequately for Niigata and Noshiro, Japan, but tends to overpredict many of the displacements measured at U.S. sites (Figure 5-10). The $R^2$ for this model is 35.1 percent and 73 percent (329 of 448) of the predicted observations fall between the upper and lower prediction bounds. 

To be consistent with the techniques used by Hamada et al. in developing this model, we modified our liquefaction analysis program to calculate the thickness of the liquefied layer, $H$, using the liquefaction susceptibility curves outlined by the Japanese Code of Bridge Design (Section 2.7.2). We also measured $\theta$ in a manner that was consistent with the definition proposed by Hamada et al. (Figures 2-2a and 2-2b.) There are a few possible reasons why the Japanese model tends to perform poorly at many U.S. sites. First, the earthquakes that generated lateral spread in the U.S. were significantly different from those that struck Niigata and Noshiro. Niigata and Noshiro experienced very similar earthquakes ($M = 7.5$ and $7.7$, respectively) and the seismic sources were located approximately the same distance from the two cities (approximately 21 and 27 km, respectively). In contrast, the U.S. case studies include earthquakes that range from $6.6 \leq M \leq 9.2$, and lateral spread sites that were located at varying distances from the seismic source ($0.2 \leq R \leq 100$ km). Second, the liquefied sediments in Niigata and Noshiro tend to be relatively clean compared to many U.S. sediments that are more silty. Third, our techniques of measuring $H$ and $\theta$ may not be entirely consistent with those used by the Japanese investigators in reducing their data.

Based on the performance of Equation 4.1.9 as shown in Figure 5-10, we conclude that our attempt to formulate a more comprehensive MLR model for predicting lateral spread displacement has been successful. Because our model is derived from and adjusted for a wider range of seismic, site, and soil conditions than the previously proposed empirical models, it is more general and will yield better results if properly applied.
Figure 5.10 Comparison of $D_h$ versus $D_{h\text{hat}}$ for LSI model, Hamada et al. thickness-slope model, and Equation 4.1.9 showing improved performance of Equation 4.1.9.
SECTION 6
CONCLUSIONS

Considerable progress has been made since the 1964 Alaska and 1964 Niigata earthquakes in understanding liquefaction and predicting its occurrence. However, only limited progress has been made in developing practical models for estimating the liquefaction-induced, horizontal ground displacement. The thickness-slope model proposed by Hamada et al. (1986) emphasizes certain site factors, such as slope and thickness of the liquefiable layer, but it does not directly address the effects that earthquake and soil factors have on displacement. In contrast, the LSI model (Youd and Perkins, 1987) is based on earthquake factors, but it is not adjusted for the influence of topographical and soil factors. In this study, we used an extensive database derived from Japanese and U.S. lateral spread sites to formulate a more comprehensive empirical model for predicting lateral spread displacement. Because our model was developed from a wider range of seismic and site conditions than previously proposed models, it is more general and will yield better results. In summary, Equation 4.1.9 yields the best predictions and is applicable for magnitude 6 to 8 earthquakes and at sites underlain by relatively thick (T<sub>ls</sub> > 1 m), continuous, shallow (Z<sub>rel</sub> < 20 m) layers of liquefiable sand and silty sand (F<sub>s</sub> < 50%) having SPT (N<sub>L</sub>)<sub>60</sub> values ≤ 15.

In developing our model, we observed two general types of failures: (1) lateral spread toward a free face (i.e., river channel or some other abrupt topographical depression), and (2) lateral spread down gentle ground slopes. Our regression analyses indicate that the topographical parameters fitted for free face failures differ significantly from those fitted for ground slope failures, thus we formulated our final MLR model with separate components for each type of failure. These analyses also indicate that ground displacement is strongly influenced by the height and proximity of the free face and that this influence decays logarithmically with increasing distance. In contrast to free face failures, ground slope failures generally produces smaller, more uniform displacements and typically occur on slopes that are less than 6 percent. Displacement produced by ground slope failure is strongly correlated with the steepness of the ground slope and generally occurs in the direction of the maximum topographical gradient.

In addition to these topographical factors, the thickness, silt content and mean grain-size of the liquefied layer are strongly to moderately correlated with displacement. Our study suggests that significant lateral spread is almost always restricted to saturated cohesionless sediments having (N<sub>L</sub>)<sub>60</sub> values ≤ 15 for M < 8 earthquakes. We also found that lateral spread is generally restricted to liquefiable sediments that are thicker than 1 m and found in the upper 15 to 20 m of the soil profile. Our analyses also indicates that horizontal displacement tends to markedly decrease as the percentage of fines and mean grain-size of the liquefied sediment increase.

Because our MLR models performed poorly at predicting displacement for soils with D<sub>50</sub> values > 2 mm, we concluded that gravelly sediments behave differently than fine-grained sediments in some nonlinear fashion that our model does not accommodate. Also, silty sediments appear to displace much less than clean, sandy sediments. More research and case histories are needed to better understand and model the dynamic displacement behavior of gravels and silts. In addition, because our model is heavily dependent on Japanese case histories, where liquefaction was widespread and boundary effects were small, it may tend to overpredict displacements occurring near the margins of failures. More detailed subsurface investigations are needed from locales where liquefaction and lateral spread are influenced by lateral boundaries. Finally, as more case studies become available from past and future earthquakes, the performance of this model should be re-evaluated and the model parameters adjusted as necessary.
SECTION 7
REFERENCES


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Finn, W. D. L., and Yogendrakumar, M., 1989, "TARA-3FL: Program for Analysis of Liquefaction Induced Flow Deformations, Department of Civil Engineering, University of British Columbia, Vancouver, B. C., Canada.


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Utermohle, G. E., 1965, "Foundation Investigation, Portage River No. 1 (#630) and Portage River No. 2 (#631), Project No. ERFO-20(1), Anchorage District," Alaska Department of Transportation, Anchorage, Alaska, 8 p.


APPENDIX 1

DISCUSSION OF INDEPENDENT VARIABLES USED IN MLR ANALYSES

This appendix discusses the procedures used to measure the independent variables compiled by this study. All variables tested in our MLR analyses are tabulated in ASCII format on the computer disk labeled Appendix 3 in the file named "MLR.DAT." This disk and two additional disks, which comprise "Appendix 4", are available from NCEER Information Service, care of Science and Engineering Library, 304 Capen Hall, State University of New York at Buffalo, Buffalo, New York, 14260. The independent variables included in our final MLR model are also discussed in the body of this report. This appendix provides additional information about all independent variables compiled and tested by this study (see Table 3-2). It also discusses the borehole summary sheets, SPT data, and liquefaction analyses contained in Appendix 4. Finally, an example of the weighted average, which was used to assign geological and soil independent variables to individual displacement vectors, is given in Section A1.5.

The liquefaction analyses summary sheets are found in Appendix 4 for the 267 SPT boreholes compiled by this study. These summary sheets tabulate the earthquake, topographical, geological, and soil measurements for each borehole and give the results of liquefaction analyses that were done according to the "simplified procedure" (Seed and Idriss, 1971; Seed et al., 1983; 1985, NRC, 1985) and Liao's 50 percent probability of liquefaction curve (Liao, 1986). Each summary sheet lists the methods used to measure the independent variables and documents the source(s) of the information. An example summary sheet is shown in Table A1-1.

These summary sheets are grouped by earthquake on the computer disks labelled Appendix 4. Table A1-2 lists the file names for the earthquakes and lateral spreads found in Appendix 4.

A1.1 Earthquake Independent Variables

Table A1-3 lists the earthquake independent variables that are tabulated in Appendix 3 in MLR.DAT.

A1.1.1 Earthquake Magnitude

Earthquake magnitude is a measure of the amount of seismic energy released by an earthquake. Large magnitude earthquakes are capable of producing large and widespread ground failure in moderately to highly liquefiable soils that are located near the seismic source. In contrast, lateral spread displacement tends to be smaller and more limited for moderate to smaller earthquakes. Because the moment magnitude, \( M_o \), is defined in terms of energy, it is generally a better estimate of the amount of seismic energy released by a given event than other earthquake magnitude measures. The moment magnitude is calculated from the seismic moment, \( M_o \) (Kanamori, 1978):

\[
M_o = \left( \log M_o / 1.5 \right) - 10.7 \quad (A1.1.1.1)
\]

where:

\( M_o \) = seismic moment (dyne*cm).

\( M_o \) is based on elastic dislocation theory and is a function of the amount of tectonic deformation at the seismic source and the rigidity of the earth's crust:

\[
M_o = u S D \quad (A1.1.1.2)
\]

where:

\( u \) = rigidity of the ruptured material
\( S \) = surface area of the fault
\( D \) = average displacement of the fault.
### TABLE A1-1
EXAMPLE SUMMARY SHEET SHOWING CASE HISTORY
INFORMATION AND LIQUEFACTION ANALYSES
(Summary sheets are found in Appendix 4)

| BOREHOLE ID. NO.: | 13 | EARTHQUAKE: | 1964, ALASKA |
| BORING: | M-4 | SITE NAME: | RAILROAD BRIDGE MP 147.1 |
| LOCATION: | MATANUSKA RIVER, ALASKA |

#### SOURCE OF HORIZONTAL DISPLACEMENT DATA

<table>
<thead>
<tr>
<th>METHOD</th>
<th>DOCUMENTATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>DISPLACEMENT OF BRIDGE PIER</td>
<td>MCCULLOCH AND BONILLA, 1970</td>
</tr>
</tbody>
</table>

#### EARTHQUAKE MEASUREMENTS

| MAGNITUDE, Mw: | 9.2 | METHOD | DOCUMENTATION |
| DIST. TO FAULT, km: | 100 | SCALED FROM 0 CONTOUR, p. D7 | KANAMORI, 1978 |
| EPICENTRAL DIST. (km): | 95 | SCALED FROM p. D7 | MCCULLOCH AND BONILLA, 1970 |
| PEAK ACCEL., A (g): | 0.21 | MAG-DIST. (INCREASED 2.33 X) | McCULLOCH & BONILLA, 1970 |
| DURATION, D (s): | 79 | MAGNITUDE-DISTANCE RELATION | JOYNER AND BOORE, 1981 |

#### GEOLOGICAL AND TOPOGRAPHICAL MEASUREMENTS

| AGE OF SEDIMENTS (yrs): | RECENT | METHOD | DOCUMENTATION |
| DEPTH TO GROUNDWATER (m): | 0.0 | MODERN CHANNEL DEPOSITS | MCCULLOCH & BONILLA, 1970 |
| SOURCE OF SLOPE DATA | | AT OR NEAR SURFACE, p. D11B | MCCULLOCH AND BONILLA, 1970 |
| SOURCE OF FREE FACE DATA | | TOPOGRAPHICAL SURVEY | BARTLETT AND YOUD, 1991 |
| AREA OF FAILURE (sq. m): | | 1998 PROFILE BY ALASKA RR | NOT ESTIMATED |
| | | | REPORTED |

#### STANDARD PENETRATION AND SOIL MEASUREMENTS

| SPLIT SPOON 2" O.D. | | DONUT ROPE AND PULLEY | \( u = 0 \) |
| SOURCE OF % FINES DATA | | SIEVE ANALYSIS | UTERMÖHLE, 1963 |
| SOURCE OF D50 DATA | | SIEVE ANALYSIS | UTERMÖHLE, 1963 |
| THICKNESS N(50) < 10; T(10), (m): | 0.0 | AVG FINES IN T10, F10, (%) | 0 | AVG D50 IN T10, D5010, (%) | 10.85 |
| THICKNESS N(50) < 15; T(15), (m): | 13.3 | AVG FINES IN T15, F15, (%) | 16 | AVG D50 IN T15, D5015, (%) | 3.615 |
| THICKNESS N(50) < 20; T(20), (m): | 15.2 | AVG FINES IN T20, F20, (%) | 14 | AVG D50 IN T20, D5020, (%) | 3.871 |

| LOWEST N IN PROFILE, N: | 11.0 | DEPTH TO LOWEST N, ZN, (m): | 4.6 |
| LOWEST N(50) IN PROFILE, N(50): | 10.4 | DEPTH TO LOWEST N(50), ZN(50), (m): | 10.7 |

#### LIQUEFACTION ANALYSIS (SIMPLIFIED PROCEDURE)

| LIQUEFACTION POTENTIAL, IS: | 6.32 |
| ACCUMULATIVE THICKNESS OF LIQUEFIED ZONE, TS, (m): | 16.0 |
| DEPTH TO TOP OF LIQUEFIED ZONE, ZTL, (m): | 0.0 |
| DEPTH TO BOTTOM OF LIQUEFIED ZONE, ZBL, (m): | 19.8 |
| AVG. FACTOR OF SAFETY IN LIQUEFIED ZONE, KS: | 0.60 |
| AVG. N(50) IN LIQUEFIED ZONE, OS: | 13.6 |
| LOWEST FACTOR OF SAFETY, IS: | 0.35 |
| DEPTH TO LOWEST FACTOR OF SAFETY, ZS, (m): | 3.0 |
| AVG. FINES IN LIQUEFIED ZONE, F5, (%) | 14 |
| AVG. CLAY IN LIQUEFIED ZONE, (%) | 14 |
| AVG. D50 IN LIQUEFIED ZONE, D50, (mm): | 3.732 |

#### LIQUEFACTION ANALYSIS (Liao's 50% PROBABILITY CURVE)

<p>| LIQUEFACTION POTENTIAL, IL: | 6.04 |
| ACCUMULATIVE THICKNESS OF LIQUEFIED ZONE, TL, (m): | 17.5 |
| DEPTH TO TOP OF LIQUEFIED ZONE, ZTL, (m): | 0.1 |
| DEPTH TO BOTTOM OF LIQUEFIED ZONE, ZBL, (m): | 26.8 |
| AVG. FACTOR OF SAFETY IN LIQUEFIED ZONE, KL: | 0.64 |
| AVG. N(50) IN LIQUEFIED ZONE, OL: | 11.5 |
| LOWEST FACTOR OF SAFETY, JL: | 0.45 |
| DEPTH TO LOWEST FACTOR OF SAFETY, ZL, (m): | 3.0 |
| AVG. FINES IN LIQUEFIED ZONE, FL, (%) | 20 |
| AVG. CLAY IN LIQUEFIED ZONE, (%) | 20 |
| AVG. D50 IN LIQUEFIED ZONE, D50L, (mm): | 3.366 |</p>
<table>
<thead>
<tr>
<th>File Name</th>
<th>Earthquake and Case History Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>SANFRAN.DAT</td>
<td>1906 San Francisco Earthquake&lt;br&gt;Coyote Creek Bridge&lt;br&gt;Mission Creek Zone&lt;br&gt;Salinas River Bridge&lt;br&gt;South of Market Street Zone</td>
</tr>
<tr>
<td>ALASKA.DAT</td>
<td>1964 Alaska Earthquake&lt;br&gt;Bridges 141.1, 147.4, 147.5, 148.3, Matanuska R.&lt;br&gt;Bridges 63.0, 63.5, Portage Creek&lt;br&gt;Highway Bridge 629, Placer River&lt;br&gt;Snow River Bridge 605A, Snow River&lt;br&gt;Bridges 3.0, 3.2, 3.3, Resurrection River</td>
</tr>
<tr>
<td>NIIGATA.DAT</td>
<td>1964 Niigata, Japan Earthquake&lt;br&gt;Many lateral spreads in Niigata, Japan</td>
</tr>
<tr>
<td>SANFERN.DAT</td>
<td>1971 San Fernando Earthquake&lt;br&gt;Jensen Filtration Plant, San Fernando, Ca.&lt;br&gt;Juvenile Hall, San Fernando, Ca.</td>
</tr>
<tr>
<td>IMPERIAL.DAT</td>
<td>1979 Imperial Valley Earthquake&lt;br&gt;Heber Road near El Centro, California&lt;br&gt;River Park near Brawley, California</td>
</tr>
<tr>
<td>BORAH.DAT</td>
<td>1983 Borah Peak, Idaho Earthquake&lt;br&gt;Whiskey Springs near Mackay, Idaho&lt;br&gt;Pence Ranch near Mackay, Idaho</td>
</tr>
<tr>
<td>NIHONKAI.DAT</td>
<td>1983 Nihonkai-Chubu Earthquake&lt;br&gt;Many lateral spreads in Noshiro, Japan</td>
</tr>
</tbody>
</table>
TABLE A1-3
SUMMARY OF EARTHQUAKE INDEPENDENT VARIABLES USED IN MLR

<table>
<thead>
<tr>
<th>Earthquake Variables</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>M*</td>
<td>Earthquake moment magnitude, $M_w$.</td>
</tr>
<tr>
<td>R*</td>
<td>Nearest horizontal distance to seismic energy source or fault rupture, (km).</td>
</tr>
<tr>
<td>A</td>
<td>Peak horizontal ground acceleration, (g).</td>
</tr>
<tr>
<td>D</td>
<td>Duration of strong ground motion (&gt;0.05 g), (s).</td>
</tr>
</tbody>
</table>

* Indicates independent variables used in final MLR models.

In this study, we tabulated values of $M_w$ as the variable $M$. For some of the smaller earthquakes included in this study, $M_w$ was not reported by the original researchers, and the surface wave magnitude, $M_s$, or the local magnitude, $M_l$, was used to represent $M$ instead of $M_w$ (see documentation of earthquake magnitude in Appendix 4). Nonetheless, $M_s$ and $M_l$ are comparable to $M_w$, especially for smaller magnitude earthquakes (Kanamori, 1978).

A1.1.2 Distance to Seismic Energy Source or Fault Rupture

During large earthquakes, severe and damaging lateral spread occurs close to the seismic energy source. The amount of liquefaction-induced ground displacement tends to diminish logarithmically with increasing distance from the seismic source (Yod and Perkins, 1978, 1987). For this study, the distance to the seismic energy source or fault rupture, $R$, was tabulated the horizontal distance (km) from the site in question to the nearest point on a surface projection of the fault rupture zone (see Section 4.1).

A1.1.3 Peak Ground Acceleration

The peak horizontal ground acceleration, $A$, (g), is often used to characterize the intensity of strong ground motion. Earthquake shaking must have a sufficient amplitude to generate excess pore pressure and initiate liquefaction. Once the soil has liquefied, the resulting ground displacement also increases with increasing values of $A$. For sites where a strong ground motion accelerometer was nearby, we recorded the instrumented value of $A$; for sites without a strong motion record, a magnitude-distance, empirical relationship was used to estimate $A$ (Joyner and Boore, 1988):

$$ \log A = a + b(M-6) + c(M-6)^2 + d(\log r) + kr + s $$  \hspace{1cm} (A1.1.3.1)

where:
- $A$ = the larger value of the two horizontal components of peak horizontal acceleration measured on rock or stiff soil sites (g)
- $M$ = moment magnitude
- $a = 0.49$
- $b = 0.23$
- $c = 0.0$
- $h = 8.0$
- $d = -1.0$
- $k = -0.0027$
- $s = 0.0$
- $r = r_{so}^2 + h^2\frac{1}{2}$

A1-4
The value of \( r_0 \) in Equation A1.1.3.1 is the shortest horizontal distance, (km), from the recording site to the surface projection of the fault rupture. The summary sheets found in Appendix 4 document those sites having instrumented values of \( A \) and those sites where \( A \) was estimated from Equation A1.1.3.1.

We also attempted formulating a M-A model by using \( A \) values estimated from Equation A1.1.3.2 for our non-instrumented sites (Idriss, in press) and modifying those bedrock estimates of \( A \) for stiff soil conditions by using Figure 5-5. For this analysis, we applied a faulting factor, \( F \), of 0.5; but, we also found that \( A \) is not very sensitive to the selected value of \( F \). The attempt to formulate M-A models using \( A \) estimated from either Equation A1.1.3.1 or from Equation A1.1.3.2 and modified for stiff soil conditions using Figure 5-5 yielded poorer correlations than models based directly on \( M \), \( \log R \), and \( R \) (see Sections 4.1 and 5.1).

\[
\begin{align*}
\ln (A) &= [a_1 + \exp(a_1 + a_2 M) + \\
&[B_1 - \exp(B_1 + B_2 M)] \ln(R+20) + 0.2 F \quad \text{(A1.1.3.2)}
\end{align*}
\]

where:
\( A \) = peak bedrock horizontal acceleration (g)
\( M \) = moment magnitude, \( M_w \)
\( R \) = closest distance to source in km; however for \( M \leq 6 \) the hypocentral distance is used.
\( F \) = style of faulting factor; \( F = 0 \) for strike slip fault; \( F = 1 \) for a reverse fault and \( F = 0.5 \) for an oblique source.
\( a_1 = -0.150 \) for \( M \leq 6 \) and \( -0.050 \) for \( M > 6 \)
\( a_2 = 2.262 \) for \( M \leq 6 \) and \( 3.477 \) for \( M > 6 \)
\( B_1 = -0.083 \) for \( M \leq 6 \) and \( -0.284 \) for \( M > 6 \)
\( B_2 = 0 \) for both \( M \leq 6 \) and \( M > 6 \)
\( B_3 = 1.602 \) for \( M \leq 6 \) and \( 0 \) for \( M > 6 \)
\( B_4 = -0.142 \) for \( M \leq 6 \) and \( -0.286 \) for \( M > 6 \)

**A1.1.4 Duration of Strong Ground Motion**

Long-lived strong ground motion tends to increase the extent of liquefaction in the soil and prolong the time that the mobilized soil mass is subjected to downslope translation by earthquake and gravitational forces. Page et al. (1972) used the bracketed interval, \( (s) \), between the first 0.05 g peak to the last 0.05 g peak to measure the duration of strong ground motion, \( D \). For this study, we used the same definition of \( D \) at lateral spread sites where strong ground motion records were available. For noninstrumented sites, we used an empirical relationship developed by Krinitzsky and Chang (1988b) to estimate \( D \):

\[
\log D = -2.36 + 0.43 M + 0.30 \log (r/10) \quad \text{(A1.1.4.1)}
\]

where:
\( D \) = the bracketed interval, \( (s) \), defined by Page et al.
\( M \) = earthquake magnitude
\( r \) = epicentral distance, (km).

Equation A1.1.4.1 was applied to hard and soft soil sites for earthquakes with a focal depth > 19 km. Equation A1.1.4.1 was also applied to hard soil sites for earthquakes with a focal depth ≤ 19 km. For soft soil sites and for earthquakes with a focal depth ≤ 19 km, the following equation was used (Krinitzsky and Chang, 1988b):

\[
\log D = -2.06 + 0.43 M + 0.60 \log (r/10). \quad \text{(A1.1.4.2)}
\]
where:
D, M, r are defined above.

The summary sheets in Appendix 4 document the methods used to estimate D.

A1.2 Topographical Independent Variables

The topographical variables listed in Table A1-4 are compiled in MLR.DAT in Appendix 3.

<table>
<thead>
<tr>
<th>TABLE A1-4</th>
</tr>
</thead>
<tbody>
<tr>
<td>SUMMARY OF TOPOGRAPHICAL INDEPENDENT VARIABLES USED IN MLR</td>
</tr>
<tr>
<td><strong>Topographical Variables</strong></td>
</tr>
<tr>
<td>S*</td>
</tr>
<tr>
<td>L</td>
</tr>
<tr>
<td>H</td>
</tr>
<tr>
<td>W*</td>
</tr>
</tbody>
</table>

* Indicates independent variables used in final MLR models.

A1.2.1 Ground Slope

Figure 3-11, in Section 3.5, shows the techniques we used to measure ground slope, S, for all ground slope failures. The technique shown in Figure 3-11, Case 1, was used to measure S for all uniform slopes:

\[ S (\%) = 100 \frac{Y}{X} \]  
(A1.2.1.1)

where:
Y = the change in vertical elevation
X = the horizontal distance of the slope.

In gently undulating topography, like that found in Noshiro, Japan, many slopes were undulating and required a slightly modified method of measuring S (see Figure 3-11, Cases 2 and 3).

In the case of free face failures, S was used to measure the slope of the floodplain into or away from the channel. The value of S was recorded as a positive value for ground sloping toward the free face (Section 3.4, Figure 3-8, Case 1) and was recorded as negative value for ground sloping away from the free face (Figure 3-8, Case 2). However, our regression analyses showed that S was not significant when included in the free face model, thus S was omitted from the free face component of our final MLR model.

A1.2.2 Distance to and Height of Free Face

A free face is any abrupt topographical depression such as an escarpment, river channel, canal, or road cut. Lateral spread displacement markedly increased with the proximity of the free face (see Section 3.4). In modeling the influence of the free face, Hamada et al. (1986) artificially steepened their estimate of the ground slope, \( \theta \), to the base of the free face (Figure 2-2b). We decided not to steepen our estimate of S to the base of the free face, but used the free face ratio, W, to represent the effects of the free face in our MLR models. To calculate W, we tabulated the height of the free face, H, (m), and the distance from the displacement vector to the free face, L, (m). In order to normalize L
with respect to $H$, we formed the free face ratio, $W_1$, ($\%$), where: $W = 100H/L$ (see Section 3.4). If a free face was not present (e.g., ground slope failures), the values $W$ and $L$ are set equal to zero in MLR.DAT found in Appendix 3.

A1.3 Geological and Soil Independent Variables

In post-earthquake liquefaction investigations, the thickness and depth of the liquefied zone(s) cannot be determined by direct observation. Thus, empirical analyses are commonly used to detect layers in the subsurface that are susceptible to liquefaction. Layer(s) that are shown to be susceptible by these analyses are often assumed to be the same layer(s) that liquefied during the earthquake. In this study, two techniques were used to estimate the thickness of and depth to the liquefied layer: (1) the so-called "simplified procedure of liquefaction analysis" by Seed and Idriss, 1971; Seed et al. 1983, 1985, 1986a, and (2) Liao's 50 percent probability of liquefaction curve (Liao, 1986).

Table A1-5 lists the geological independent variables compiled in Appendix 3 and 4. Variables that are subscripted with a "S" were determined from the simplified procedure curves; variables subscripted with a "L" were determined from Liao's 50% probability of liquefaction curve. A liquefaction analysis program was written in dBase III to perform the liquefaction analyses. The dBase program was used to calculate all subsurface measures listed in Tables A1-5 and A1-6. The results of these analyses are found in Appendix 4. The program constructs a continuous SPT N and soil profile for each SPT log by linearly interpolating (at 0.1 m increments) between measured values of $N$, $D_{50}$, and the percentage of fines and clay in subsurface sample (Figure A1-1). However, linear interpolation is not done across a soil boundary. The soil parameters for the last, or deepest sample in a given layer are kept constant to until the next layer is encountered. Likewise, the soil parameters for the first sample in the underlying layer are transferred upward to the layer boundary, forming a discontinuity in the interpolated profile at the layer boundary. The computer program also accumulates the thickness of cohesionless sediments below the water-table that have $(N1)_w$ values $\leq 10$, $\leq 15$, and $\leq 20$. These thickness measures, (m), are tabulated as the independent variables $T_{10}$, $T_{15}$, and $T_{20}$ respectively (Appendix 4). Soils with a clay content $\geq 15$ percent are not considered to be liquefiable and are not accumulated in $T_{10}$, $T_{15}$, and $T_{20}$. An example of the $T_{20}$ layer(s) is shaded in Figure A1-1; see also Table A1-1, which is the summary sheet for this borehole. Table A1-1 indicates that $T_{20}$ for Figure A1-1 is 13.3 m.

Measured $D50$ values, (mm), are averaged in $T_{20}$, $T_{15}$, $T_{10}$, and $T_{20}$ and assigned to the variables $D50_{20}$, $D50_{15}$, $D50_{10}$, and $D50_{20}$, respectively (see Table A1-6, see also Table A1-1). Likewise, measured values for the percentage of fines, $F$, (particle size $< 0.075$ mm) are averaged in $T_{20}$, $T_{15}$, $T_{10}$, and $T_{20}$ and assigned to the variables $F_{20}$, $F_{15}$, $F_{10}$, and $F_{20}$, respectively. Values of $D50$ and $F$ that are interpolated between sampling intervals are not used in these averages. Also, layers that are above the water-table or soils having a clay content $\geq 15$ percent are not included in these averages.
<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_s$</td>
<td>Thickness of liquefied zone(s) (Simplified procedure), (m).</td>
</tr>
<tr>
<td>$T_L$</td>
<td>Thickness of liquefied zone(s) (Liao's 50% probability curve), (m).</td>
</tr>
<tr>
<td>$T_{10}$</td>
<td>Thickness of saturated cohesionless soils with $(N1)_{90} \leq 10$, (m).</td>
</tr>
<tr>
<td>$T_{15}$</td>
<td>Thickness of saturated cohesionless soils with $(N1)_{90} \leq 15$, (m).</td>
</tr>
<tr>
<td>$T_{20}$</td>
<td>Thickness of saturated cohesionless soils with $(N1)_{90} \leq 20$, (m).</td>
</tr>
<tr>
<td>$I_s$</td>
<td>Index of Liquefaction Potential (Simplified procedure).</td>
</tr>
<tr>
<td>$I_L$</td>
<td>Index of Liquefaction Potential (Liao's 50% probability curve).</td>
</tr>
<tr>
<td>$z_{da}$</td>
<td>Depth to top of liquefied zone (Simplified procedure), (m).</td>
</tr>
<tr>
<td>$z_{db}$</td>
<td>Depth to top of liquefied zone (Liao's 50% prob. curve), (m).</td>
</tr>
<tr>
<td>$z_{ab}$</td>
<td>Depth to bottom of liquefied zone (Simplified procedure), (m).</td>
</tr>
<tr>
<td>$z_{ab}$</td>
<td>Depth to bottom of liquefied zone (Liao's 50% prob. curve), (m).</td>
</tr>
<tr>
<td>$z_a$</td>
<td>Depth to the lowest factor of safety (Simplified procedure), (m).</td>
</tr>
<tr>
<td>$z_L$</td>
<td>Depth to the lowest factor of safety (Liao's 50% prob. curve), (m).</td>
</tr>
<tr>
<td>$z$</td>
<td>Depth to lowest SPT N value in saturated cohesionless soil, (m).</td>
</tr>
<tr>
<td>$z_{N160}$</td>
<td>Depth to lowest SPT $(N1)_{90}$ value in saturated cohesionless soil, (m).</td>
</tr>
</tbody>
</table>

- $N$: Lowest SPT N value in saturated cohesionless sediments.
- $N_{160}$: Lowest SPT $(N1)_{90}$ value in saturated cohesionless sediments.
- $J_s$: Lowest factor of safety below water table (Simplified procedure).
- $J_L$: Lowest factor of safety below water table (Liao 50% prob. curve).
- $N_{160s}$: $(N1)_{90}$ value corresponding to $J_s$.
- $N_{160L}$: $(N1)_{90}$ value corresponding to $J_L$.
- $K_s$: Average factor of safety in $T_s$.
- $K_L$: Average factor of safety in $T_L$.
- $O_s$: Average $(N1)_{90}$ in $T_s$.
- $O_L$: Average factor of safety in $T_L$.

* Indicates independent variables used in final MLR models.
Figure A1-1 A sample SPT borehole taken at Railroad Bridge Mile Post 147.1, Matanuska River, Alaska.
The liquefaction analysis program also tabulates the lowest SPT N and (N1)0 values for each soil log. These values are assigned to variables \( N \) and \( N_{10} \), respectively (for example, compare Table A1-1 and the borehole shown in Figure A1-1). The depth corresponding to \( N \) and \( N_{10} \) are also tabulated as the variables \( Z_N \) and \( Z_N10 \). If two intervals have equivalent \( N \) or \( N_{10} \) values for a given profile, the shallower depth is assigned to \( Z_N \) and \( Z_N10 \). Values of \( N \) and \( N_{10} \) that are interpolated between sampling intervals not considered for \( N \), \( N_{10} \), \( Z_N \), and \( Z_N10 \). Also, \( N \) and \( N_{10} \) values found above the water-table or in soils with a clay content \( \geq 15 \) percent are not considered.

### Table A1-6

**SUMMARY OF SOIL INDEPENDENT VARIABLES USED IN MLR**

<table>
<thead>
<tr>
<th>Variables</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>D50 (_s)</td>
<td>Average ( D_{50} ) in ( T_s ), (mm).</td>
</tr>
<tr>
<td>D50 (_l)</td>
<td>Average ( D_{50} ) in ( T_l ), (mm).</td>
</tr>
<tr>
<td>D50 (_s)</td>
<td>Average ( D_{50} ) in ( T_s ), (mm).</td>
</tr>
<tr>
<td>D50 (_0)</td>
<td>Average ( D_{50} ) in ( T_0 ), (mm).</td>
</tr>
<tr>
<td>D50 (_f)</td>
<td>Average ( D_{50} ) in ( T_f ), (mm).</td>
</tr>
<tr>
<td>D50 (_m)</td>
<td>Average ( D_{50} ) IN ( T_m ), (mm).</td>
</tr>
<tr>
<td>( F_s )</td>
<td>Average fines content (&lt;0.075 mm) in ( T_s ), (%)</td>
</tr>
<tr>
<td>( F_l )</td>
<td>Average fines content (&lt;0.075 mm) in ( T_l ), (%)</td>
</tr>
<tr>
<td>( F_s )</td>
<td>Average fines content (&lt;0.075 mm) in ( T_s ), (%)</td>
</tr>
<tr>
<td>( F_0 )</td>
<td>Average fines content (&lt;0.075 mm) in ( T_0 ), (%)</td>
</tr>
<tr>
<td>( F_f )</td>
<td>Average fines content (&lt;0.075 mm) in ( T_f ), (%)</td>
</tr>
<tr>
<td>( F_m )</td>
<td>Average fines content (&lt;0.075 mm) in ( T_m ), (%)</td>
</tr>
</tbody>
</table>

* Indicates independent variables used in final MLR models.

### A1.3.1 Geological and Soil Measures from the Simplified Procedure

For a given \( (N1)0 \) value, the liquefaction analysis program compares the cyclic stress ratio generated by the earthquake, CSRQ, to the cyclic stress ratio required to generate liquefaction in the soil profile, CSRL. If CSRQ exceeds CSRL, then the soil is assumed to have liquefied. CSRQ is a function of the earthquake magnitude, maximum ground acceleration, and the total vertical and effective vertical in-situ stresses. Based on laboratory cyclic shear tests, Seed and Idriss (1982) defined the average cyclic shear stress developed on a horizontal plane during cyclic loading as \( \tau_h \). They showed that \( \tau_h \) can be approximated by:

\[
\tau_h = 0.65\left(a_{\text{max}}/g\right)\sigma_v\rho_d
\]  
(A1.3.1.1)

where:
- \( a_{\text{max}} \) = maximum ground acceleration, (g)
- \( \sigma_v \) = in-situ vertical stress, (force/length²).
- \( \rho_d \) = stress reduction factor, (unitless).

The stress reduction factor, \( \rho_d \), is used to decrease \( \tau_h \) with increasing depth. The value of \( \rho_d \) is a function of depth, \( z \), and differs according to soil type. An average value for \( \rho_d \) is calculated from the following equations (Liao, 1986):

\[
\rho_d = 1 - 0.00765 \ z \text{ (for } z \leq 10 \text{ m)}
\]  
(A1.3.1.2)

\[
\rho_d = 1.174 - 0.0276 \ z \text{ (for } z > 10 \text{ m)}
\]  
(A1.3.1.3)
To calculate the cyclic stress ratio induced by the earthquake, CSRQ, (unitless), the value of $\tau_b$ is divided by the in-situ effective stress:

$$CSRQ = \frac{\tau_b}{\sigma'_e} = 0.65(a_{\max}/g) \times (\sigma'_e/\sigma'_e) \times \tau_b.$$  \hspace{1cm} (A1.3.1.4)

The value of CSRL, (unitless), is calculated from $(Nl)_{cw}$ and is adjusted for $\sigma'_e$, the amount of energy delivered by the driving hammer, and the fines content of the soil (particle size $\leq 0.075$ mm). To calculate CSRL, each SPT N must first be normalized to an effective stress of 100 kPa by using the equation:

$$N_i = C_N \times N$$  \hspace{1cm} (A1.3.1.5)

where:

- $N_i$ = the normalized blow count
- $C_N$ = an overburden correction factor.

The value of $C_N$ is estimated from (NRC, 1985):

$$C_N = (\sigma'_e)^{1/2}/10$$  \hspace{1cm} (A1.3.1.6)

where:

- $\sigma'_e$ = the in-situ effective stress, kPa.

Each $N_i$ value must also be corrected for the measured hammer energy delivered to the drill stem. A hammer energy ratio of 60 percent is used as the standard:

$$(Nl)_{cw} = N/60 \times ER_m$$  \hspace{1cm} (A1.3.1.7)

where:

- $ER_m$ = the hammer energy ratio, (%).

The value obtained from Equation A1.3.2.7 is the corrected, blow count (i.e., $(Nl)_{cw}$) used to calculate CSRL. The liquefaction analysis program also has the capability to convert cone penetration test (CPT) data to SPT $(Nl)_{cw}$ values by using a relationship developed by Seed and DeAlba (1986) and extended to larger grain sizes by Andrus and Youd (1989).

The CSRL for a given earthquake magnitude is determined from CSRL versus SPT $(Nl)_{cw}$ curves develop by Seed et al. (1983, 1985; NRC 1985; see Figure A1-2). These curves are only valid for clean sands and must be adjusted for the fines content of the soil (particle size $\leq 0.075$ mm). Figure A1-3 shows how CSRL varies with fines content for a $M = 7.5$ earthquake. The liquefaction analysis program uses linear interpolation to adjust these CSRL curves for various earthquake magnitudes. It also corrects CSRL for the percentage of fines in the soil by linearly interpolating between the CSRL curves in shown Figure A1-3. (Currently, no guidelines exist on how to extrapolate the CSRL curves for soils with a fines content $> 35$ percent. For these soils, CSRL values are not extrapolated beyond the 35 percent fines content CSRL curve, but are set equal to this curve by the liquefaction analysis program.)

The liquefaction analysis program also calculates a factor of safety against liquefaction, FS (unitless), for each $(Nl)_{cw}$ value:

$$FS = \frac{CSRL}{CSRQ}.$$  \hspace{1cm} (A1.3.1.8)

where:

- CSRL and CSRQ are previously defined.
Increments in granular layers located below the water table and with FS < 1.0 are identified as potentially liquefiable. Increments not found to be susceptible to liquefaction according to the criteria given by Seed and Idriss (1983; see also Section 2.2) are not marked as liquefiable.

Figure A1-4 is an illustration of a liquefaction analysis for the borehole shown in Figure A1-1 and summarized in Table A1-1. The left side of the diagram shows SPT (N1) values versus depth. The right side of the diagram is a plot of CSRQ and CSRL versus depth. The solid line on the right side of the diagram is CSRQ, and the dotted line is CSRL. Layers that are susceptible to liquefaction are indicated in zones where the dotted line falls on the left side of the solid line (i.e., FS < 1).

Table A1-7 is an example of the computer printout for the liquefaction analysis shown in Figure A1-4. Printouts of this type are included in Appendix 4 for all of SPT boreholes included in this study. These analyses are printed in ASCII format and are grouped together according to earthquake (see Table A1-2). Each printout lists the borehole identification number, earthquake, site name, SPT and soil data, and the input parameters for the analysis. The last three columns of the printout give the values of CSRQ, CSRQ, and the factor of safety against liquefaction, FS. The start of each liquefiable zone is identified by the letter "S", and the end of each liquefiable zone is marked with an "E" (Table A1-7).

We reduced and tested several thickness, depth, SPT N, factor of safety, and soil measures (i.e., T_s, I_s, Z_{LS}, Z_{RLS}, Z_{St}, J_{Ss}, N_{140}, K_p, O_s, D_{50s}, and F_s) using the simplified procedure for liquefaction analysis. The accumulative thickness of liquefiable sediments was tabulated as the variable, T_s. (For example, the value of T_s for the borehole shown in Figure A1-4 is 16.0 m; see also Tables A1-1 and A1-7.) Soils that are not liquefiable according to the criteria given by Seed et al. (see Section 2.2) were not accumulated. We also calculated an "Index of Liquefaction Potential," I_s, for each SPT profile (Hamada et al., 1986). This index measures the liquefaction susceptibility of the profile and is a function of FS and T_s.
\[ I_s = \Sigma (1 - FS) dz \]

(A1.3.1.9)

where:
\( \Sigma (1 - FS) \) is evaluated at 0.1 m increments for all liquefiable zone(s) in the profile.

In general, thick, highly susceptible profiles have high values of \( I_s \) and thin, marginally susceptible profiles have low values. If liquefaction analyses indicated that no liquefaction occurred in a given profile, then \( I_s \) was set equal to zero.

We also compiled several depth measures and tested them in our MLR models. The variables \( Z_{TLS} \) and \( Z_{RSL} \) were assigned to the depth, \( Z \), to the top and bottom of the liquefied zone, respectively. If more than one zone was shown to be potentially liquefiable, then the tabulated value of \( Z_{TLS} \) corresponded to the top of the uppermost zone, and \( Z_{RSL} \) corresponded to the bottom of the deepest zone. (The values of \( Z_{TLS} \) and \( Z_{RSL} \) are 0.0 and 19.8 m, respectively for the analysis shown in Table A1-7.) We also compiled and tested the depth, \( Z_s \), (m), that corresponded to the lowest FS in the profile. (For example, \( Z_s \) is 3.05 m for the borehole given in Table A1-7.)

Figure A1-3 Relationship between CSRL and \( N_{IBo} \) values for silty sands for \( M = 7.5 \) earthquakes (NRC, 1985).
Figure A1-4  SPT \((N1)_{60}\) values and cyclic stress ratios (CSR) for SPT borehole at Railroad Bridge Milepost 147.1, Matanuska River, Alaska.
Several low and average FS, N, (N1)0 variables were also tested in our MLR models. The lowest factor of safety in the liquefied profile was tabulated as the variable Jk. (The value of Jk for the borehole in Table A1-7 is 0.35.) We also defined N1000 as the (N1)0 value corresponding to Jk. (N1000 for our example is 10.8 (Table A1-7)). We also calculated the average factor of safety in Tg and tabulated it as the variable Kg. Values of FS corresponding to interpolated values of N were not included in this average. (Kg for the sample borehole is 0.60 (see Tables A1-7 and A1-1)). Additionally, we calculated the average (N1)0 value in Tg and tabulated it as the variable Og. (The value of Og for the example borehole is 13.6, see Tables A1-7 and A1-1). Interpolated (N1)0 values were not included in this average.

Measured values of the mean grain size, D90 (mm), were also averaged in Tg and assigned to the variable D50g (see Tables A1-1 and A1-7). Likewise, measured values of the percentage of fines (particles size < 0.075 mm) were averaged in Tg and assigned to the variable Fg. Interpolated values of D90 and the percentage of fines were not used in these averages. Also layers located above the water-table or soils with a clay content ≥ 15 percent were not included in these averages.

### A1.3.2 Liao's Probability Curves for the Simplified Procedure

Liao (1986) analyzed 278 sites of liquefaction or non-liquefaction to develop a best-fit, probabilistic model to predict liquefaction susceptibility. In Liao's procedure, CSRQ is normalized for the magnitude of the earthquake instead of CSRL:

\[
CSRQN = \frac{CSRQ}{MNF} \tag{A1.3.2.1}
\]

where:

- CSRQN = normalized cyclic stress ratio, (unitless)
- MNF = magnitude normalization factor, (unitless).

The value of MNF is given by:

\[
MNF = 0.032 M^2 - 0.631 M + 3.9334 \tag{A1.3.2.2}
\]

where:

- M = earthquake magnitude.

Liao formulated a set of probability curves to determine CSRL based on SPT (N1)0 values (Figure A1-5). For this study, we selected the 50 percent probability of liquefaction curve to calculate our independent variables. The 50 percent probability curve for soils with a percentage of fines ≤ 12 percent is:

\[
CSRL = \exp((0.39760((N1)0) - 16.447)/6.4603). \tag{A1.3.2.3}
\]

The 50 percent probability of liquefaction curve for soils with a percentage of fines > 12 percent is:

\[
CSRL = \exp((0.18190((N1)0) - 6.4831)/2.6854). \tag{A1.3.2.4}
\]

The liquefaction program uses these curves to calculate FS. Zones in the profile below the water table where FS < 1.0 are marked as liquefiable.
TABLE A1-7
EXAMPLE OF SPT DATA AND LIQUEFACTION ANALYSIS FOUND IN APPENDIX 4

LIQUEFACTION ANALYSIS
BOREHOLE ID. NO.: 13
EARTHQUAKE: 1964 ALASKA
SITE: RAILROAD BRIDGE MP 147.1
HAMMER ENERGY RATIO: 45?
DEPTH TO GROUNDWATER:
MAX. ACCELERATION AT SITE:
MAGNITUDE: 9.2

Simplified Procedure Curves

FIELD BLOW COUNT DATA, BOUNDARIES AND LIQUEFIABLE ZONES

<table>
<thead>
<tr>
<th>DEPTH (M)</th>
<th>SOIL DESCRIPTION</th>
<th>N</th>
<th>N160</th>
<th>%E</th>
<th>%C</th>
<th>D50 (MM)</th>
<th>DRY MOIST RATIO</th>
<th>LIQ. RATIO</th>
<th>FS</th>
<th>WT (KN/m3)</th>
<th>WT (KN/m3)</th>
<th>LIQ. EARTHQUIKE</th>
<th>CSRQ</th>
<th>CSRL</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>.F. BOUNDARY</td>
<td>13.0</td>
<td>11.7</td>
<td>2 0</td>
<td>3.269</td>
<td>14.150</td>
<td>18.080</td>
<td>0.109</td>
<td>0.137</td>
<td>0.80</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-2.44</td>
<td>.F. FINE GRAVEL - C. SAND</td>
<td>13.0</td>
<td>11.7</td>
<td>2 0</td>
<td>3.269</td>
<td>14.150</td>
<td>18.080</td>
<td>0.109</td>
<td>0.282</td>
<td>0.39</td>
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* = DATA LINE INPUT BY USER
? = ESTIMATED OR INTERPOLATED VALUE
L = LIQUEFACTION CODE, T = MARKED BY USER AS NONLIQUEFIABLE LAYER,
F = MARKED BY USER AS POSSIBLY LIQUEFIABLE
%E = PERCENT FINES (<0.075mm), %C = PERCENT CLAY (<0.005mm)
S = START OF LIQUEFIED ZONE, E = END LIQUEFIED ZONE

LIQUEFACTION POTENTIAL: 6.32
Figure A1-5 Contours of equal probability of liquefaction for: (a) clean sands (i.e., fines content ≤ 12%), and (b) silty sands (i.e., fines content > 12%) (Liao, 1986).
We used the 50 percent probability of liquefaction curves to determine the values of $T_L$, $I_L$, $Z_{TLL}$, $Z_{ELL}$, $Z_L$, $J_L$, $N_{MLR}$, $X_L$, $Q_L$, $D50_L$, and $F_L$ in the same manner as we did for the simplified procedure (see Section A1.3.1). The results of these analyses are tabulated on the summary sheets in Appendix 4 (for an example of these summary sheets, see Table A1-1). In general, preliminary MLR models based on variables determined from the simplified procedure curves and Liao's 50 percent probability curves were comparable. We have not included the liquefaction analyses printouts using Liao's 50 percent probability curve in Appendix 4 because none of the variables determined from this procedure were used in our final MLR model. However, the summary sheets found in Appendix 4 summarize the results of these analyses (for example, see the bottom of Table A1-1).

A1.4 Weighted Averaging Routine

For many of the liquefaction sites, more than one SPT borehole was located within the failure zone. Thus, we needed a systematic method of assigning borehole measures to individual displacement vectors. We selected an inverse-distance, linearly-weighted average to interpolate all geological and soil factors listed in Tables A1-5 and A1-6 between boreholes. This averaging scheme assigns the largest weight to the borehole(s) located closest to the displacement vector:

$$X_{AVG} = W_1^*_X_1 + W_2^*_X_2 + \ldots + W_n^*_X_n$$  \hspace{1cm} (A1.4.1)

where:

$X_{AVG}$ = weighted average
$W_1,\ldots,W_n$ are the weights.

These weights are calculated from:

$$W_i = 1/d_i/z(1/d_i)$$  \hspace{1cm} (A1.4.2)

where:

d_i = distance from $i^{th}$ borehole to the displacement vector of interest (see Figure A1-6).

The geological and soil measures tabulated in MLR.DAT were averaged using Equation A1.4.1. A dBase program was written to perform the interpolation and was designed to include up to 4 boreholes in the weighted average. Appendix 3, MLR.DAT, lists the borehole identification numbers (BOREID#) for each of the boreholes used in the weighted average. The corresponding SPT logs and liquefaction analyses are found in Appendix 4 and are identified by the same identification number. Also, the distances from the displacement vector to each of the boreholes (i.e., the d_i's) are tabulated in MLR.DAT in Appendix 3 as the variables BOREDIST1, BOREDIST2, BOREDIST3, and BOREDIST4.
BOREHOLE #1
X1 = 2

BOREHOLE #2
X2 = 5

BOREHOLE #3
X3 = 3.5

\[ X_{\text{avg}} = \frac{W_1 X_1 + \ldots + W_n X_n}{\sum_{i=1}^{n} (1/di)} \]

where
\[ W_i = \frac{1}{d_i} \]

example:
\[ W_1 = \frac{1}{60}/(1/60+1/25+1/70) = 0.235 \]
\[ W_2 = \frac{1}{25}/(1/60+1/25+1/70) = 0.564 \]
\[ W_3 = \frac{1}{70}/(1/60+1/25+1/70) = 0.201 \]

\[ X_{\text{avg}} = \sum_{i=1}^{n} W_i X_i = 0.235(2) + 0.564(5) + 0.201(3.5) \]

Figure A1-6 Weighted average used to assign borehole measurements to displacement vectors.
APPENDIX 2
MINITAB ANALYSES

A2.1 Minitab Analysis for Equation 3.4.10

MINITAB OUTPUT FOR EQUATION 3.4.10 (FREE FACE MODEL FOR NIIGATA, JAPAN)

MTB > REGRESS LOG DH ON LOG W, T15, F15, D5015, N160S

The regression equation is

\[
\text{LOG DH} = 0.610 + 0.572 \text{ LOG W} + 0.0247 \text{T15} - 0.0278 \text{ F15} - 1.61 \text{ D5015} - 0.0315 \text{ N160S}
\]

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\[s = 0.1870 \quad \text{R-sq} = 72.4\% \quad \text{R-sq(adj)} = 71.3\%\]

Analysis of Variance

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Unusual Observations

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R denotes an obs. with a large st. resid.  
X denotes an obs. whose X value gives it large influence.
MTB > STANDARDIZED RESIDUAL PLOT FOR EQUATION 3.4.10

MTB > STANDARDIZED RESIDUAL PLOT FOR EQUATION 3.4.10

MTB > STANDARDIZED RESIDUAL PLOT FOR EQUATION 3.4.10

A2-2
### A2.2 Minitab Analysis for Equation 3.5.3

**Minitab Output for Equation 3.5.3 (Ground Slope Model for Niigata and Noshiro, Japan)**

MTB > REGRESS LOG DH ON 4 LOG S, T15, F15, D5015

The regression equation is

\[
\text{LOG DH} = 0.698 + 0.378 \text{ LOG S} + 0.0362 \text{ T15} - 0.0326 \text{ F15} - 0.929 \text{ D5015}
\]

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\( s = 0.1454 \quad R^2 = 54.2\% \quad R^2\text{(adj)} = 53.4\% \)

**Analysis of Variance**

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R denotes an obs. with a large st. resid.
X denotes an obs. whose X value gives it large influence.
MTB > STANDARDIZED RESIDUAL PLOT FOR EQUATION 3.5.3

2.0+  *  *  2  *  *  4  2
-  4
es -  3  2  *  2  3  4  *  3  *  2
-  4  3  4  4  2  *  *  *  3
-  5  *  4  3  *  *  *  *  2  6  *  6
0.0+ 7  2  *  6  2  3  3  3  *  2  5
-  4  4  *  3  4  *  5  5  2
-  7  4  *  4  *  *  5  5  *  3  *  3
-  *  *  *  *  3  *  4  *
-  *  *  *  *  4  *
-2.0+ 2
-  *  *  *
-  *
-  *
0.0  2.5  5.0  7.5  10.0  12.5

MTB > STANDARDIZED RESIDUAL PLOT FOR EQUATION 3.5.3

2.0+  *  2
-  4
es -  22  *  3  *  5  *  4  *  2
-  *  *  *  *  2
-  36  *  3  *  4  *  2
0.0+ 34  *  7  *  2  2  2
-  *  2  *  3  *  2  4  *  2
-  *  *  *  22  *  *  *  2  3
-  *  *  *  *  2  *  *  *
-2.0+  *  *  2
-  *
-  *
-  *
0.160  0.240  0.320  0.400  0.480  0.560

A2-6
A2.3 Minitab Analysis for Equation 4.1.9

Minitab Output for Equation 4.1.9 (Combined Model for U.S. and Japan Data)

MTB > REGRESS LOG(DH+0.01) ON BOFF, M, LOG R, LOG Wff, LOG Sgs,
     LOG T15, LOG(100-F15), D5015

The regression equation is:

\[ \text{LOG DH} = -15.8 - 0.579 \text{ Boff} + 1.18 \text{ M} - 0.927 \text{ LOG R} - 0.0133 \text{ R} + 0.657 \text{ LOG Wff} \\
+ 0.429 \text{ LOG Sgs} + 0.348 \text{ LOG T15} + 4.53 \text{ LOG(100-F15)} - 0.922 \text{ D5015} \]

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\[ s = 0.2086 \quad R^2 = 82.6\% \quad R^2(\text{adj}) = 82.3\% \]

Analysis of Variance

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R denotes an obs. with a large st. resid.
X denotes an obs. whose X value gives it large influence.
MTB > STANDARDIZED RESIDUAL PLOT FOR EQUATION 4.1.9

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  -     *
    4* ***
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    *85*4 6+++33*
  0.0+ *28 3*9++6++5* *2 *
  -     *25 *28+86+433
    5*23347427 *2 **
    *  ** 3  **
    *2** **2
-3.0+ ** *
   -     *
   -     ** *
   -     * *
-6.0+ -------------------------------D5015
  0.00  0.30  0.60  0.90  1.20  1.50
```

A2-12