Seismic Analysis and Design of Bridge Abutments Considering Sliding and Rotation

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K.L. Fishman and R. Richards, Jr.

Technical Report NCEER-97-0009
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PREFACE

The National Center for Earthquake Engineering Research (NCEER) was established in 1986 to develop and disseminate new knowledge about earthquakes, earthquake-resistant design and seismic hazard mitigation procedures to minimize loss of life and property. The emphasis of the Center is on eastern and central United States structures, and lifelines throughout the country that may be exposed to any level of earthquake hazard.

NCEER’s research is conducted under one of four Projects: the Building Project, the Nonstructural Components Project, and the Lifelines Project, all three of which are principally supported by the National Science Foundation, and the Highway Project which is primarily sponsored by the Federal Highway Administration.

The research and implementation plan in years six through ten (1991-1996) for the Building, Nonstructural Components, and Lifelines Projects comprises four interdependent elements, as shown in the figure below. Element I, Basic Research, is carried out to support projects in the Applied Research area. Element II, Applied Research, is the major focus of work for years six through ten for these three projects. Demonstration Projects under Element III have been planned to support the Applied Research projects and include individual case studies and regional studies. Element IV, Implementation, will result from activity in the Applied Research projects, and from Demonstration Projects.
Research under the **Highway Project** develops retrofit and evaluation methodologies for existing bridges and other highway structures (including tunnels, retaining structures, slopes, culverts, and pavements), and develops improved seismic design criteria and procedures for bridges and other highway structures. Specifically, tasks are being conducted to: (1) assess the vulnerability of highway systems and structures; (2) develop concepts for retrofitting vulnerable highway structures and components; (3) develop improved design and analysis methodologies for bridges, tunnels, and retaining structures, with particular emphasis on soil-structure interaction mechanisms and their influence on structural response; and (4) review and improve seismic design and performance criteria for new highway systems and structures.

**Highway Project** research focuses on one of two distinct areas: the development of improved design criteria and philosophies for new or future highway construction, and the development of improved analysis and retrofitting methodologies for existing highway systems and structures. The research discussed in this report is a result of work conducted under the new highway construction project, and was performed within Task 112-D-3.4, “Develop Analysis and Design Procedures for Retaining Structures” of the project as shown in the flowchart on the following page.

*The objective of this task is to generalize the currently-used sliding block procedure to determine seismic displacements of walls and abutments which include mixed-mode behavior with rotation and/or bearing capacity movement. In this report, a revised procedure for determining permanent displacement of rigid walls due to earthquake motion is described. This procedure was verified via shake table testing of model bridge abutments and retaining walls which fail by a coupled sliding/rotation mode. The report concludes with recommendations regarding this mixed-mode behavior for inclusion in a seismic analysis or design of bridge abutments.*
ABSTRACT

Current displacement based seismic design of gravity retaining walls utilizes a sliding block idealization, and considers only a translation mode of deformation. However, results from recent studies demonstrate the possibility of seismic loss of bearing capacity and subsequent rotation or mixed mode of deformation. The purpose of the task described herein is to generalize the sliding block procedure to determine seismic displacements of walls and bridge abutments to include mixed-mode behavior with rotation due to bearing-capacity movement.

Others have proposed this more complex scenario be described with coupled equations of motion cast in terms of relative acceleration between the retaining wall, and the foundation soil. Equations of motion consider the seismic resistance of the retaining wall and coupling between rectilinear and angular accelerations. Coupled equations of motion are double integrated with respect to time to compute relative displacements and rotations.

The authors of this report have updated, and extended the coupled equations of motion that appear in the literature. A newly developed fundamental theory on seismic bearing capacity of soils is incorporated. The theory is used to compute the seismic resistance of a retaining wall or bridge abutment and the resisting moment offered by the foundation soil. Also, equations presented are extended to consider the case of bridge abutments and load transfer from the bridge decks.

Algorithms for predicting permanent deformations were applied to a number of test cases that were modeled in the laboratory. Model bridge abutments were constructed within a seismic testing chamber, and seismic loading was applied to the models via a shaking table. Compared to previous studies described in the literature, models were unique in the sense that they were not constrained to a particular mode of failure. Failure was possible by sliding, tilting or a combination of both. The mode of failure could be accurately predicted and depended on model parameters and properties of the backfill and foundation soil. Comparisons between observed and computed model responses serve to verify the ability of the proposed algorithms to predict sliding, tilting or mixed modes of deformation. Thus displacement based seismic design is now possible for all modes of wall movement and not just translation.
TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>SECTION</th>
<th>TITLE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>1.1 Displacement Based Design</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>1.2 Settlement and Rotation</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>1.3 Need for Model Tests</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>THEORY AND METHOD OF ANALYSIS</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>2.1 Equations of Motion</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>2.2 Seismic Bearing Capacity</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>2.3 Determination of Threshold Levels of Acceleration</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>2.4 Foundation Soil Moment Resistance</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>2.5 Numerical Integration of Accelerations</td>
<td>13</td>
</tr>
<tr>
<td>3</td>
<td>SHAKING TABLE EXPERIMENTS</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>3.1 Description of Model Tests</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>3.2 Results from Experiments</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>3.2.1 Data Validation</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>3.2.1.1 Data Reduction</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td>3.2.2 Translation Mode</td>
<td>23</td>
</tr>
<tr>
<td></td>
<td>3.2.3 Rotation Mode</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td>3.2.4 Mixed Mode</td>
<td>29</td>
</tr>
<tr>
<td></td>
<td>3.3 Discussion of Results</td>
<td>33</td>
</tr>
<tr>
<td>4</td>
<td>CONCLUSIONS</td>
<td>35</td>
</tr>
<tr>
<td></td>
<td>4.1 Summary of Conclusions</td>
<td>35</td>
</tr>
<tr>
<td></td>
<td>4.2 Recommendation for Seismic Analysis</td>
<td>36</td>
</tr>
<tr>
<td></td>
<td>4.3 Recommendations for Future Research</td>
<td>37</td>
</tr>
<tr>
<td>5</td>
<td>REFERENCES</td>
<td>39</td>
</tr>
</tbody>
</table>

APPENDIX

<table>
<thead>
<tr>
<th>APPENDIX</th>
<th>TITLE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>DERIVATION OF EQUATIONS OF MOTION</td>
<td>A-1</td>
</tr>
<tr>
<td>B</td>
<td>ABUTMENTS WITH CONSTRAINT</td>
<td>B-1</td>
</tr>
<tr>
<td>C</td>
<td>DISPLACEMENT AND ROTATION COMPUTER PROGRAM</td>
<td>C-1</td>
</tr>
</tbody>
</table>
# LIST OF ILLUSTRATIONS

<table>
<thead>
<tr>
<th>FIGURE</th>
<th>TITLE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-1</td>
<td>Free Body Diagram of Bridge Abutment with Free Connection to Bridge Deck.</td>
<td>5</td>
</tr>
<tr>
<td>2-2</td>
<td>Free Body Diagram of Bridge Abutment with Fixed Connection to Bridge Deck.</td>
<td>5</td>
</tr>
<tr>
<td>2-3</td>
<td>Assumed Failure Mechanism for Seismic Bearing Capacity.</td>
<td>7</td>
</tr>
<tr>
<td>2-4</td>
<td>Seismic Bearing Capacity Factors.</td>
<td>8</td>
</tr>
<tr>
<td>2-5</td>
<td>Limit State for a Bridge Abutment Due to Seismic Loading.</td>
<td>10</td>
</tr>
<tr>
<td>3-1</td>
<td>Schematic of Bridge Abutment Model.</td>
<td>16</td>
</tr>
<tr>
<td>3-2</td>
<td>Inertial Reference Frames for Simple Relative Motion.</td>
<td>18</td>
</tr>
<tr>
<td>3-3</td>
<td>Observed Time History of Relative Displacement for Bridge Abutment Models.</td>
<td>20</td>
</tr>
<tr>
<td>3-4</td>
<td>Verification of Model I Relative Displacement Measurements.</td>
<td>21</td>
</tr>
<tr>
<td>3-5</td>
<td>Verification of Model II Relative Displacement Measurements.</td>
<td>21</td>
</tr>
<tr>
<td>3-6</td>
<td>Verification of Model III Relative Displacement Measurements.</td>
<td>22</td>
</tr>
<tr>
<td>3-7</td>
<td>Backfill Sand Height vs. Applied Horizontal Acceleration For Model I.</td>
<td>23</td>
</tr>
<tr>
<td>3-8</td>
<td>Predicted Threshold Accelerations for Model I.</td>
<td>24</td>
</tr>
<tr>
<td>3-9</td>
<td>Typical Wave Amplification for Model I.</td>
<td>25</td>
</tr>
<tr>
<td>3-10</td>
<td>Cutoff Acceleration for Sliding Block Model.</td>
<td>26</td>
</tr>
<tr>
<td>3-11</td>
<td>Comparison of Predicted and Observed Relative Displacement for Model I.</td>
<td>27</td>
</tr>
<tr>
<td>3-12</td>
<td>Comparison of Predicted and Observed Relative Displacement for Model III for Acceleration Pulse Input.</td>
<td>28</td>
</tr>
<tr>
<td>3-13</td>
<td>Comparison of Predicted and Observed Relative Displacement for Model III for Sine Ramp Input No. 1.</td>
<td>28</td>
</tr>
<tr>
<td>3-14</td>
<td>Comparison of Predicted and Observed Relative Displacement for Model III for Sine Ramp Input No. 2.</td>
<td>29</td>
</tr>
<tr>
<td>3-15</td>
<td>Comparison of Predicted and Observed Relative Displacement for Model II for Acceleration Pulse Input.</td>
<td>30</td>
</tr>
<tr>
<td>3-16</td>
<td>Comparison Between Observed and Predicted Angular Acceleration for Model II.</td>
<td>31</td>
</tr>
<tr>
<td>3-17</td>
<td>Predicted and Observed Time History of Displacement for an Acceleration Pulse Applied to Model II.</td>
<td>32</td>
</tr>
<tr>
<td>3-18</td>
<td>Time History of Acceleration for a Sine Ramp Applied to the Base of Model II.</td>
<td>32</td>
</tr>
<tr>
<td>3-19</td>
<td>Comparison of Predicted and Observed Relative Displacement for Model II from Sine Ramp Input.</td>
<td>33</td>
</tr>
</tbody>
</table>
# LIST OF TABLES

<table>
<thead>
<tr>
<th>TABLE</th>
<th>TITLE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-I</td>
<td>Model I Parameters</td>
<td>23</td>
</tr>
<tr>
<td>3-II</td>
<td>Model III Parameters and Estimated Threshold Accelerations</td>
<td>27</td>
</tr>
<tr>
<td>3-III</td>
<td>Model II Parameters and Estimated Threshold Accelerations</td>
<td>29</td>
</tr>
</tbody>
</table>
Section 1
INTRODUCTION

1.1 Displacement Based Design

Richards and Elms (1979) introduced the concept of displacement based seismic design to free standing, gravity retaining walls or bridge abutments. The design procedure is well documented in the AASHTO specifications and commentary (1993), and involves finding the wall weight required to limit permanent displacement to tolerable levels. Analysis of seismic induced permanent displacement is an integral part of the design procedure.

Seismic induced permanent deformation is controlled by the seismic resistance of the retaining structure. Seismic resistance is quantified by a threshold level of acceleration, beyond which the wall yields and permanent deformation is initiated. The wall will translate progressively with respect to the foundation soil whenever the ground acceleration exceeds the threshold. The total displacement is found by integrating the relative acceleration between the wall and foundation soil twice until the relative velocity is zero.

The dynamic active earth pressure transferred to the wall by the backfill, the inertia of the bridge abutment or retaining wall, and the shearing resistance between the base of the wall and the foundation soil relates to seismic resistance. Dynamic active earth pressure is calculated using equations derived by Mononobe and Matsuo (1929) and Okabe (1926). The Mononobe-Okabe (M-O) equations assume that the backfill behaves as a rigid plastic material, and seismic forces are pseudostatic.

The original work by Richards and Elms (1979) considered only the possibility of a sliding mode of deformation. However, as cited by Siddharthan et al. (1992), past earthquake damage reports and laboratory tests indicate that wall failure by rotation is quite common (Seed and Whitman (1970), Paruvokat (1984), Bolton and Steedman (1989)).

1.2 Settlement and Rotation

Based on the work of Nadim and Whitman (1984), Siddharthan et al. (1992) extended the displacement analysis by incorporating seismic induced tilting of rigid walls. Coupled equations of motion were implemented describing the relationships between wall translation, rotation, and the forces and moments acting on the wall system. Moment resistance applied to the base of the wall was considered in the analysis and depended on the bearing capacity of the foundation soil. Bearing capacity was evaluated with the bearing capacity factors presented by Vesic (1974) including effects of eccentric loading and shear transfer. These bearing capacity factors are for static loads, and seismic reduction of bearing capacity was not included in the analysis.
Results from recent analytic and laboratory studies demonstrate the possibility of a seismic reduction in bearing capacity beneath bridge abutments and retaining walls founded on spread footings. Based on an assumed "Coulomb type" failure mechanism beneath the footing seismic bearing capacity factors are determined from a limit equilibrium analysis which includes inertial body forces within the failed region of foundation soil (Richards et al (1993), Shi (1993)).

In this report a revised procedure for determination of permanent displacement of rigid walls due to earthquake excitation will be presented. Coupled sliding and rotation will be described, as before by Siddharthan et al. (1992), but the possibility of seismic loss of bearing capacity will also be included in the analysis.

1.3 Need for Model Tests

Although the ability to predict permanent seismic displacement with a sliding mode of failure has already been verified in the laboratory (Lai (1979), Lai and Berrill (1979), Aitken (1982), Steedman (1984), Elms and Woods (1986), Uwabe and Moriya (1988), Elms and Richards (1990), Whitman (1990)) there is a need to extend this data base to include studies of coupled sliding and rotation. An important contribution contained in this report is to present results from shake table testing of model bridge abutments and retaining walls which fail by a coupled sliding/rotation mode. The tests described herein are an improvement over previous studies (Aitken (1982), Musante and Ortigosa (1984), Anderson et al (1987), and Whitman (1990)), in the sense that the foundation soil beneath the abutment is included, and the model is not constrained to a tilting mode of failure, but rather any possible mode of failure is allowed.
Section 2
THEORY AND METHOD OF ANALYSIS

Guidelines and recommendations for aseismic design and analysis of bridge abutments are based on the Richards and Elms (RE) limited displacement method (AASHTO, 1993). The RE rigid-plastic model assumes that a bridge abutment fails by excessive translation, and does not consider the possibility of rotation. A practical method for predicting coupled rotation and translation of bridge abutments due to seismic loading has not been verified. In what follows a simple and more general method for estimating seismically induced permanent deformation of bridge abutments and retaining walls is presented and verified. The method is based on the original work of Nadim and Whitman (1984), and Siddharthan et al. (1990, 1991, and 1992).

2.1 Equations of Motion


Figure 2-1 is a free body diagram of a retaining wall subjected to seismic forces which induce active earth pressures in the backfill. Inertial forces are applied according to d'Alembert's principle. Much like the Richards and Elms (1979) approach to translating walls Newton's fundamental laws of motion are applied to arrive at the coupled equations of motion proposed by Siddharthan et al. (1991). Complete details of the derivation are provided in Appendix A. Equations 2.1 and 2.2 are the coupled equations of motion described by Siddharthan et al. (1992).

\[
\frac{W}{g} (\ddot{X}) + \frac{WR}{g} (\dot{\theta} \frac{\sin(\gamma + \phi_b)}{\cos(\theta)}) = \frac{W}{g} (\dot{X}_g(t)) + P_{AE} \cos(\delta) - \left[ \frac{W}{g} (\dot{Y}_g(t)) + P_{AE} \sin(\delta) \right] \tan(\delta_t)
\]

\[(2.1)\]

\[
\frac{W}{g} \left( \dddot{X} \right) \sin(\eta) + \left[ I_g + \left( \frac{WR^2}{g} \right) \right] \ddot{\theta} = \frac{W}{g} (\dot{X}_g(t)) \sin(\eta) - \left[ 1 - \frac{\dot{Y}_g(t)}{g} \right] WR \cos(\eta)
\]

\[P_{AE} (h) \cos(\delta) - P_{AE} \sin(\delta) \left[ R \cos(\eta) + a \right] - M_o
\]

\[(2.2)\]

where:
\( \ddot{X}_g(t) \) = horizontal ground acceleration,

\( \ddot{Y}_g(t) \) = vertical ground acceleration,

\( \ddot{x} \) and \( \ddot{\theta} \) = relative horizontal and angular acceleration of the wall through and about the center of gravity (CG), respectively,

\( x \) and \( \theta \) = relative wall displacement and rotation through or about the center of gravity (CG), respectively,

\( M_0 \) = soil moment resistance,

\( I_{CG} \) = mass moment of inertia about the CG,

\( W \) = wall weight,

\( P_{AE} \) = total backfill thrust,

\( \delta_f \) = wall base/foundation soil interface friction angle

And the geometric variables are:

\( a \) = horizontal distance between the center of gravity and the wall backface,

\( mH \) = height from the abutment base to the line of action of \( P_{AE} \),

\( R \) = radial distance from the center of rotation (CR) to the (CG),

\( \eta \) = angle that \( R \) makes with the horizontal.

An advantage to equations 2.1 and 2.2 is that they involve much less computational work than those originally proposed by Nadim and Whitman (1984). The only unknown variables in equations 2.1 and 2.2 are (\( \ddot{x} \)) and (\( \ddot{\theta} \)).

Siddharthan's equations of motion apply to a retaining wall, but not a pin-connected bridge abutment shown in Figure 2-2. Derivation of the equation of motion for an abutment pinned at the top is presented in Appendix B. The resulting equation of motion is equation 2.3.

\[
\left[ I_{gs} + \frac{W}{g} R^2 \right] \ddot{\theta} = \frac{W}{g} \ddot{X}_g(t) R \sin(\eta) - WR \cos(\eta) + P_{AE} h \cos(\delta) + Nb - SH
\]  

(2.3)

where:

\( N \) = vertical soil resistance,

\( S \) = horizontal soil resistance,

Values for the normal and shear forces at the abutment foundation-soil interface, \( N \) and \( S \), respectively, must be determined. The sliding threshold represents the acceleration that the abutment can resist before sliding. Beyond the sliding threshold acceleration the shear force, \( S \), is:

\[
S = P_{AE} \cos(\delta_w) + k_s W
\]  

(2.4)
Figure 2-1. Free Body Diagram of Bridge Abutment with Free Connection to Bridge Deck

Figure 2-2. Free Body Diagram of Bridge Abutment with Fixed Connection to Bridge Deck
where \( P_{AE} \) is at the limit described by the threshold acceleration level for sliding, \( k_{sl} \).

A newly developed theory describing seismic reduction of bearing capacity (Richards et al. (1993), and Shi (1993)) provides the limit to the normal force, \( N \), at a given level of acceleration.

**2.2 Seismic Bearing Capacity**

Seismic reduction in bearing capacity has been studied by Richards et al. (1990), and (1993), and Shi (1993). Seismic bearing capacity factors are developed considering shear tractions transferred to the soil surface as well as the effect of inertial loading on the soil in the failed region below the footing (Figure 2-3). For simplicity a "Coulomb-type" of failure mechanism is considered within the foundation consisting of an active wedge directly beneath the abutment and a passive wedge which provides lateral restraint with the angle of friction between them of \( \phi/2 \). This empirical choice of \( \delta = \phi/2 \) for the Coulomb Mechanism was first proposed by Richards and Shi (1991) as it is shown to give static bearing capacity within 10% of standard value for \( 10^\circ \leq \phi \leq 40^\circ \) and seismic which is in even closer agreement to a Prandtl mechanism. The seismic bearing capacity by the Coulomb mechanism with \( \delta = \phi/2 \) for variable shear transfer was further verified by Shi (1993) by comparison to the solution by the method of characteristics.

The bearing capacity is evaluated by limit equilibrium analysis whereby critical orientations \( \rho_{AE} \) and \( \rho_{PE} \) of the failure planes are determined to minimize the vertical resistance. Shear transfer between the footing and foundation soil is conveniently described by the coefficient:

\[
\beta = \frac{S}{k_{sl} F_v}
\]  

(2.5)

where:

- \( S \) is the shear traction,
- \( k_{sl} \) is coefficient of horizontal acceleration, and
- \( F_v \) is the normal force applied to the foundation.

The analytic solution gives a bearing capacity formula in terms of seismic bearing capacity factors \( N_{vE}, N_{vE}, N_{vE} \) as

\[
q_{lE} = c N_{vE} + \gamma D N_{qE} + \frac{1}{2} \gamma B N_{vE}
\]  

(2.6)

similar to its counterpart for the static case. For the simplest case of surface footing on sand, only \( N_{vE} \) provides bearing capacity. Figure 2-4 presents the ratio of \( N_{vE}/N_{vE} \), where \( N_{vE} \) is the static case bearing capacity factor, as a function of the friction angle of the foundation soil, \( \phi \), seismic acceleration coefficient, \( k_{sl} \), and the shear transfer coefficient, \( \beta \). Note that the \( \beta = 0 \) curves show the contribution of inertial forces in the foundation soil to the seismic degradation of bearing capacity. Thus the difference between \( \beta = 0 \) and the curve for the actual \( \beta \) value is...
Figure 2-3. Assumed Failure Mechanism for Seismic Bearing Capacity
Figure 2-4. Seismic Bearing Capacity Factors
the contribution of the interface shear traction to the loss in bearing capacity. The corresponding curves for \( N_{ce}/N_{ca} \) and \( N_{ae}/N_{qa} \) are given by (Shi 1993). If vertical accelerations, \( k_v \), are to be included, \( k_h \) should be replaced by \( \tan \theta = k_h/(1-k_v) \). The weight of the wall \( W \) becomes \((1-k_v)W\) for equilibrium calculations.

### 2.3 Determination of Threshold Levels of Acceleration

The seismic vulnerability of gravity wall bridge abutments involves the determination of a threshold acceleration beyond which permanent deformation of the gravity wall will occur. A thorough seismic analysis must investigate the possibility of both a sliding mode of failure as well as a bearing capacity failure introducing rotation. The analysis for the sliding failure mode has been well documented (AASHTO 1993)). Seismic bearing capacity is a new development as applied to gravity wall bridge abutments so details of the analysis follow.

Since seismic bearing capacity factors are dependant on ground acceleration, determination of the threshold acceleration requires an iterative procedure. Referring to Figure 2-5, it is assumed that there is no cohesion or depth of embedment and that \( k_v = 0 \). For walls free to move at the top \( F_{dh} \) is zero or a known value and we can:

1. Assume a trial value for \( k_h \) and determine \( P_{ae} \) from the M-O analysis.
2. Compute the vertical force resultant, \( F_v \), as:

   \[
   F_v = F_{dv} + P_{ae} \sin(\delta_v + \beta) + W_w \tag{2.7}
   \]

3. Compute the resultant of the shear traction to be transferred to the foundation soil as:

   \[
   S = F_{dh} + P_{ae} \cos(\delta_v + \beta) + k_h W_w \tag{2.8}
   \]

4. Compute the factor \( f \) from equation 2.5

5. Sliding will occur if \( S = F_v \tan \delta_f \) and therefore

   \[
   F_v S_{slide} = \frac{\tan \delta_f}{k_h f} \tag{2.9}
   \]

6. Given the friction angle of the foundation soil, \( \phi_f \), and the \( f \) factor from step 4, find the seismic bearing capacity factor, \( N_{re} \) from figure 2-3.

7. Compute the seismic bearing capacity \( q_{re} \) from equation 2.6.

8. Bearing capacity failure will occur when \( F_v = q_{re} B_f \) and therefore:
Figure 2-5. Limit State for a Bridge Abutment Due to Seismic Loading
\[ F.S._{BC} = \frac{q_{le} B_e}{F_v} \]  

(2.10)

Iterate on \( k_b \) to determine the threshold values given when \( F.S. = 1 \). That is:

(9a) If \( F.S._{BC} \) determined in step (8) is nearly equal to one, and \( F.S._{slide} \) from step (5) is greater than one, stop the iteration procedure since the assumed value for \( k_b \) is the threshold value for bearing capacity failure, \( k_{th} \), which occurs first.

(9b) If \( F.S._{slide} \) determined in step (5) is nearly equal to one and \( F.S._{BC} \) is greater than one, stop the iteration procedure since the assumed value for \( k_b \) is the threshold value for sliding failure, \( k_{th} \). In this case when sliding occurs first there is still the potential for a bearing capacity failure at a higher acceleration introducing a mixed mode. To estimate \( k_{th} > k_{th} \) set, \( F_v \) and \( S \) at their constant values for sliding, compute \( N_{re} \) from Equation (2.21) with \( q_{le} = F_v / B \) and determine \( k_{th} \) corresponding to \( N_{re} \) from figure 2-3.

(9c) If neither of the conditions in 9a or 9b is met, select higher trial value for \( k_b \) and return to step (1).

For abutments not free to move outward at the top due to the girder connection details or other reasons, \( F_{Dh} \) will not be zero and the analysis procedure must involve the moment equilibrium equation even if lines of action for \( P_{AE} \) and \( F_v \) are assumed. For the extreme case, the top can be considered pinned and the wall must rotate about the top (RT mode).

However, until the base moves creating the active situation, it acts as a rigid wall. For this case the seismic lateral pressure increment is parabolic giving a thrust \( P_{RE} \) roughly twice the M-O value and the wall/backfill interface friction angle, \( \delta_w \), is close to zero (Wood, 1975).

Therefore, to modify the analytic procedure for walls restrained at the top (where \( F_{Dh} \neq 0 \)) for determining threshold values:

a) In step 2 use \( P_{RE} = 2P_{AE} \) from step 1 and \( \delta_w = 0 \). For a wall with vertical interface

\[ F_v = F_{DV} + W_w \]  

(2.11)

independent of \( k_b \).

b) Assume \( P_{RE} = 2P_{AE} \) acts 0.375H from the top and \( F_v \) acts at the midpoint of the base, \( B_l / 2 \).
c) Take moments about the top of the wall to determine $S$ rather than using horizontal equilibrium (Equation 2.8). For a wall with a vertical interface with its center of gravity at $\bar{Z}$ and $\bar{X}$ from the top:

\[
S = 2P_{ke}(0.375H) + k_sW_w\bar{Z} + F_s(0.5B) - W_w\bar{X}
\]

(2.12)

d) If the value of $F_{dh}$ is desired it can now be computed from Eqn. 2.8.

2.4 Foundation Soil Moment Resistance

The soil resisting moment, $M_0$, is used in equation 2.2. Siddharthan (1991) proposed to determine $M_0$ by modeling the abutment base as a strip foundation resting on Winkler springs. Both full contact and partial contact (lift-off) conditions were considered. However, the use of the Winkler spring model is questionable once the soil reaches its ultimate bearing pressure.

Nadim and Whitman (1983) proposed setting the soil moment resistance equal to zero, considering rotation only about the toe as an initial estimate. However Nadim (1980) shows that this assumption is not necessarily conservative. As with Siddharthan, Nadim and Whitman do not consider a seismic reduction in bearing capacity. However, Nadim and Whitman's (1983) assume that the foundation soil has an ultimate moment capacity, beyond which wall rotation takes place. We will also employ this assumption to compute $M_o$.

We will assume that the soil has a maximum normal resistance and that its line of action is maximized at the toe of the wall. In this manner a limit to the soil moment resistance is calculated. Exceeding this limit creates wall rotation about any CR along the base. Physically, a line of action at the toe is not valid since it implies an infinite stress beneath the toe. However, the model's purpose is to provide a simple, idealized case useful for estimating deformation. In what follows the rational for this assumption is decribed. Implicit within this discussion is that ground motion is such that active pressures develop within the backfill.

Summation of moments about any point can be used to determine the line of action of the normal force required for equilibrium. It is assumed that $P_{AE}$ acts at the wall mid-height. At accelerations beyond the threshold the seismic bearing capacity defines a limit to the normal resistance offered by the foundation soil. The maximum normal force resisted by the foundation decreases as the soil responds to increasing levels of ground acceleration. At the same time, to satisfy moment equilibrium, the line of action of the normal force must move incrementally towards the toe. Theoretically, at this limit the soil resisting moment is maximum since the line of action cannot move beyond the toe. At this instant static equilibrium is no longer possible and angular acceleration results for increasing levels of ground acceleration.
2.5 Numerical Integration of Accelerations

The wall acceleration components, $\ddot{\theta}$ and $\ddot{x}$, need to be determined from equations 2.1 and 2.2. The center of rotation, CR, is not known but is assumed to exist at a point along the wall base. It is also assumed that the correct CR renders the highest wall rotation. The CR is found by iteration as equations 2.1 and 2.2 are solved for different trial locations of CR. Solving these two equations requires a step-by-step solution procedure, in the time domain, as follows:

1. At time ($t$), corresponding to a certain ground acceleration, calculate $P_{AE}$ according to the MO analysis.

2a. Uncouple and evaluate equation 2.1 (i.e., set $\dot{\theta} = 0$). If $\ddot{x}$ is positive, than the wall begins to translate.

2b. Uncouple and evaluate equation 2.2 (i.e., set $\ddot{x} = 0$). If $\ddot{\theta}$ is positive, than the wall begins to rotate.

3a. If $\ddot{x}$ and $\ddot{\theta}$ are both positive at the same time ($t$), equations 2.1 and 2.2 remain coupled and must be solved as such. Integrating both $\ddot{x}$ and $\ddot{\theta}$ in the time domain produces relative wall velocity. Solve the coupled equations until the relative velocity component, for either $\ddot{x}$ or $\ddot{\theta}$, is zero. Once a velocity component is zero, the equations become uncoupled. Solve the remaining equation until its relative velocity component is zero. Again integrate both $\ddot{x}$ and $\ddot{\theta}$ in the time domain to calculate the total wall permanent displacement. The bottom wall displacement is $x$ [units], while the top wall displacement is $x$ [units] + $H \tan(\theta)$ [units].

3b. If only $\ddot{x}$ or $\ddot{\theta}$ is positive, than only that mode of deformation exists. Solve only the corresponding uncoupled equation. The equations only become coupled when both of the wall responses are positive. However, until such time, the equations remain uncoupled and are integrated in the time domain following the above procedure to achieve that particular deformation displacement.

At any time ($t_1$), when either $\ddot{x}$ or $\ddot{\theta}$ are positive, the corresponding $P_{AE}$ at time ($t_1$) is constant until the wall's relative velocity is zero. Since wall motion begins at time ($t_1$), the $P_{AE}$ cannot increase since the wall backfill is yielding by driving the wall outward. In essence, the backfill cannot exceed its threshold acceleration for the Newmark's sliding block analysis. This same assumption is adopted by Richards and Elms (1979) and Siddharthan (1991). This procedure is simplified compared to Nadim and Whitman's (1984) model where they assume that the locations and magnitudes of the wall forces vary as a function of sliding and rotating wall acceleration. Although their approach may be more exact, the additional computational effort required to solve the wall displacement outweighs the increased accuracy one may achieve, since each time step requires an iterative procedure. Using the simplified model, one may achieve an upper bound or conservative wall displacement/rotation prediction with limited computational effort.
4. If neither $\ddot{x}$ or $\ddot{\theta}$ is positive, the time step increments to the next value and the
procedure repeats. If throughout the time history neither $\ddot{x}$ nor $\ddot{\theta}$ is positive, the wall
does not translate or rotate.

The above algorithm was programmed into a FORTRAN computer code. The program
applies to a single sine pulse acceleration. A simple code modification would enable one to
input any time history of acceleration. The program used to predict the case of mixed
sliding/tilting deformation (RBT mode) is included in Appendix III. The case of rotation
about a fixed point at the top of the wall (RT mode) requires a different solution
procedure.

The predictions for the RT mode follow a procedure which is similar to that for the RBT
mode. The major difference is that the CR is known and only one equation of motion is
required; as opposed to two coupled equations which describe the RBT mode. The step-
by-step outlined procedure, in the time domain, is as follows:

1. At time ($t$), corresponding to a certain ground acceleration, calculate $P_{AE}$
according to the MO analysis. Since the assumed line of action of $P_{AE}$ is at $H/2$ only
half of the $P_{AE}$ is resisted at the base of the abutment, while the remaining half is
resisted by the connection to the bridge deck. This also applies to the horizontal wall
inertia force. Since the abutment is pin-connected, the center of rotation is known a
priori. Therefore, one does not need to investigate the possibility of other centers of
rotation.

2. Evaluate equation 2.3 at time ($t$). If $\ddot{\theta}$ is positive, then the wall begins to rotate.
Once wall rotation occurs, the lateral earth pressure remains constant until the relative
angular velocity is zero. The $\ddot{\theta}$ component is integrated once in the time domain to
calculate the relative angular velocity. Integrating $\dot{\theta}$ again in the time domain
produces the total wall rotation about the pin. Calculating the total bottom abutment
displacement requires multiplying $\tan(\theta)$ by the abutment height.

3. If the $\ddot{\theta}$ component is negative, increment the time and repeat the above
procedure. If $\ddot{\theta}$ remains negative throughout the time history, then the abutment
does not rotate.
Section 3

SHAKING TABLE EXPERIMENTS

3.1 Description of Model Tests

Model retaining walls and bridge abutments were constructed in a seismic testing chamber. The test chamber was placed on a shaking table and subjected to horizontal base acceleration. Details of the design, construction, and response of the test chamber are described by Fishman et al. (1994), Divito (1994), Fishman et al. (1995), Richards et al. (1995) and Fishman and Richards (1996). A bridge deck load was provided at the top of the model abutment. The bridge deck consisted of two W8 x 10 girders. A variety of connections between the top of the abutment and the bridge deck could be implemented. The connection detail enforced the fixity conditions at the top of the wall, and the shear transfer between the deck and the abutment. Figure 3-1 is a schematic representation of the model bridge abutment.

Model bridge abutments were 46 cm high and had footing widths between 15 and 20 cm. Foundation soil beneath the footing was 46 cm deep so that development of a failure region, necessary for seismic loss of bearing capacity, was not inhibited. As indicated in Figure 3-1 measurements include input base acceleration, and acceleration and relative displacement at the top and near the base of the wall. In Figure 3-1 accelerometers are designated as A and numbered 1 through 17. Displacement transducers 1 through 8 are designated as T. The alignment of the instruments, horizontal or vertical, is also depicted.

Results from three different model wall configurations will be presented in this report. The first bridge abutment model (Model I) was designed to fail by excessive sliding. The base of the model in contact with the foundation soil was smooth, cold-rolled steel. The second and third models (Models II and III) had increased frictional resistance at the base from coarse sand paper glued to the smooth steel surface. Models II and III are heavier than Model I with dead weights secured to the wall stem. Thus, with increased frictional resistance Models II and III are less vulnerable to sliding failure than Model I but could still fail from seismic loss of bearing capacity.

Models I and II had a free connection to the bridge deck permitting relative motion between the top of the bridge deck and the abutment. Model III featured a fixed connection detail forcing the abutment to fail by rotation about the top.

Dry Ottawa Sand (ASTM C-109) was used for both foundation soil and backfill. Engineering properties of this Ottawa Sand are consistent and well established. Pluviation as described by Richards et al. (1990) was used to place the soil in the test chamber. This resulted in a nearly uniform, medium dense soil deposit. Soil parameters and interface friction angles used in analysis of the model tests were determined from laboratory direct shear and interface tests conducted independent of the model testing as described by Fishman et al (1997).
Figure 3-1. Schematic of Bridge Abutment Model
Model walls were subjected to a series of acceleration pulses. Acceleration pulses were applied in increments of 0.05g through a range of 0.05g to 0.7g. At each level of acceleration pulses were repeated three times. Subsequent to pulse testing each model bridge abutment was subjected to acceleration time functions which included cycles of loading, reverse loading and reloading. Both a ramped sine functions and scaled records of the 1940 El Centro California earthquake were applied.

3.2 Results from Experiments

3.2.1 Data Validation

Inevitably, laboratory data is associated with a certain range of error. Establishing confidence in test data is crucial before arriving at any meaningful conclusions. Herein, a straightforward and simple validation procedure using kinematics is applied to validate the observed relative abutment displacements.

Permanent displacement of the model bridge abutments may be obtained by integrating the observed relative acceleration twice. Therefore, consider the two reference frames shown in Figure 3-2. Then by definition

$$A_{\text{(Inertial Reference)}} = A_{xyz(A16)} \quad (3.1)$$

$$A_{\text{(Inertial Reference)}} = A_{xyz(ALAT)} \quad (3.2)$$

Note that the inertial reference is the earth. Consider what happens if the acceleration between the abutment wall and test box (i.e., A16 and ALAT as shown in Figure 3-1) differ:

$$A_{\text{(wall acceleration w.r.t testbox)}} = A_{(ALAT)} - A_{(A16)} \quad (3.3)$$

This establishes the abutment acceleration relative to the test box by simply subtracting the measured test box acceleration (ALAT) with the measured abutment acceleration (A16). The theory of simple relative motion enables the wall acceleration with respect to the test box, A, to be integrated twice producing the permanent wall displacement. Referring to Figure 3-2, one may visualize XYZ as the inertial reference, xyz as the test box and particle V as the abutment.

Considering two possible scenarios:

1. If the applied ground acceleration does not overcome the wall shearing resistance, the acceleration vector b is zero and there is no relative displacement between the test box and wall.
Figure 3-2. Inertial Reference Frames for Simple Relative Motion
2. If the applied ground acceleration does overcome the shearing resistance, the acceleration vector \( \mathbf{b} \) increases in magnitude, producing permanent relative displacement between the wall and test box.

The ultimate goal is to validate the test data. Referring to Figure 3-1, the T4 and T3 transducers measure the actual displacement at the top and bottom, respectively, of the gravity wall bridge abutment. Double integrating \( A_{\text{wall acceleration wrt test box}} \):

\[
x(t) = \int \int A_{\text{(wall Testbox)}} \ dt + C_1 t + C_2
\]  

(3.4)

renders the abutment displacement relative to the test box, which is compared to directly measured values from T3 or T4. The integration constants are zero since \( x(0) = 0 \) and \( x(0) = 0 \). This provides a relatively simple method to check the test-data quality.

3.2.2.1 Data Reduction

Figure 3-3 displays the measured time history of abutment displacement for Models I, II and III. Permanent abutment displacements due to single amplitude sinusoidal acceleration pulses with 5 Hz frequency are presented. The acceleration is towards the backfill, creating active backfill pressures. Unless otherwise noted, these are the typical accelerations employed throughout testing. Note that at each applied horizontal acceleration, \( k_h \), there are three separate pulses of the same magnitude. Therefore, the graphs presented throughout this report will contain three marks with each \( k_h \) to represent the three individual pulses. The only deviations are for Model I which was pulsed only once at \( k_h = 0.225g \). The following salient observations are made with respect to the three models:

- **Model I** - The abutment experiences excessive translation; the greatest magnitude among the three models. Negligible difference between the top and bottom abutment displacement is observed, i.e., very little rotation.

- **Model II** - The abutment undergoes rotation followed by coupled sliding and tilting due to a bearing capacity failure.

- **Model III** - The abutment is pin-connected at the top forcing rotation about this point. The figure displays displacement at the base of the abutment. Large displacements occur for high accelerations.
Figure 3-3. Horizontal Abutment Top and Bottom Displacements For All Three Models

S3B = Model III Bottom, S2T/B = Model II Top and Bottom,
and S1T/B = Model I Top and Bottom

Figures 3-4 through 3-6, present results from integrating measured relative acceleration. Note the strong correlation between observations and calculated displacements for Model I. Model II and III results do have some slight deviation. Since these abutments rotated, the motion, on a finite scale, is not purely rectilinear. At the point where the accelerometer records the rectilinear horizontal acceleration (A16), there is also a component due to the rotation. Although slight, this component is not accounted for, consequently, the displacement calculations are slightly inaccurate.

Figure 3-3 presents only the permanent wall displacement, while Figures 3-4 to 3-6 are the peak displacement. Models II and III required additional dead weight, supported by a rod extending from the wall stem. This extra inertia force produced sufficient driving moments about the base, to significantly increase the flexure of the stem. Idealized, this system represents a cantilever beam with a weak impulsive force. The force elastically deforms the wall stem.
Figure 3-4. Verification of Model I Relative Displacement Measurements

Figure 3-5. Verification of Model II Relative Displacement Measurements
Figure 3-6 Verification of Model III Relative Displacement Measurements
3.2.2 Translation Mode

Parameters used in the analysis of Model I are shown below in Table 3-1.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Table 3-1. Model I Parameters</strong></td>
<td></td>
</tr>
<tr>
<td>Backfill Unit Weight, ( \gamma )</td>
<td>15.71 kN/m(^3)</td>
</tr>
<tr>
<td>Backfill Friction Angle, ( \phi )</td>
<td>30°</td>
</tr>
<tr>
<td>Soil-Wall Interface Friction Angle, ( \delta_w = \delta_f )</td>
<td>20°</td>
</tr>
<tr>
<td>Deck Load on Wall</td>
<td>0.85 kN/m</td>
</tr>
<tr>
<td>Wall Weight</td>
<td>0.29 kN/m</td>
</tr>
</tbody>
</table>

Due to excessive sliding of Model I the backfill settlement was significant and as testing progressed affected the seismic resistance of the abutment. Figure 3-7 displays the variation of backfill height with levels of base acceleration.

![Graph showing backfill sand height vs applied horizontal acceleration for Model I](image)

Figure 3-7. Backfill Sand Height vs Applied Horizontal Acceleration for Model I

Given the model parameters and height of backfill threshold accelerations are predicted at different stages of testing. Figure 3-8 provides the graphical representation of the estimated threshold accelerations for Model I.
Some additional considerations must be given to the nature of the ground motions before abutment displacements may be computed. Waves generated at the base of the test box take time to propagate through the foundation soil to the base of the abutment. Due to this time lag accelerations at the base of the abutment are out of phase with those at the base of the test box. The shear wave velocity of the foundation soil was found to be 91.4 m/sec (Fishman et al. (1995)), and with a foundation soil depth of 0.457 m this renders a time lag of 0.005 seconds. Therefore, it takes approximately 0.005 seconds for a shear wave to propagate from the base of the test box to the base of the abutment.

Wave amplification is also another consideration. Figure 3-9 depicts the amplification of ground acceleration from a typical Model I pulse. The applied ground acceleration is 0.33g, while the backfill response (A7 from Figure 3-1) is amplified to 0.41g. The time lag discussed above is readily apparent in the backfill response.
Newmark's (1965) description for the behavior of dams and embankments is very similar to the procedure adopted here considering the phase difference, between the applied ground and backfill accelerations, as well as amplification effects. The following details, which refer to Figure 3-10, describe the displacement prediction procedure.

The applied ground acceleration is a generated sine wave with the peak amplitude equal to the maximum recorded from ALAT. The acceleration at the level of the abutment is determined from A7. To account for the time lag, the abutment acceleration is delayed 0.005 seconds. Once the abutment acceleration reaches the predicted threshold it remains constant until the velocity relative to the test chamber is zero. The relative velocity component is integrated to predict the permanent abutment displacement.
Comparisons between Model I observed and predicted horizontal permanent displacements are presented in Figure 3-11. Displacements are underpredicted, which seems a little dubious, since classically this represents an upper bound prediction. However, results from Model I do contain some uncertainty regarding the wall and soil parameters. Refer to Divito (1994) for further discussion. Overall the displacement predictions provide satisfactory results.
3.2.3 Rotation Mode

Parameters for Model III and corresponding threshold accelerations are presented in Table 3-2.

Table 3-2. Model III Parameters and Estimated Threshold Accelerations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Backfill Unit Weight, $\gamma$</td>
<td>16.65 kN/m$^3$</td>
</tr>
<tr>
<td>Backfill Friction Angle, $\phi$</td>
<td>36°</td>
</tr>
<tr>
<td>Soil-Wall Friction Angle, $\delta_w = \delta_f$</td>
<td>$\delta_w = 22^\circ$, $\delta_f = 30^\circ$</td>
</tr>
<tr>
<td>Deck Load on Wall</td>
<td>0.39 kN/m</td>
</tr>
<tr>
<td>Wall Weight</td>
<td>1.32 kN/m</td>
</tr>
<tr>
<td>Sliding Threshold</td>
<td>0.60g</td>
</tr>
<tr>
<td>Bearing Capacity Threshold</td>
<td>0.46g</td>
</tr>
</tbody>
</table>

Displacement predictions are presented in Figure 3-12. Note that although the applied ground acceleration ended at 0.65g, the predictions only include up to 0.60g. During the 0.65g accelerations the wall failed in an observed deep-seated shear failure. Subsequent to
the "pulse" testing, two separate sine ramps, were also applied to the abutment and predictions are presented in Figures 3-13 and 3-14.

![Graph](image)

Figure 3-12. Comparison of Predicted and Observed Displacements for Model III

![Graph](image)

Figure 3-13. Comparison of Predicted and Observed Displacements for Model III for Sine Ramp Input No. 1
Figure 3-14. Comparison of Predicted and Observed Displacements for Model III for Sine Ramp Input No. 2

3.2.4 Mixed Mode

Model II failed due to excessive rotation, indicative of a bearing capacity failure. Parameters for Model II and the corresponding estimates of threshold acceleration are presented in Table 3-3.

Table 3-3. Model II Parameters and Estimated Threshold Accelerations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Backfill Unit Weight, $\gamma$</td>
<td>16.65 kN/m$^3$</td>
</tr>
<tr>
<td>Backfill Friction Angle, $\phi$</td>
<td>36°</td>
</tr>
<tr>
<td>Soil-Wall Friction Angle, $\delta_w$ and $\delta_f$</td>
<td>$\delta_w = 24°$, $\delta_f(\text{peak}) = 34°$, $\delta_f(\text{residual}) = 31°$</td>
</tr>
<tr>
<td>Deck Load on Wall</td>
<td>0.39 kN/m</td>
</tr>
<tr>
<td>Wall Weight</td>
<td>1.27 kN/m</td>
</tr>
<tr>
<td>Sliding Threshold</td>
<td>0.38g</td>
</tr>
<tr>
<td>Bearing Capacity Threshold</td>
<td>0.22g</td>
</tr>
</tbody>
</table>
The foundation soil's normal force resistance is maximum at 0.22g, the bearing capacity threshold. From the above analysis, the normal force is simply:

\[ N = \text{Deck Load} + P_AE\sin(\delta_w) + W_w \]  

(3.5)

where \( P_AE \) corresponds to a horizontal acceleration of 0.22g. The soil resisting moment is \( N \) multiplied by the distance between the assumed center of rotation and the toe. The center of rotation moves incrementally towards the heel (assuming one starts from the toe) while the foundation soil's normal line of action remains at the toe. Therefore, the soil resisting moment increases in magnitude as the abutment's center of rotation moves towards the heel. Correspondingly, the predicted abutment rotation and displacement must vary since the soil resisting moment varies. The largest predicted abutment rotation is the abutment design rotation.

Displacements including predictions and observations are presented in Figure 3-15. The displacements are not presented as commutative, and are the responses for each pulse. The displacements shown are for the top of the abutment wall.

![Figure 3-15. Comparison of Predicted and Observed Displacements for Model II](image)

Although the displacement predictions appear rather erroneous, closer inspection reveals that at the lower and upper accelerations, the predicted displacements compare well with the actual results. Significant prediction errors occur during the 0.35 to 0.40g predictions. This corresponds to the sliding threshold prediction. An explanation to this phenomenon will be discussed later in this report.
Observed and predicted angular accelerations are compared in Figure 3-16. Note the close agreement between the predicted and actual angular acceleration. Figure 3-17 presents the predicted and observed time history of displacement during a single pulse.

Subsequent to pulse testing a sine ramp acceleration function was applied to Model II. Figure 3-18 shows the time history of acceleration applied to the base of the model. Predictions are more difficult, since both active and passive pressures develop in the backfill. During the excitation, the foundation soil may strain-soften. The normal resistance may correspondingly decrease causing the higher displacement observations (Figure 3-19). These predictions represent the complete time history of acceleration, so errors are commutative.

![Angular Acceleration vs Time](image)

Figure 3-16. Comparison Between Observed and Predicted Angular Acceleration for Model II
Figure 3-17. Predicted And Observed Time History Of Displacement for an Acceleration Pulse Applied to Model II

Figure 3-18. Time History of Acceleration for Sine Ramp Applied to the Base of Model II
Figure 3-19. Comparison of Predicted and Observed Displacements for Model II from Sine Ramp Input

3.3 Discussion of Results

The validity of measured relative displacement between abutment models and foundation soil was established. Measured accelerations were integrated twice and the results provided a useful check of the measured relative wall displacement. Laboratory and numerical errors account for the slight discrepancies. Validating the test box data by this redundancy in measurement established the quality of the test data.

Although the case of a sliding failure mode (Model I) has already received considerable attention it serves as a useful starting point for the present study. Results from testing Model I, and corresponding analysis, are a benchmark documenting the credibility of this study. Model I is also a point of reference to which behaviors observed with more complex failure modes (Models II and III) may be referred. Predicted permanent deformation for the sliding failure mode compared favorably with the observed response of Model I. Considering uncertainty, and possible errors, associated with input parameters the R&E model predicts the behavior of bridge abutments subject to a sliding mode of failure exceptionally well. This is consistent with results from previous studies reported in the literature.

Compared with observed responses of Models II and III the equations of motion described in this report predict abutment displacements reasonably well. The case of rotation about the top of the wall, RT mode, (Model III) is easier to characterize analytically than mixed mode deformation (Model II). For the RT mode the center of rotation is known apriori. Rotations are easily obtained by integrating a single dynamic
equation for moment equilibrium. Results obtained with the algorithm proposed for the RT mode rendered results in excellent agreement with the observed response of Model III.

Model II, mixed mode, displacements were overpredicted during the early part of the deformation history. At higher base accelerations, in excess of the sliding threshold, good agreement between observed and predicted deformation was realized. These results are consistent with Z, the the line of action of \( P_{AE} \), assumed in the analysis. A constant value for \( Z \) was assumed in the analysis although in reality it varies during deformation.

Previous investigators have studied how the line of action of \( P_{AE} \) varies with deformation mode and level of acceleration (Prakash and Basavanna (1969), Ishibashi and Fang (1987), Matsuzawa et al. (1994)). In the RB mode the line of action of \( P_{AE} \) varies from \( Z \leq 0.33H \) at \( k_b = 0 \) to \( Z \approx 0.4H \) at \( k_b \geq 0.45 \). During mixed mode deformation \( Z \) is higher compared to the RB mode, and \( Z \approx 0.5H \) at \( k_b \geq 0.45 \). Model II was observed to deform initially by rotation about the base (RB) followed by mixed mode, translation and tilting, (Divito (1994)). The algorithm for predicting deformation proposed herein assumes \( Z = 0.50H \) for all levels of acceleration. Under this assumption Model II overturning moments are initially overpredicted, and therefore, so is the amount of wall rotation. At higher levels of acceleration rotations are better predicted when compared to observations because \( Z = 0.50H \) is a better approximation.

Analysis of mixed mode behavior is more complicated than that for the unadulterated sliding or tilting modes. The solution involves two dynamic equations of equilibrium in which rectilinear and angular accelerations are coupled. Uncertainties exist regarding the location of the center of rotation, and the shear strength of the foundation soil. For mixed mode deformation the center of rotation is not known apriori. The center of rotation remains uncertain throughout the analysis, and is found by trial and error. The predicted rotation is sensitive to the location of the center of rotation used in the calculation.

Mixed mode deformations occur as the result of a bearing capacity failure beneath the retaining wall or bridge abutment. Bearing capacity is strongly dependent on the shear strength of the foundation soil which may not be constant. Following initial yielding of the foundation soil yield stresses may vary during cyclic loading due to strain hardening. Strain hardening continues until a peak shear resistance is mobilized. Post peak response of the foundation soil may be strain softening until at higher acceleration levels the residual strength of the soil is attained. At this point in the deformation history the response of the system becomes steady state. Uncertainty regarding the shear resistance of the foundation soil exists throughout steady state response until the ultimate or residual strength of the soil is realized.
Section 4
CONCLUSIONS

4.1 Summary of Conclusions

Whitman (1992) and Siddharthan, et al. (1990, 1991, and 1992) proposed the use of coupled equations of motion to predict seismic induced permanent deformation of retaining walls. These equations can be used to describe mixed modes of deformation including sliding and/or tilting. Equations of motion are cast in terms of relative acceleration between the retaining wall and foundation soil. Relative displacement and rotations are computed by double integration of the equations of motion with respect to time, similar to Newmark (1965) and Richards and Elms (1979). The coupled equations of motion as they appear in the literature were modified and implemented into this project task. Modifications include:

- calculation of seismic bearing capacity,

- estimation of the moment resistance of the foundation soil, and

- extension of the equations to consider bridge abutments that may be forced to rotate about a point of fixity at the top.

Equations of motion include terms that describe the seismic resistance of the facility and the moment resistance of the foundation soil. Both of these quantities require an evaluation of seismic bearing capacity of the foundation soil. A recently developed theory to describe seismic bearing capacity (Richards and Shi (1991), Richards et al. (1993), Shi(1993)) was incorporated into the analysis. An analytic procedure is described to compute the seismic resistance of retaining walls or bridge abutments that considers modes of failure including both sliding and loss of bearing capacity. A simple method to estimate the moment resistance of the foundation soil is proposed. The method utilizes the seismic bearing capacity, and an assumed line of action for the resultant of the contact pressure beneath the footing.

The problem as formulated by Siddharthan et al. (1990, 1991, and 1992) considered a center of rotation along the base of the abutment or retaining wall. However, rotation will occur about the point of fixity when an abutment is constrained by the bridge deck. The equations of Siddharthan et al. (1990, 1991, and 1992) were modified to consider a point of rotation at the top of the wall. Thus, the seismic induced rotation of an abutment fixed to a bridge deck near the top can now be evaluated.

Algorithms for predicting seismic induced permanent deformations were applied to a number of test cases that were modeled in the laboratory. Model bridge abutments were constructed within a seismic testing chamber, and seismic loading was applied to the
models via a shaking table. Compared to previous studies described in the literature models were unique in the sense that they were not constrained to a particular mode of failure. Failure was possible by sliding, tilting or a combination of both. The mode of failure could be accurately predicted and depended on model parameters and properties of the backfill and foundation soil.

Three model bridge abutments are described in this report. Modes of failure include sliding, rotation about the top, and mixed sliding/tilting. It was demonstrated that the proposed algorithms could successfully predict modes of failure including sliding or rotation about the top.

Responses during mixed mode deformation were not as successfully predicted. The proposed algorithm overpredicted displacements during the early part of the deformation history. At higher base accelerations, in excess of the sliding threshold, good agreement between observed and predicted deformations was realized. Beyond the sliding threshold acceleration evidence of a decrease in seismic resistance was apparent. This may be the result of a change in shear strength parameters of the foundation soil from peak to residual strength values.

4.2 Recommendations for Seismic Analysis

For the sliding mode of deformation, the design procedure proposed by Richards and Elms (1979) is recommended. The procedure makes use of a semiempirical equation to estimate permanent seismic induced displacement. Relevant earthquake parameters and the seismic resistance of the facility are required as input. For design, the required seismic resistance of the facility is determined for a given allowable permanent displacement. To complete the seismic design, the weight of the facility, which provides the required seismic resistance, is found by analysis.

New facilities should be proportioned to avoid seismic loss of bearing capacity, and corresponding rotation mode of deformation. When this is not possible, or for assessment of the seismic risk of existing facilities, the contribution of tilting to the overall deformation must be assessed. At this time, no semiempirical equation for estimating seismic induced rotation has been proposed. Therefore, a more rigorous seismic analysis must be undertaken.

An earthquake record must be selected for the analysis. The record should have characteristics representative of the geology and seismicity of the region where the site is located. Characteristics important to the analysis include the duration of ground motion, peak ground acceleration, frequency content, and the time interval between cycles of strong ground motion. A representative earthquake may be selected based on worldwide published data, an existing regional data base of earthquake records, or synthetically generated.
With a given earthquake record, the algorithms described in Section 2 of this report may be applied. The earthquake record provides values of $X_g(t)$. Other required input parameters are needed to describe the bridge abutment or retaining wall, and the surrounding soil. Data required for the bridge abutment or retaining wall include the wall geometry, and load transfer and fixity to the bridge deck. Required soil parameters include the shear strength and unit weight of the backfill and foundation soil, and the shear resistance at the interface between the backfill and wall, and between base of the wall (footing) and the foundation soil. The analysis is sensitive to the soil shear strength and the shear behavior at the interface with the foundation soil. Soils at the site must be well characterized and shear strength parameters determined as accurately as possible. In-situ and/or laboratory testing is recommended.

4.3 Recommendations for Future Research

A variable in the coupled equations of motion is the soil moment resistance. A simple and practical method to calculate the soil moment resistance is presented in this report. However, the methodology is not based on a rigorous theoretical understanding of the problem. Computing the soil moment resistance is an area that needs further research.

The resultant line of action of the active earth pressure during a seismic event is also an area that requires further attention. Richards and Elms (1979) suggest for practical purposes that the resultant acts at 0.50H, while Seed and Whitman (1970) state that the static component acts at 0.33H, while the dynamic increment acts at 0.60H. Siddharthan et al. (1992) considers the line of action to be at 0.52H. Further research should be done to clarify this issue.
Section 5

REFERENCES


Appendix A

DERIVATION OF EQUATIONS OF MOTION

Consider the free body diagram of Figure 2-1 and equilibrium in both the vertical and horizontal directions:

\[
\sum F_x = \frac{W}{g} (\dot{x}) + S - \frac{W}{g} (\ddot{x} (t)) \cdot P_{AE} \cos(\delta_w) + \frac{WR}{g} \dot{\theta} \sin(\eta)
\]

(A.1)

\[
\sum F_y = W - N - \frac{W}{g} (\ddot{y} (t)) + P_{AE} \sin(\delta_w) + \frac{WR}{g} \dot{\theta} \cos(\eta)
\]

(A.2)

Sliding occurs when:

\[
S = N \tan \delta_f
\]

(A.3)

Solving equations A.1 and A.2 for S and N, respectively, one obtains:

\[
S = \frac{W}{g} (\ddot{x} (t)) - \frac{W}{g} (\dot{x}) + P_{AE} \cos(\delta_w) - \frac{WR}{g} \dot{\theta} \sin(\eta)
\]

(A.4)

\[
N = W - \frac{W}{g} (\ddot{y} (t)) - P_{AE} \sin(\delta_w) + \frac{WR}{g} \dot{\theta} \cos(\eta)
\]

(A.5)

Combining equations A.4 and A.5 into equation A.3 produces equation A.6:

\[
\frac{W}{g} (\ddot{x} (t)) - \frac{W}{g} (\dot{x}) + P_{AE} \cos(\delta_w) - \frac{WR}{g} \dot{\theta} \sin(\eta) = \left[ W - \frac{W}{g} (\ddot{y} (t)) + P_{AE} \sin(\delta_w) + \frac{WR}{g} \dot{\theta} \cos(\eta) \right] \tan \delta_f
\]

(A.6)

Rearranging equation A.6 yields equation A.7:
\[
\frac{W}{g} \dot{X}_e(t) - \frac{W}{g} (\dot{x} + p_{ae} \cos(\delta_\eta)) - \left[ \frac{W}{g} - \frac{W}{g} (Y_e(t) + p_{ae} \sin(\delta_\eta)) \right] \tan \delta_\eta = \frac{WR}{g} \dot{\theta} \sin(\eta) + \frac{WR}{g} \dot{\theta} \cos(\eta) \tan \delta_\eta
\]

(A.7)

Looking at only the right side of equation A.7, or:

\[
\frac{WR}{g} \dot{\theta} \sin(\eta) + \frac{WR}{g} \dot{\theta} \cos(\eta) \tan \delta_\eta
\]

(A.8)

and rearranging equation A.8 by noting that \( \tan(\delta_\eta) = \frac{\sin(\delta_\eta)}{\cos(\delta_\eta)} \) produces:

\[
\frac{WR}{g} \dot{\theta} \sin(\eta) \left( \frac{\cos(\delta_\eta)}{\cos(\delta_\eta)} \right) + \frac{WR}{g} \dot{\theta} \cos(\eta) \left( \frac{\sin(\delta_\eta)}{\cos(\delta_\eta)} \right)
\]

(A.9)

Simplifying further yields:

\[
\frac{WR}{g} \dot{\theta} \left[ \sin(\eta) \left( \frac{\cos(\delta_\eta)}{\cos(\delta_\eta)} \right) + \cos(\eta) \left( \frac{\sin(\delta_\eta)}{\cos(\delta_\eta)} \right) \right]
\]

(A.10)

The trigonometric identity, \( \sin(\eta \pm \delta_\eta) = \sin(\eta) \cos(\delta_\eta) \pm \cos(\eta) \sin(\delta_\eta) \) reduces equation A.10 to:

\[
\frac{WR}{g} \dot{\theta} \left( \frac{\sin(\eta + \delta_\eta)}{\cos(\delta_\eta)} \right)
\]

(A.11)

Combining equation A.11 with the left side of equation A.7 produces the first equation of motion presented by Siddharthan et al (1991), equation A.12:
\[
\frac{W \ddot{x}}{g} + \frac{WR}{g} \left( \sin(\eta + \delta_t) \right) \left( \frac{\sin(\eta + \delta_t)}{\cos(\delta_t)} \right) = \frac{W}{g} \dot{X}_a(t) + P_{AE} \cos(\delta_w) - \left[ \frac{W}{g} (\dot{Y}_a(t) - P_{AE} \sin(\delta_w)) \right] \tan(\delta_t)
\]

(A.12)

Examining equation A.12, one will note that the equation is indeterminate since there are two unknowns, mainly (\ddot{x}) and (\dot{\theta})

Thus taking moments about the CR yields:

\[
\sum M_{CR} = -\left[ \frac{W}{g} \right] R \sin(\eta) - I_{eg} \ddot{\theta} - \left[ \frac{WR}{g} \right] \bar{\theta} R + \left[ \frac{W}{g} \right] (\ddot{X}_a(t) \sin(\eta) - WR \cos(\eta) + \left[ \frac{W}{g} \right] (\ddot{Y}) R \cos(\eta) + P_{AE} (\dot{h}) \cos(\delta_w) - P_{AE} \sin(\delta_w) \left[ R \cos(\eta) + a \right] - M_a
\]

(A.13)

Simplifying and rearranging equation A.13 in a convenient form:

\[
\left[ \frac{W}{g} \right] (\ddot{x}) R \sin(\eta) + \left[ I_{eg} + \left( \frac{WR^2}{g} \right) \right] \bar{\theta} = \left[ \frac{W}{g} \right] (\ddot{X}_a(t) \sin(\eta) - \left[ 1 - \frac{\ddot{Y}_a(t)}{g} \right] WR \cos(\eta) + P_{AE} (\dot{h}) \cos(\delta_w) - P_{AE} \sin(\delta_w) \left[ R \cos(\eta) + a \right] - M_a
\]

(A.14)

Equation A.14 is Siddharthan's second and final equation of motion.
Appendix B

ABUTMENTS WITH CONSTRAINT

Consider the force diagram, Figure 2-2, with the abutment pinned connected at the top. Summing moments about the pin, produces equation B.1.

$$\sum M_{\text{pin}} = I_{\text{cg}} \ddot{\theta} + \frac{W}{g} R^2 \dddot{\theta} - \frac{W}{g} \dddot{X}_s(t) R \sin(\eta) + W R \cos(\eta) - P_{A_E} h \cos(\delta) - N_b + S_H$$

(B.1)

Rearranging into a suitable form:

$$\left[ I_{\text{cg}} + \frac{W}{g} R^2 \right] \dddot{\theta} = \frac{W}{g} \dddot{X}_s(t) R \sin(\eta) - W R \cos(\eta) + P_{A_E} h \cos(\delta) + N_b - S_H$$

(B.2)
Appendix C

DISPLACEMENT AND ROTATION COMPUTER PROGRAM

III.1 Program Code

The displacement and rotation predictions presented in Section 3 were computed with the following computer program. The program's application is for model bridge abutments tested on the shake table at the State University of New York at Buffalo. It is strongly recommended that anyone who intends to use this code for any purpose consult the author first.

```
PROGRAM DR
* CREATED BY WALTER G. KUTSCHKE
* MASTER'S THESIS - FEBRUARY 1995

* THIS PROGRAM SOLVES SIMULTANEOUS EQUATIONS OF MOTION FOR
* A SINGLE SINUSOIDAL PULSE ACCELERATION APPLIED TO A BRIDGE
* ABUTMENT. THE ACCELERATIONS ARE DOUBLE INTEGRATED, PRODUCING
* TOP AND BOTTOM ABUTMENT DISPLACEMENTS. THE BACKFILL AND
* FOUNDATION SOIL ARE COHESIONLESS MATERIAL.

* APPLICABLE TO THE SERIES TWO (FEB. 1994) TEST

* VERIFY ALL RESULTS - THE AUTHOR ACCEPTS NO RESPONSIBILITY

* VARIABLE DECLARATION
IMPLICIT NONE
INTEGER ADD, COUNTER, FLAG, I, II
REAL A, BASE, BOTTOM, CR, DB, DBF, DEN, DRR, FREQ, FV, G, GAMMA
REAL GM(150), H, HZ, ICG, INTERVAL, M, MH, NU, NUM, PAE(150)
REAL PBF, PEAK, R, R1, R2, ROT(150), RRIGHT, RSIMP, STEP, T, T1, T2
REAL TEMP, THETA(150), TIME, TOP, TRANS(150), TRIGHT, TSIMP
REAL VR(150), VROT, VT(150), VTRANS, W, WB, X, Y

* VARIABLE EXPLANATION
* DUMMY VARIABLES - ALL INTEGERS, NUM, T, T1, T2, TEMP
* GROUND MOTION - FREQ, FV, GM(150), HZ, M, PAE(150), PEAK, THETA(150)
* SOIL PARAMETERS - DB, DBF, GAMMA, PBF
* WALL MOTION PARAMETERS - DEN, R1, R2, RRIGHT, STEP, T1, T2, TIME, TRIGHT
* ACCELERATION - ROT(150), TRANS(150)
* VELOCITY - RSIMP, TSIMP, VR(150), VROT, VT(150), VTRANS
* DISPLACEMENT - BOTTOM, DRR, DTT, TOP

* ESTABLISH GEOMETRIC PARAMETERS (PER UNIT INCH)
```

C-1
* WHEN APPLICABLE, PARAMETERS ARE WRT THE TOE
  ICG = 0.0392
  W = 7.2917
  WB = 9.5139
  G = 32.174
  X = 3.625
  Y = 6.4183
  H = 18
  MH = (H+0.5)/12
  A = (6-X)/12

* USER INPUTS SOIL PARAMETERS
  WRITE(*,*)
  WRITE(*,*)
  PRINT*, "WHEN APPLICABLE - ALL CALCULATIONS ARE W.R.T. THE TOE"
  WRITE(*,*)
  PRINT*, "INPUT THE SOIL BACKFILL FRICTION ANGLE (DEGREE)"
  READ*, PBF
  DBF = 2*PBF/3
  PRINT*, "INPUT THE SOIL-FOUNDATION FRICTION ANGLE (DEGREE)"
  READ*, DB
  PRINT*, "INPUT THE SOIL BACKFILL FRICTION ANGLE"
  READ*, GAMMA
  PRINT*, "INPUT THE PEAK GROUND ACCELERATION (G)"
  READ*, PEAK
  PRINT*, "INPUT THE FOUNDATION LENGTH FROM THE TOE TO THE HEEL (IN)"
  READ*, BASE
  PRINT*, "INPUT THE DESIRED INTERVAL SPACING ALONG THE FOUNDATION (IN)"
  READ*, INTERVAL
  PRINT*, "INPUT THE FOUNDATION VERTICAL FORCE AT BEARING CAPACITY"
  READ*, FV

* CENTER OF ROTATION, MOMENT, OTHER GEOMETRIC VALUES
  DO 115 CR=0,BASE,INTERVAL
  M=(FV*CR)/12
  R=(SQRT((X-CR)**2+Y**2))/12
  NU=ASIND(Y/(12*R))

* GROUND MOTION AND ACTIVE EARTHQUAKE PRESSURE
  HZ=5
  FREQ=2*HZ**3.1415926536
  TIME=0

  DO 10 I=1,150,1
    GM(I)=PEAK*SIN(FREQ*TIME)
    THEAT(I)=ATAND(GM(I))
    GM(I)=GM(I)**G
    T=(1+SQRT((SIND(PBF+DBF)**2+DBF**2)/COSD(DBF+THEAT(I))))**2
    T1=T*COSD(THEAT(I)*COSD(DBF+THEAT(I)))
    T2=(COSD(DBF+THEAT(I))**2)/T1
    PAE(I)=GAMMA*H**2*T2/3456
    TIME=TIME+0.001
  CONTINUE

  DO 15 I=1,150,1
    TRANS(I)=0

C-2
ROT(I)=0
VT(I)=0
VR(I)=0
15 CONTINUE

* DETERMINE THE TYPE OF MOTION
* I.E., TRANSLATION, ROTATION, OR COUPLED MOTION
T1=W/G
T2=W*R*SIND(DB+NU)/(G*COSD(DB))
R1=W*R*SIND(NU)/G
R2=(ICG+W*R*R/G)
DEN=T1*R2-R1*T2
II=1

DO 30 I=1,51,1
TRIGHT=W/G*GM(I)+PAE(I)*COSD(DBF)-(WB+PAE(I)*SIND(DBF))*TAND(DB)
TEMP=W/G*R*GM(I)*SIND(NU)-WB*R*COSD(NU)+PAE(I)*MH*COSD(DBF)
RRIGHT=TEMP-PAE(I)*SIND(DBF)*(R*COSD(NU)+A)-M

IF(TRIGHT.LE.0.AND.RRIGHT.LE.0)THEN
   II=II+1
   GOTO 20
ENDIF
IF(TRIGHT.LE.0)THEN
   ROT(I)=RRIGHT/R2
   TRANS(I)=0
   FLAG=1
ELSEIF(RRIGHT.LE.0)THEN
   ROT(I)=0
   TRANS(I)=TRIGHT/T1
   FLAG=2
ELSE
   ROT(I)=(T1*RRIGHT-R1*TRIGHT)/DEN
   TRANS(I)=(TRIGHT*R2-RRIGHT*T2)/DEN
   FLAG=3
ENDIF
IF(TRANS(I).LE.0.OR.RRIGHT.LE.0)THEN
   ROT(I)=0
   TRANS(I)=0
   FLAG=2
ENDIF
IF(TRANS(I).LE.0.OR.TRIGHT.LE.0)THEN
   ROT(I)=0
   TRANS(I)=0
   FLAG=1
ENDIF
ENDIF
ENDIF
20 CONTINUE
30 CONTINUE

* SOLVE THE EQUATIONS OF MOTION
* INTEGRATION - SIMPSON'S RULE

ADD = 3
COUNTER = 1
STEP = 0.001
RSIMP = 0
VROT = 0
TSIMP = 0
VTRANS = 0
VT(0) = 0
VR(0) = 0
DRR = 0
DTH = 0

IF(FLAG.EQ.1)THEN
   NUM = 52
   GOTO 35
ELSEIF(FLAG.EQ.2)THEN
   NUM = 52
   GOTO 50
ELSEIF(FLAG.EQ.3)THEN
   NUM = 52
   GOTO 65
ENDIF

WRITE(*,*)**** NO ABUTMENT MOVEMENT OCCURS ****
GOTO 150

*  SOLVER - ROTATION ONLY
35  DO 40 I = NUM, 150
    TEMP = WIG*R*GM(I)*SIND(NU)-WB*R*COSD(NU)+PAE(I)*MH*COSD(DBF)
    RRIGHT = TEMP-PAE(I)*SIND(DBF)*(R*COSD(NU)+A)/2
    ROT(I) = RRIGHT/2
40  CONTINUE

IF(FLAG.EQ.1)THEN
   NUM = II
ENDIF

41  CONTINUE
DO 45 I = NUM, 150, 2
   RSIMP = STEP*(ROT(I)+4*ROT(I+1)+ROT(I+2))/3
   VROT = VROT + RSIMP
IF(VROT.LE.0) GOTO 90
   COUNTER = COUNTER + 1
VR(ADD) = VROT
VR(I) = VT(2)/2
VR(ADD-1) = (VR(ADD-2)+VR(ADD))/2
ADD = ADD + 2
45  CONTINUE

*  TRANSLATION
DO 60 I = NUM, 150, 2
   TSIMP = STEP*(TRANS(I)+4*TRANS(I+1)+TRANS(I+2))/3
   VTRANS = VTRANS + TSIMP
IF(VTRANS.LE.0) GOTO 90
   COUNTER = COUNTER + 1
VT(ADD) = VTRANS
VT(I) = VT(2)/2
VT(ADD-1) = (VT(ADD-2)+VT(ADD))/2
ADD = ADD + 2
60  CONTINUE
* SOLVE - COUPLED TRANSLATION AND ROTATION

65 DO 70 I = NUM, 150
TRIGHT = GW*G(II)+PAE(II)*COSD(DBF)-
(WB+PAE(II)*SIND(II)*SIND(DBF)*TAND(DB))
TEMP = GW*G(II)*SIND(NU)-WB*R*COSD(NU)+PAE9(RI)*M*COSD(DBF)
RRIGHT = TEMP-PAE(II)*SIND(DBF)*(R*COSD(NU)+A)-M
ROT(I) = (T1*RRIGHT-R1*TRIGHT)/DEN
TRANS(I) = (TRIGHT*R2-RRIGHT*T2)/DEN

70 CONTINUE

DO 75 I = II, 150, 2
TSIMP = STEP*(TRANS(I)+4*TRANS(I+1)+TRANS(I+2))/3
RSIMP = STEP*(ROT(I)+4*ROT(I+1)+ROT(I+2))/3
VTRANS = VTRANS + TSIMP
VROT = VROT + RSIMP

IF(VTRANS.LE.0) THEN
   NUM = I + 1
   GOTO 35
ENDIF

IF(VROT.LE.0) THEN
   NUM = I + 1
   GOTO 50
ENDIF

COUNTER = COUNTER + 1
VT(ADD) = VTRANS
VR(ADD) = VROT
VT(1) = VT(2)/2
VR(1) = VR(2)/2
VT(ADD-1) = (VT(ADD-2) + VT(ADD))/2
VR(ADD-1) = (VR(ADD-2) + VR(ADD))/2
ADD = ADD + 2

75 CONTINUE

* INTEGRATION - DISPLACEMENT

90 DO 100 I = 1, ADD, 2
TSIMP = STEP*(VT(I)+4*VT(I+1)+VT(I+2))/3
DTT = DTT + TSIMP
RSIMP = STEP*(VR(I)+4*VR(I+1)+VR(I+2))/3
DRR = DRR + RSIMP

100 CONTINUE

TOP = 18*DRR + DTT*12
BOTTOM = 3*DRR + DTT*12
PRINT 105, CR, BOTTOM, TOP

105 FORMAT(1X,'CR =',F5.2,1X,'T3 =',F5.2,' IN','S5X','T4 =',F5.2,' IN')
PRINT 110, BOTTOM*25.4, TOP*25.4

110 FORMAT(11X,'T3 =',F5.2,' MM','S5X','T4 =',F5.2,' MM')

115 CONTINUE

150 STOP

END